The rest mass of the electron charge

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Summary

We recap our electron model, which is based on the ring current model. In this paper, we will calculate (1) the real radius of the ring current and (2) the real velocity and rest mass of the pointlike Zitterbewegung charge, which is at the core of the model. We show its rest mass is very near to zero, but not quite zero. We calculate it to be equal to about 1.7% of the electron’s rest mass. Needless to say, all formulas are based on – and explained by – the fine-structure constant.

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The rest mass of the electron charge

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Introduction
Any realist interpretation of quantum mechanics should be able to explain what a photon, an electron and a proton actually are. Here we will recap our electron model, which is based on the ring current model. For the photon and proton model, we refer to previous papers.¹

In this paper, we will calculate the rest mass of the pointlike Zitterbewegung charge.

The four quantum numbers for electron orbitals
Any realist electron model must explain the properties of an electron as used in mainstream quantum physics, including its mass, magnetic moment and the anomaly. In a realist interpretation of quantum mechanics, these properties are not to be considered as mysterious intrinsic properties of a pointlike electron: the model should generate them.

We must also be able to relate the model to the four quantum numbers that define electron orbitals. We may usefully remind the reader of the basics as discussed in Feynman’s derivation of the structure of the elementary atom (¹H) based on Schrödinger’s equation²:

1. As usual in quantum mechanics, we have discrete energy states or energy levels, and the principal quantum number (n) refers to the energy of the nth energy level. It is used in the formula for the allowed energy levels, which is equal to $E_n = -E_R/n^2$ (E_R is the Rydberg energy).³ It is often conveniently referred to as a shell⁴. The principal quantum number is always a simple natural number: $n = 1, 2, 3, etc.$

We will have no need for this number because we will be discussing a free electron only – which has one energy state only.⁵

2. The orbital angular momentum (l) is expressed in units of ħ. It may also be zero. In fact, $l = 0$ is associated with spherically symmetric solutions: these states have no angular dependence.⁶ They are referred to as an s-state $– s$ for spherical $–$ which is, perhaps, somewhat confusing because the same

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² See: Feynman’s Lectures on Physics, Vol. III, Chapter 19. The reader should note we will also use Feynman’s notation (n, l, and mz). The use of s for the spin quantum number is somewhat confusing because s is also used to refer to the first energy level.
³ Note that the energy is negative and lowest for $n = 1$. The energy concept used is the potential energy, and we assume the electron has zero (potential) energy when it is not in an electron orbital.
⁵ Its energy (potential and kinetic) depends on the reference frame, obviously, but we are not concerned with other reference frames now.
⁶ Feynman solves Schrödinger’s equation using polar (spherical) coordinates. Hence, the coordinates are expressed in terms of the distance from the proton (r), a polar angle (θ) and an azimuthal angle (φ). The spherical symmetric solutions only depend on the distance from the proton (r).
symbol is used for the much more general concept of spin. We will, effectively, also use it to designate the spin of the \textit{Zitterbewegung} charge in our electron model.

The non-spherical solutions for Schrödinger’s equation are associated with proper multiples of \( \hbar \). If \( l = 1 \), for example, then we have a number of \( p \)-states, which are defined by the magnetic quantum number \((m_z)\) as a function of \( l \) (see the next section). For \( l = 2 \), we have \( d \)-states. When \( l = 3, 4, 5, \ldots \) we get \( f, g, h, \ldots \) states.\(^7\)

The orbital angular momentum of an electron in an electron orbital should be distinguished from the orbital angular momentum as discussed in the context of an electron model (ring current, \textit{Zitterbewegung}, or Kerr-Newman). We, therefore, find the oft-used term \textit{subshell} for this number very convenient. The subshell number \( l \) will always be less than the number of energy states. To be precise, we can write: \( l = 0, 1, 2, \ldots n - 1 \).

If we have one energy state only, then we have only state: \( l = 0 \). Hence, this number may also not seem to be relevant in the context of a model for a free electron. However, we should immediately add that the concept of orbital angular momentum in the context of an electron model is very relevant. We will argue that the angular momentum of a free electron is also expressed in units of \( \hbar \).\(^8\)

The quantum-mechanical law that angular momentum must come in units of \( \hbar \) is a direct implication of – or equivalent to – the Planck-Einstein law: \( E = hf = \hbar \omega \). However, we will bring some nuance to this statement in a moment: we believe the Planck-Einstein relation holds, but the anomalous magnetic moment tells us that the angular momentum of the electron might be slightly off.

At this point, we request the reader to start thinking of a free electron as having some \textit{structure}. More specifically, we are requesting the reader to think of the electron as a ring current: a structure which consists of a pointlike charge orbiting around some center at (nearly) the speed of light. These electron models may also be referred to as \textit{Zitterbewegung} models.\(^9\)

\section*{3.}

The magnetic quantum number \((m_z)\) corresponds to the orientation of the shape of the subshell. It is defined by the following formula:

\[-l \leq m_z \leq l\]

The \( l = 0 \) state is a state which is defined in the context of atomic structure. Hence, it is not relevant to our discussion. However, we do think the electron – as a ring current – has orbital angular momentum. We will denote this as \( L \) and its theoretical value (making abstracting of the anomaly\(^{10}\)) will be equal to \( L \)

\begin{footnotesize}
\begin{enumerate}[7. Feynman’s dictionary of quantum numbers (III-19-3, Table 19-1) is very useful.
8. The reader will wonder: full or half units of \( \hbar \)? The electron is a spin-1/2 particle, right? We will try to answer this question in a moment.
9. It is always worth quoting Dirac’s summary of this: “The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment.” (Paul A.M. Dirac, Theory of Electrons and Positrons, Nobel Lecture, December 12, 1933)
10. We will explain the anomalous magnetic moment in a moment. We do not think of it as an anomaly. It is part of the model.
\end{enumerate}
\end{footnotesize}
\( \pm \hbar \). The plus (+) and minus (−) signs have a physical meaning: they refer to the direction of the current. As such, they are directly related to the magnetic moment of the (free) electron. We may remind the reader to a formula he should be very familiar with:

\[
\mu = -g \left( \frac{q_e}{2m} \right) L \iff \frac{q_e}{2m} \hbar = g \frac{q_e}{2m} \hbar \iff g = 2
\]

We like the vector notation for \( \mu \) and \( L \) (boldface) because they draw our attention to the physical interpretation of this equation: the minus sign (−) is there because, in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions. In contrast, we do not like the concept of a \( g \)-factor. When thinking about the anomalous magnetic moment, for example, we will want to analyze this in terms of the magnetic moment itself rather than in terms of \( g \)-factors. We will come back to this. As for now, the reader should just briefly contemplate the ring current model, as illustrated below. He will understand the current may have one of two directions: clockwise or counterclockwise, or up or down, or plus or minus—whatever name the reader will want to give this binary distinction.

As we are here, the reader should note the idea of a circulating charge implies the idea of a centripetal force and momentum. We effectively believe that the pointlike charge inside the electron acquires some relativistic mass as its velocity is equal to or – as we will argue later – very near to the speed of light. We refer to this mass as the effective mass of the pointlike charge. A geometric analysis shows the effective mass will be equal to 1/2 of the total mass of the electron. Hence, we write:

\[
m_r = m_e/2
\]

This concept of effective mass pops up very naturally in the quantum-mechanical analysis of the linear motion of electrons. In his famous Lectures on quantum mechanics\(^\text{11}\), Richard Feynman limits his comments on it to a rather confusing connection to the non-relativistic kinetic energy formula \((mv^2/2)\) and – if I am not mistaken – this connection to a non-relativistic formula is one of the reasons why Schrödinger’s equation is not supposed to be relativistically correct. I am not so sure about that, but that is another topic for discussion. Right now, we just want to note that one of the reasons why we feel our model makes sense is that it also explains or incorporates the effective mass concept.

The 1/2 ratio between the effective mass of the \( zbw \) charge and the total mass of the electron makes intuitively sense because of the energy equipartition theorem: we may think of the total energy of the electron to be distributed – in equal parts – over (i) the energy of the oscillation, and (ii) the relativistic

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\(^\text{11}\) See: [https://www.feynmanlectures.caltech.edu/III_16.html](https://www.feynmanlectures.caltech.edu/III_16.html).
mass of the pointlike charge. We will come back to this and we will, therefore, let the matter rest for the

time being.\(^{12}\)

4. The fourth and last quantum number is usually referred to as the spin. It explains why we can have
two electrons in any configuration—say, the \(2p^6\) configuration for the neon atom. It also explains the
finer structure of the hydrogen spectral lines.

The term ‘spin’ is a very simple but, at the same time, also a very confusing term because so many things
are spinning here. Indeed, besides the electron that is spinning inside an atom, and the pointlike
Zitterbewegung charge that is spinning inside the electron, we will now also want to think of the
Zitterbewegung charge spinning around its own axis.

In how many directions can it spin around its own axis? The honest answer is: we do not know, but the
quantum-mechanical analysis strongly suggests that, here also, the spin will be either up or down. In
light of the geometry of the situation, we will, of course, want to define the up or down here in terms of
the orientation of the plane of the ring current.

The electron as a fractal structure?

We are now ready to tune our electron model which – as we mentioned above – is based on a simple
ring current model. However, before we do so, the reader may want to think briefly about the following
question: could we continue this line of reasoning? Can we imagine the zbwb charge is – perhaps – some
kind of ring current too? So we would have ring currents within ring currents? The answer is positive: we
could, effectively, imagine a fractal structure of the electron.\(^{13}\)

In fact, the spectral lines within or near the main spectral lines of the hydrogen atom strongly suggest
this is the case. We also think it explains why Schwinger’s \(\alpha/2\pi\) factor explains most of the anomaly of
the magnetic moment but not all of it: about 0.15% is explained by second-, third- etc. order factors,
which – as we know – have been calculated to amazing precision using quantum field theory. We think
our electron model offers an alternative classical explanation. Because we allow for the possibility of
ring current within ring currents, we call our model a fractal ring current model or – what amounts to
the same – a fractal Zitterbewegung model.

The ring current model and the Compton radius

In our previous papers, we assumed we derived the Compton radius of an electron \(\textit{directly}\) from (i)
accepting Einstein’s mass-energy equivalence relation \((E = m \cdot c^2)\) for what it is, (ii) interpreting \(c\) as the
tangential velocity of the zbwb charge\(^{14}\) \((c = a \cdot \omega)\) and, then (iii) just uses the Planck-Einstein relation \((E = \hbar \cdot \omega)\) to find the Compton radius:

\[
\frac{a}{\omega} = \frac{c}{mc^2} = \frac{\hbar}{mc} = \frac{\lambda_c}{2\pi} \approx 0.386159268 \ldots \text{ pm}
\]

\(^{12}\) As we are explaining matter as the motion of a charge, the expression is rather weird, but so be it.
\(^{13}\) See, for example, Oliver Consa’s 2018 \textit{Helical Solenoid Model of the Electron} paper (https://vixra.org/abs/1809.0567), in
which he expresses the following deep thought: “The universe generally behaves in a fractal way, so the most natural solution
assumes that the electron’s substructure is similar to the main structure.”

\(^{14}\) We also referred to the zbwb charge as a naked charge: it has no properties except its charge. It has, therefore, zero rest mass
and that is why it moves around at lightspeed: the slightest force on it will cause an infinite acceleration.
Note that the Compton radius is inversely proportional to the mass. The Compton radius of a proton, for example, is much smaller than the Compton radius of an electron. Can the idea be applied to a proton? Of course. One can do it in a very classical way by discussing it in the context of photon scattering, which can also be done with protons. Let me quote from a very standard textbook here\textsuperscript{15}:

“The only difference is that the proton is heavier. We simply replace the electron mass in the Compton wavelength shift equation with the proton mass. [...] The maximum shift is \( \Delta \lambda_{\text{max}} = 2\hbar/m_p c \approx 2.64 \text{ fm} \). Fantastically small. This is roughly the size attributed to a small atomic nucleus, since the Compton wavelength sets the scale above which the nucleus can be localized in a particle-like sense.”\textsuperscript{16}

This also shows even mainstream physicists do think of the Compton wavelength as effectively defining some space. To be precise, we think its reduced form – the Compton radius \( a = \lambda_C/2\pi \) – effectively defines the space in which the Zitterbewegung charge is actually moving.

The hypothesis is further reinforced by noting we can also calculate a Compton radius for a muon-electron using the same relation. The CODATA value for the Compton wavelength is equal to:

\[
\lambda = 1.17344110(26) \times 10^{-14} \text{ m}
\]

When dividing this by 2\( \pi \), one gets a radius. The reader can now calculate the \( a = \hbar/m_c \) value for the muon-electron and will find this value is well within the confidence interval of the above-mentioned number. This adds credibility to our electron model. Any discrepancy is likely the result of the muon-electron being unstable: as an unstable particle, we think it does not quite respect the Planck-Einstein relation. Its cycle is, therefore, not stable: it decays in a (stable) electron and two (stable) neutrinos.

**Spin and the energy equipartition theorem**

Our proton model\textsuperscript{17} inspired us to think that, perhaps, we should add a 1/2 factor in our model. In addition, most of the other Zitterbewegung theorists have such 1/2 factor in their model, and think the Zitterbewegung radius of an electron is only half the \( 2\hbar/m_c \) Compton radius. What do we get if we would, effectively, assume that half of the energy of the electron is in the ring current, and the other half of the energy is in the pointlike charge? It is just another application of the energy equipartition theorem, and it would be very consistent with our calculations of the effective mass of the pointlike charge, which we wrote as \( m_\gamma = m_e/2 \). The new calculation gives this:

\[
a = \frac{c}{\omega} = \frac{c\hbar}{E/2} = \frac{2c\hbar}{m_c^2} = \frac{2\lambda_C}{2\pi} = \frac{\lambda_C}{\pi} \approx 0.7723 \ldots \text{ pm}
\]

We do not get half of the Zitterbewegung radius from, say, Hestenes’ model – but twice its value. This value is equal to the maximum shift of the wavelength in Compton scattering experiments. Hence, the new value may make some sense – even if it would set us apart from other Zitterbewegung or ring current models.

\textsuperscript{15} See: Prof. Dr. Patrick LeClair, Physics 253 course (http://pleclair.ua.edu/PH253/Notes/compton.pdf), p. 10
\textsuperscript{16} Italics in the quote were added by the author of this paper.
\textsuperscript{17} See our solution to the proton radius (https://vixra.org/abs/2001.0685).
However, let us not worry about $1/2$ factors right now. I would like to paraphrase Feynman here\textsuperscript{18}: when doing this kind of ‘dimensional analysis’, we need not trust our answer to within factors like $2$, $\pi$, etc.

**The electron radius as calculated from its magnetic moment**

This pointlike $zbw$ charge is supposed to whizz around at the speed of light but that assumption is – most likely – a mathematical idealization. This is why the anomalous magnetic moment is *not* an anomaly: the assumption that the elementary charge has no dimension or structure whatsoever is bound to result in an ‘anomaly’ between our measurements and these ‘good theories’ we have about the structure of electrons, photons and protons.\textsuperscript{19}

Let us do some calculations. Because $\hbar$ and $c$ have *precisely defined* values since the 2019 revision of SI units, we can now calculate the Compton wavelength from the mass—not approximately, but *exactly*.\textsuperscript{20} Newton’s law defines a force in terms of the *inertia* to a change in its state of motion. Hence, the CODATA value for the electron mass is something to hold onto—literally:

$$m_{\text{CODATA}} = 9.1093837015(28) \times 10^{-31} \text{ kg}$$

That is a *measured* value. There were a zillion experiments. This is a valid measure.

We have a CODATA value for the radius too. Again: zillion experiments, no problem. Well… No… I should be precise, we do not: we have a CODATA value for the Compton wavelength, based on which we calculated the radius, including a $1/2$ factor or not. If we include it, we get:

$$a = \frac{c}{\omega} = \frac{c \hbar}{E/2} = \frac{2 \hbar}{mc^2} = \frac{2 \lambda_c}{2 \pi} = \frac{\lambda_c}{\pi} \approx 0.7723 \ldots \text{ fm}$$

The mass is, of course, the mass: $m = m_{\text{CODATA}}$. No weird anomaly stories about the mass. Let us now see if our ring current model is consistent. We should, effectively, be able to calculate the radius from the magnetic moment, for which we also have a *measured* CODATA value\textsuperscript{21}:

$$\mu_{\text{CODATA}} = 9.2847647043(28) \times 10^{-24} \text{ J} \cdot \text{T}^{-1}$$

How can we use this value to calculate a radius? The logic is self-evident. The magnetic moment is the product of the current and the area of the loop, and the current is the product of the elementary charge and the frequency. The frequency is, of course, the velocity of the charge divided by the circumference of the loop. Because we assume the velocity of our charge is equal to $c$, we get the following radius value:

\[\text{---}\]

\textsuperscript{18} See: https://www.feynmanlectures.caltech.edu/III_02.html#Ch2-S4.

\textsuperscript{19} Mathematical idealizations are just what they are: we need the math and the mathematical ideas that come with it (including the ideas of nothingness and infinity) to describe reality – math was Wittgenstein’s ladder to understanding – but Planck’s quantum of action, and the finite speed of light, effectively tell us our mathematical ideas are what they are: idealized notions we use to describe a reality which is, in the end, quite finite. Something that has no dimension whatsoever probably exists in our mind only.

\textsuperscript{20} Note that the radius is inversely proportional to the mass. The Compton radius of a proton, for example, is much smaller than the Compton radius of an electron.

\textsuperscript{21} We should put a minus sign as per the convention but, because we are interested in magnitudes here, we will omit it. It will, hopefully, confuse the reader less, rather than more.
\[
\mu = l\pi a^2 = q_e f\pi a^2 = q_e \frac{c}{2\pi a} \pi a^2 = \frac{q_e c}{2} - a \Leftrightarrow a = \frac{2\mu}{q_e c} \approx 0.3866607 \ldots \text{pm}
\]

The first thing that we should note is that the radius we get is not twice the Compton radius. Should we, therefore, stick to our original \( a = \frac{\hbar}{mc} \) radius? We think so, but let us explore the question by reflecting some more and – most importantly – by doing some more calculations.

**The Pauli matrix of the electron as a two-state system**

This is a rather grand title for a rather simple reflection. The point is this: if we assume the *zbw* charge has spin of its own – which it probably should have in light of the above-mentioned quantum number logic – then we can think of the magnetic moment of an electron consisting of the addition of the magnetic moment generated by the spinning *zbw* charge and the magnetic moment generated by the ring current.\(^{22}\) Hence, if we measure these magnetic moments in 1/2 values of the total magnetic moment, we effectively get a sort of Pauli spin matrix for a two-state system here:

<table>
<thead>
<tr>
<th><em>zbw</em> spin vs. ring current</th>
<th>clockwise (+1/2)(^{23})</th>
<th>counterclockwise (−1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>up (+1/2)</td>
<td>½ + ½ = 1</td>
<td>½ − ½ = 0</td>
</tr>
<tr>
<td>down (−1/2)</td>
<td>−½ + ½ = 0</td>
<td>−½ − ½ = −1</td>
</tr>
</tbody>
</table>

Now, of course we know – because of real-life measurements that we cannot argue with – the zero values do not exist: our electron has a magnetic moment and it is not equal to zero: it is given – experimentally – by the CODATA value we mentioned. The zero-spin state does not exist.\(^{24}\) Hence, we must conclude only two of the above-mentioned theoretical states are relevant: clockwise/spin-up and counterclockwise/spin-down. It, therefore, does not help us to make a choice between the \( a = \frac{\hbar}{mc} \) and \( a = \frac{2\hbar}{mc} \) calculations for the theoretical radius of an electron.

However, the reflection above does tell us that, if the magnetic moment is real, then half of it is going to be generated by the orbital angular momentum, while the other half will be generated by the spin of the *zbw* charge. We write:

\[ \mu = \mu_L + \mu_s = \mu/2 + \mu/2 \]

It is an extremely simple formula. However, we want to highlight it because it explains why an electron is being referred to as a spin-1/2 particle.

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\(^{22}\) Wikipedia offers a confusing but – as far as we can see – also quite consistent explanation for the addition of spin and orbital angular momenta. See: [https://en.wikipedia.org/wiki/Vector_model_of_the_atom](https://en.wikipedia.org/wiki/Vector_model_of_the_atom).

\(^{23}\) What is clockwise or counterclockwise depends on your reference frame, but that is the same for defining up or down. If we look from the opposite direction, both up and down as well as clockwise as well counterclockwise will swap their definition. Hence, the reference frame doesn’t matter here.

\(^{24}\) We are tempted to rant about the inconsistency of the photon being defined as spin-1 particle here which – despite the theory saying spin-1 particles should have a zero-spin state – does also not have a zero-spin state. We think the theory of bosons and fermions adds no value whatsoever. Worse, it complicates the analysis as much (or even more) as the theory of g-factors.
The anomaly

The 0.38666... value we got out of our radius calculation using the CODATA value for the magnetic moment is slightly larger than the value we get based on the mass or the Compton wavelength, which is equal to something like 0.386159 fm. So, yes, we do have an anomaly. We can use a lot of subscripts here, but they are all the same: subscripts don’t matter. The bottom line is this: we will want to think of the radius based on the mass or the Compton wavelength as some kind of theoretical radius and so we will put it in the denominator. You can write it like you want, with or without some subscript: \( a = a_{\text{CODATA}} = a_m = a_l = a_C \). In contrast, we will write the radius based on our calculation using the magnetic moment as \( a_\mu \). We can then write the anomaly as:

\[
\frac{a_\mu - a}{a} \approx 0.00115965 \iff \frac{a_\mu}{a} = 1.00115965 \ldots
\]

As you can see, it doesn’t matter if we multiply the radius by 2 or by whatever factor. So you should not worry about it—not for now, at least. As for now, you should just verify the relations you are familiar with: the anomaly is, effectively, equal to about 99.85% of Schwinger’s factor:

\[
\frac{\alpha}{2\pi} = 0.00116141 \ldots
\]

Let us, for good order, also recalculate the anomaly of the magnetic moment. We will follow a slightly different presentation than the usual one but you will see the logic is not very different. We first calculate a new theoretical value for the magnetic moment using the Compton radius, which we will denote as \( \mu_a \). When writing it all out, we get this:

\[
\mu_a = I_\pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a} \pi a^2 = \frac{q_e c}{2} a = \frac{q_e c}{2m} \hbar \approx 9.27401 \ldots \times 10^{-24} \text{ J} \cdot \text{T}^{-1}
\]

We can now calculate the anomaly – against the CODATA value – once more:

\[
\frac{\mu_a - \mu}{\mu} = 0.00115965 \ldots
\]

We get the same anomaly—not approximately but exactly. That is what we would expect: in the zbw or ring current model, the anomaly is not only related to the actual magnetic moment but to the actual radius as well. This should not surprise us: the magnetic moment is, of course, proportional to the radius of the loop. Hence, if the actual magnetic moment differs from the theoretical one, then the actual radius must also differ from the theoretical one.

What do we get if we use the g-factors themselves? The intellectually honest reply to this is that it depends on your assumption in regard to the angular momentum of the electron, which we cannot directly measure. Let me write down the formula for the gyromagnetic ratio:

---

25 You should watch out with the minus signs here – and you may want to think why you put what in the denominator – but it all works out!

26 We have a squared radius in the numerator of the formula for the magnetic moment, and a non-squared radius factor in the denominator.
\[ g = \frac{\mu}{L} = \frac{1}{m_e} \cdot \pi a^2 = \frac{\frac{q_e}{2} \cdot \frac{a^2}{2a}}{m} = \frac{q_e}{m} \]

You will say this doesn’t look like a \( g \)-factor: we’re missing a \( \frac{1}{2} \) factor, and – can you remind me? – what is that weird \( m_\gamma \) concept? And you are right. It is not quite clear. It is the relativistic mass of our pointlike Zitterbewegung charge. It is also an angular mass. Purists will also say relativistic and angular mass may not be the same, and they are right: definitions and concepts do become quite fuzzy here.

As far as we are concerned, we think our geometric argument\(^\text{27}\) shows that the effective mass of the point charge is \textit{half} of the rest mass of the electron. However, we also acknowledge the proof depends on (i) the assumption that the velocity of the pointlike charge is, effectively, equal to \( c \) and (ii) some pre-conceived notion that the electron is a spin-1/2 particle. If one does not want to use these assumptions, one has to simply assume that, somehow, the mass of the electron is spread over a disk, so we can use the formula for the angular mass of a disk rather than a hoop.\(^\text{28}\) It is the same \( \frac{1}{2} \) factor that we see in the conventional definition of the \( g \)-factor, which writes \( g \) as a multiple of \( \frac{q_e}{2m} \) based on the assumption that the angular momentum is equal to \( \hbar/2 \), which may or may not be the case\(^\text{29}\):

\[ \mu = -g \left( \frac{q_e}{2m} \right) L \iff \frac{q_e}{2m} \hbar = g \left( \frac{q_e}{2m} \right) \hbar / 2 \iff g = 2 \]

However, as mentioned above, we really think the concept of a \( g \)-factor obscures the matter, and so we will just stick to ratios of magnetic moments or radii. That also deals with the question of the unknown or undefined angular momentum which – the reader should note – should cancel out anyway when taking the \textit{ratio} of two \( g \)-factors: the angular momentum is the angular momentum, right?

The anomaly (2)

Let us do some more calculations in an attempt to clarify the discussion—even if we think it will confuse the reader even more. Our assumption is that the anomaly is, somehow, the result of our mathematical idealizations. We cannot really assume the pointlike \textit{zbw} charge is whizzing around at the speed of light. It can be very near \( c \), but not equal to \( c \).\(^\text{30}\) Hence, its theoretical rest mass will also be very close to zero, but not \textit{exactly} zero. Of course, because everything is related to everything in this model, the anomalies also suggest we have some real radius \( r \) that is probably \textit{not quite} equal to the Compton radius \( a = \hbar/mc \).

Let us write it all out. What should we put where? It is not easy to figure out, but the greater value – based on the greater radius – should be in the denominator, so we write:

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\(^{27}\) See our paper on the electron as a harmonic electromagnetic oscillator (https://vixra.org/abs/1905.0521).

\(^{28}\) Our particular flavor of \textit{Zitterbewegung} model assumes the circular motion is equivalent to the motion of two linear oscillators working in tandem. We readily admit this \( \frac{1}{2} \) factor looks like the Achilles heel of the model. However, we think we were able to convincingly demonstrate why the assumption makes sense in our previous papers.

\(^{29}\) It should be clear by now that we think the Planck-Einstein law tells us angular momentum comes in units of \( \hbar \), not in units of \( \hbar/2 \). However, this is a statement which applies to the level of electron-photon interaction and similar events. \textit{Inside of the electron}, something else might be going on.

\(^{30}\) In fact, we do not even want to try to preempt the discussion here: the effective mass of the pointlike charge is not photon-like at all. We’re not talking about a few eV here. Half the mass of the electron is half the mass of the electron.
\[
\frac{\mu_r}{\mu_a} = \frac{\frac{q_e}{2m} \frac{\hbar}{v \cdot r}}{\frac{q_e v}{r}} = \frac{\hbar}{m \cdot v \cdot r} = \frac{c \cdot a}{v \cdot r}
\]

Note that, from the \( v = r \cdot \omega \) and \( c = a \cdot \omega \) relations\(^{31}\), we can also get the following result:

\[
\frac{\mu_r}{\mu_a} = \frac{c \cdot a}{v \cdot r} = \frac{\omega \cdot a^2}{\omega \cdot r^2} = \frac{a^2}{r^2} \iff \frac{c}{v} = \frac{a}{r}
\]

This helps us to with interpretation of our results: because \( v \) must be smaller than \( c \), the identity shows the real radius \( r \) must also be slightly smaller than \( a = \hbar/mc \). If there would be no anomaly – in other words: if our mathematical idealization would match reality – then the formulas just becomes unity (everything is equal to \( 1 \)). However, we know the anomaly exists, and it is very nearly equal to \( 1 + \alpha/2 \pi \). For all practical purposes – we think a 99.85% explanation is pretty good – we will just equate it and re-write the expression above as\(^{32}\):

\[
1 + \frac{\alpha}{2\pi} = \frac{2\pi + \alpha}{2\pi} = \frac{c \cdot a}{v \cdot r} \iff v \cdot r = \frac{2\pi \cdot c \cdot a}{2\pi + \alpha} = \frac{2\pi \cdot c \cdot \frac{\hbar}{mc}}{2\pi + \alpha} = \frac{\hbar}{m(2\pi + \alpha)}
\]

\[
\iff L = m \cdot v \cdot r = \frac{\hbar}{2\pi + \alpha}
\]

So now we need to answer the question: what is the real velocity \( v \) and what is the real radius \( r \) of our zbhw charge?

**The real electron radius and velocity and rest mass of its charge**

We said the quantum-mechanical law that angular momentum must come in units of \( \hbar \) is a direct implication of – or equivalent to – the Planck-Einstein law: \( E = hf = \hbar \cdot \omega \). We also said we would bring some nuance to this statement. The calculations do just that: angular momentum does not come in exact units of \( \hbar \).

However, we believe the Planck-Einstein relation to be true. Hence, we believe that the frequency \( f \) or \( \omega \) of the Zitterbewegung oscillation is, effectively equal to \( f = E/h \) or \( \omega = E/\hbar \), with or without incorporating the anomaly. If we believe that to be true, then the following relations are easily established\(^{33}\):

\(^{31}\) The reader may wonder why we take \( \omega \) to be a constant. The answer is: we take the energy (or mass) of an electron as a given, and we take the Planck-Einstein relation (\( \omega = E/\hbar \)) as a given too! The geometry of the situation gives us everything here!

\(^{32}\) The reader should note that we did use the \( a = \hbar/mc \) relation above—as opposed to the \( a = 2\hbar/mc \) relation. It makes a very significant difference. When using the \( a = 2\hbar/mc \) relation, we get this:

\[
1 + \frac{\alpha}{2\pi} = \frac{2\pi + \alpha}{2\pi} = \frac{c \cdot a}{v \cdot r} \iff v \cdot r = \frac{2\pi \cdot c \cdot a}{2\pi + \alpha} = \frac{2\pi \cdot c \cdot \frac{2h}{mc}}{2\pi + \alpha} = \frac{2h}{m(2\pi + \alpha)} \iff L = m \cdot v \cdot r = \frac{h}{\pi + \alpha/2}
\]

The difference between \( \pi + \alpha/2 \) and \( 2\pi + \alpha \) is, unsurprisingly, equal to a factor \( 2 \). Practically speaking, we have two very different form factors for the angular momentum of an electron here. We discarded the \( a = 2\hbar/mc \) hypothesis because we think the Planck-Einstein relation tells us the angular momentum comes in units of \( \hbar \) (or very nearly so), rather than in twice that amount (\( \hbar/\pi = 2\hbar \)).

\(^{33}\) We are just using the tangential velocity formula here to do the substitution that is being done: \( c = a \cdot \omega \) and \( v = r \cdot \omega \) and – yes – we assume stable particles respect the Planck-Einstein relation. We think there have been enough experiments confirming the Planck-Einstein relation so we do not want to doubt it – regardless of our crazy theories in regard to the measured magnetic moment of an electron.
\[
\frac{\mu_r}{\mu_a} = 1 + \frac{\alpha}{2\pi} = \frac{c \cdot a}{v \cdot r} = \frac{\omega \cdot a^2}{r^2} = \frac{a^2}{r^2} \iff
r = \frac{a}{\sqrt{1 + \frac{\alpha}{2\pi}}} \approx 0.99942 \cdot \frac{\hbar}{mc}
\]

We can plug this into the \( \beta = \frac{v}{c} = \frac{r}{a} \) relation to get the relative velocity:

\[
\beta = \frac{v}{c} = \frac{r}{a} = \frac{a}{\sqrt{1 + \frac{\alpha}{2\pi}}} \approx 0.99942
\]

We can now calculate the real rest mass of the pointlike \( zbw \) charge:

\[
m_0 = \sqrt{1 - \beta^2} \cdot m_e = \sqrt{1 - \beta^2} \cdot \frac{m_e}{2} = \sqrt{1 - \frac{1 - \alpha}{1 + \frac{\alpha}{2\pi}}} \cdot m_e \approx 0.017 \cdot m_e
\]

Hence, we arrive at the conclusion that the rest mass of the pointlike \textit{Zitterbewegung} charge is equal to a bit less than 2% of the rest mass of the electron. Is this a credible result? We think so, but we will let the reader re-do the calculations.

Jean Louis Van Belle, 15 February 2020