## Title

Waves of Charge.
Four Dimensional Analytic Functions.


#### Abstract

Basically, the paper concerns the generalization of the Cauchy-Riemann equations to spacetime, so (Hestenes, quote) "we can expect it to have a rich variety of solutions. The problem is to pick out those solutions with physical significance." I could expect Hopfions, EVOs and so on. I write only the conclusions, skipping all the speculations and physical meanings that led me to this.


## 1 - FOREWORD

As I said, I write only the final results of the argument, skipping all the speculations that led me right here. I only say, as far as my notations are concerned, that a brief but exhaustive explanation can be found in a few, few pages, of [1]. Other interesting readings are in Bibliography. Of other avant-garde and more or less contested topics, EVO by Shoulders, Condensed Plasmoids, Ball Lightning etc I have found no trace in the official scientific literature, therefore I have not put them in the Bibliography. However, they can be found on the Internet.

## 2 - ANALYTIC FUNCTIONS

I'll go straight to the topic.
$1, i, T j, T j i$ they commute between them, and more
(1)

$$
(1+T j)(1+T j)=2(1+T j)
$$

(2)

$$
(1-T j)(1+T j)=0
$$

That said, sums products powers of (3)

$$
a=(x+T j i y)(1+T j)
$$

and
(4)

$$
b=(z-\tau)(1+T j)
$$

they are analytic functions.
Indeed, the analytic operator $\boldsymbol{\partial}^{*}$ reduces down to

$$
\begin{equation*}
\partial^{*}[(z-\tau)(1+T \boldsymbol{j})]=\left(\boldsymbol{j} \frac{\partial}{\partial z}+\boldsymbol{T} \frac{\partial}{\partial \tau}\right)[(z-\tau)(\mathbf{1}+\boldsymbol{T} \boldsymbol{j})]=\boldsymbol{j}\left(\frac{\partial}{\partial z}+\boldsymbol{T} \boldsymbol{j} \frac{\partial}{\partial \tau}\right)[(z-\tau)(\mathbf{1}+\boldsymbol{T} \boldsymbol{j})] \tag{5}
\end{equation*}
$$

(6)

$$
\partial^{*}[(x+T j i y)(1+T j)]=\left(\frac{\partial}{\partial x}+i \frac{\partial}{\partial y}\right)[(x+T j i y)(1+T j)]
$$

and if necessary the operator does commute with $(1+\boldsymbol{T j})$.
Let's calculate for example
(7)

$$
\partial^{*} \boldsymbol{b a}=\boldsymbol{\partial}^{*}\{[(z-\tau)(\mathbf{1}+\boldsymbol{T} \boldsymbol{j})][(\boldsymbol{x}+\boldsymbol{T} \boldsymbol{j} \boldsymbol{i} \boldsymbol{y})(1+\boldsymbol{T} \boldsymbol{j})]\}=\boldsymbol{\partial}^{*}\{[(z-\tau)(\boldsymbol{x}+\boldsymbol{T} \boldsymbol{j} \boldsymbol{i} \boldsymbol{y})] \mathbf{2}(1+\boldsymbol{T} \boldsymbol{j})\}
$$

$$
\partial^{*}[(z-\tau)(x+\text { Tjiy })]=\left[\partial^{*}(z-\tau)\right](x+\text { Tjiy })+(z-\tau) \partial^{*}(x+\text { Tjiy })=(j-T)(x+\text { Tjiy })+
$$

$$
(z-\tau)(1-T j)=\hat{j}(1-T j)(x+T j i y)+(z-\tau)(1-T j)=(j x+T i y+z-\tau)(1-T j)
$$

Therefore
(8)

$$
\partial^{*} b a=2(j x+T i y+z-\tau)(1-T j)(1+T j)=0
$$

The same goes for $a^{2}, b^{2}, e t c$.

## 3 - HARMONIC FUNCTIONS

Now the argument is :
can a harmonic potential be identified?
The doubt is basically that a function $\mathcal{F}=(x+T j i y)^{m}(z-\tau)^{n}$ that is not analytic is harmonic. If this were the case, its derivative would be analytic.

I proceed by preparing some tools which are the derivatives
(9)

$$
\partial(x+T j i y)=\left(\frac{\partial}{\partial x}-i \frac{\partial}{\partial y}\right)(x+T j i y)=(1+T j)
$$

(10)
$\partial(x+\text { Tjiy })^{2}=\partial[(x+$ Tjiy $)(x+$ Tjiy $)]=[\partial(x+$ Tjiy $)](x+$ Tjiy $)+(x+$ Tjiy $) \partial(x+$ Tjiy $)$

$$
=2(x+T j i y)(1+T j)
$$

$$
\begin{equation*}
\partial(x+T j i y)^{m}=m(x+T j i y)^{m-1}(1+T j) \tag{11}
\end{equation*}
$$

In the same way, and since it is
(12)

$$
\begin{gathered}
\partial(z-\tau)=\left(-j \frac{\partial}{\partial z}-T \frac{\partial}{\partial \tau}\right)(z-\tau)=-(j-T) \\
-(j-T)=-j(1-T j)
\end{gathered}
$$

will result in the end
(13)

$$
\partial(z-\tau)^{n}=-n(z-\tau)^{n-1}(-j+T)=-n(z-\tau)^{n-1} j(1-T j)
$$

There is a difficulty that is to say that they appear in the derivative two terms, one of which has to the right $(1+T j)$ while the other has to the right $(1-T j)$.

However, the conclusion is easy.
(14)

$$
(j-T)(j-T)=-1-j T+j T+1=0
$$

That is, doing the double operation $\boldsymbol{\partial}^{*} \boldsymbol{\partial}$ the result is zero.
Final conclusion:
actually the function
(15)

$$
\mathcal{F}=(x+T j i y)^{m}(z-\tau)^{n}
$$

non- analytic, however, is harmonic.

Even more final conclusion:
any function
(16)

$$
\mathcal{F}(z-\tau, x+T j i y)
$$

it is harmonic.

## 4 - EXAMPLE

Members of this audience will recognize $\boldsymbol{\partial}^{*} \boldsymbol{f}=\boldsymbol{0}$ as a generalization of the CauchyRiemann equations to spacetime, so we can expect it to have a rich variety of solutions. The problem is to pick out those solutions with physical significance (Hestenes, quote).

Let's consider a simple case, I would say the simplest of all that is the inverse of the harmonic variable $(x+T j i y+z-\tau)$
(17)

$$
\frac{1}{(z-\tau)+(x+T j i y)}
$$

It is:
(18)

$$
\begin{gathered}
\partial \frac{1}{(z-\tau)+(x+T j i y)}=-\frac{\left(\frac{\partial}{\partial x}-i \frac{\partial}{\partial y}-j \frac{\partial}{\partial z}-T \frac{\partial}{\partial \tau}\right)(x+T j i y+z-\tau)}{(x+T j i y+z-\tau)^{2}}= \\
=-\frac{(1+T j-j+T)}{(x+T j i y+z-\tau)^{2}}
\end{gathered}
$$

and then:
(19)

$$
\begin{gathered}
\partial^{*} \frac{-(1+T j-j+T)}{(x+T j i y+z-\tau)^{2}}=\frac{(1+T j-j+T) 2(x+T j i y+z-\tau)(1-T j+j-T)}{(x+T j i y+z-\tau)^{4}} \\
=\frac{(1+T j-j+T) 2(1-T j+j-T)}{(x+T j i y+z-\tau)^{3}}
\end{gathered}
$$

(*)(see note)
The numerator reduces to:
(20) $(\mathbf{1}+\mathbf{T j}-j+T)(\mathbf{1}-\mathbf{T} j+j-T)=1-T j+j-T+T j-1-T+j-j+T+1+j T+$ $T-j+T j-1=0$

So the following function is analytic:
(21)

$$
f=\partial \frac{1}{(z-\tau)+(x+T j i y)}=-\frac{(1+T j-j+T)}{(x+T j i y+z-\tau)^{2}}
$$

To see what it is, I can (must) 'rationalize the denominator'. Another day.
But I could start ‘ab ovo’.

Considering that this $\frac{1}{(z-\tau)+(x+T j i y)}$ is harmonic, I could look for its meaning. Both of him, as a harmonic potential, and of its derivative, as an analytic field. As a harmonic potential, once rationalized, it has the form
(22)

$$
A=a+T j i b
$$

and therefore its derivative, which is analytic, has the form:

$$
\begin{align*}
& f=\left(\frac{\partial}{\partial x}-i \frac{\partial}{\partial y}-j \frac{\partial}{\partial z}-T \frac{\partial}{\partial \tau}\right)(a+T j i b)  \tag{23}\\
&=\frac{\partial a}{\partial x}-i \frac{\partial a}{\partial y}-j \frac{\partial a}{\partial z}-T \frac{\partial a}{\partial \tau}+T j i\left(\frac{\partial b}{\partial x}-i \frac{\partial b}{\partial y}-j \frac{\partial b}{\partial z}-T \frac{\partial b}{\partial \tau}\right)
\end{align*}
$$

In the specific case, to capture the physical meaning, in a first approximation we assume the potential $A$ in the form:
(24)

$$
A=a(x, z, \tau)+\operatorname{Tjib}(y)
$$

and therefore the analytic field results

$$
\begin{equation*}
f=\frac{\partial a}{\partial x}-j \frac{\partial a}{\partial z}-T \frac{\partial a}{\partial \tau}+T j i\left(-i \frac{\partial b}{\partial y}\right) \tag{25}
\end{equation*}
$$

which means, taking the conjugate, the Maxwell's field:

$$
\begin{equation*}
F=E_{x}+j E_{z}+T H_{\tau}+T \boldsymbol{j i}\left(i H_{y}\right) \tag{26}
\end{equation*}
$$

This is composed as follows:
an electromagnetic field that propagates in the $z$ direction

$$
F 1=E_{x}+T j i\left(i H_{y}\right)
$$


plus a wave of charge (scalar wave) always in the $z$ direction

$$
F 2=j E_{z}+T H_{\tau}
$$


(*) Note
All the magic comes from being able to write

$$
\partial^{*}(x+T j i y+z-\tau)^{2}=2(x+T j i y+z-\tau) \partial^{*}(x+T j i y+z-\tau)
$$

which means being able to brutally apply the rule of derivation of compound function, in this case

$$
\partial^{*} f^{n}=n f^{n-1} \partial^{*} f
$$

This fact, which is not at all obvious, is a consequence of the property that $f$ does commute with anything, in particular it commutes with derivation operations, here $\partial^{*}$

## 5 - CONCLUSIONS

Quite easy, albeit long and boring, to find many solutions. But ... let's take the following point of view. To put it in an absolutely imaginative way, this stuff, a traveling, possibly charged field

that is, a rectilinear aggregate of electrons, a strange aggregate, could he not instead be a 'thing'? A single thing? That is, a single entity (possibly charged), no more a dense aggregate in to electrons. Two observations:
one is that "gaugeless" Maxwell's equations can have a lot of similar solutions, I did some primordial exercises with the analytic functions in my Manuscripts but there are no mathematicians and electromagnetics who deal with them;
the second observation is that the " Ranada solutions" or similar or rather the corresponding experimental tests interest me because they seem to have shown that similar entities exist, whether they are filiform cylindrical spherical toroidal or what.
Making two plus two my idea is of a possible big jump.
The big jump is as follows:
we jumped from the waves to the "corpuscular nature of light'.
From the electromagnetic field as a "thing" $\qquad$ to photons.
We could now do the reverse passage for electrons. Switch from the traditional particle vision to a single entity: the charged field. Go from charges, from particles, protons, electrons, to a different single entity. Here this "thing" takes on a different light


Problems such as 'masking of Coulomb repulsion in the EVO Shoulders' are gone .... because there are no electrons. From the physical / mathematical point of view it serves to justify ... only a solution of Maxwell's equations 'gaugeless', charged. Full stop. All that is needed is a profound conceptual revolution, that is, to believe in the existence of a single entity: the charged field, in which the concept of particles, and the presence of particles, no longer exists.
Isn't it that we are ready to make this big jump?

Obvious post scriptum: after all it is not that much is needed, only a little imagination is needed.
For entirely trivial reasons, history first confronted us with light and electromagnetic waves. Nobody thought of 'photon' balls.
(I am reminded of someone, I don't remember who, who wrote 'particles exist only when you look at them').
If therefore we had been accustomed to charged waves, only charged waves, only
experimental evidence of charged waves, nobody would have thought of electrons (or protons, or particles in general). Then one day someone, observing, would say:
what ? electrons?
So there is a particle nature of charged waves!
Wow !! who would have thought.

## 6 - ACKNOWLEDGMENTS

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