The spinorial Dirac operator

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Abstract

We define a Dirac type operators called the spinorial Dirac operator

1 The Dirac operator

Let \((M, g)\) be a spin manifold, then we can define the Dirac operator \(D\) with help of the Levi-Civita connection \(\nabla\) [F].

\[ D(\psi) = \sum_i e_i \nabla_{e_i}(\psi) \]

\((e_i)\) is an orthonormal basis.

\[ D = \mu \circ \nabla \]

with \(\mu\) the Clifford multiplication.

2 The spinorial Clifford algebra

We define the spinorial Clifford algebra as the free algebra over the space of spinors with relations:

\[ \psi \cdot \psi' + \psi' \cdot \psi = 2g(\psi, \psi') \]

\[ (X \cdot \psi) \cdot \psi' = -\psi \cdot (X \cdot \psi') \]

with \(X\) a vector and \(\psi, \psi'\) two spinors. We have:

\[ \psi^{-1} = \frac{\psi}{||\psi||^2} \]

The double spinorial group \(Spin_2(n)\) is defined as the products of an even number of spinors of unit norm.

\[ Spin_2(n) = \{ \prod_i \psi_i \bar{\psi}_i', ||\psi_i|| = ||\psi_i'|| = 1 \} \]
3 The spinorial derivations

A spinor is a derivation of the space of smooth functions by the formula:
\[ \psi(f) = (df)^* \psi \]

4 The spinorial connection

A spinorial connection can be defined for a module over the spinorial Clifford algebra:
\[ \nabla \psi(fs) = \psi(f).s + f \nabla \psi(s) \]
\[ \nabla f \psi(s) = f \nabla \psi(s) \]

5 The spinorial Dirac operator

The spinorial Dirac operator is defined by the formula:
\[ D_\psi = \sum_i \psi_i \nabla \psi_i \]
with the \( \psi_i \) an orthogonal basis of the spinors.

References
