# Half Truths and Rant Against ECO Paradigm by Chandra Prakash

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One Chandra Pakash of unknown affiliation has publised his maiden preprint (vixra.org/abs/2001.0501) entitled: "Abhas Mitra and Eternally Collapsing Objects: A Review of 22 Years of Misconceptions" [1]. Although there are 20 odd peer reviewed journal papers on various aspects of the ECO paradigm Prakash cites only one old paper [2] and claims to have debunked the entire research by Mitra, Leiter, Robertson and Schild spread over almost 15 years. He even uploaded a 9 min youtube video debunking not only ECO paradigm by me as a physicist. Incidentally, he happens to be one my FB friends, and had casually raised a few points. He appeared to be satisfied with my clarifications, and never told me that he had already written a preprint "debunking" the ECO paradigm. While it is likely that one of the 20 odd related papers is invalidated, the results and conclusions behind ECO paradigm remain firmly established by series of independent papers. In fact the conclusion that a collapsing massive star should turn into an ultra-hot ball of magnetized plasma (ECO, MECO) follows from 5 papers: 1 in Physical Review (D), 2 in MNRAS Letters, 1 in MNRAS and 1 in New Astronomy [3–8]; and this result is valid irrespective of any mathematical proof for non-existence of finite mass true black holes. Incidentally, the maiden proof that a true mathematical Schwarzschild black hole should have zero gravitational mass was given by L. Bel in 1969 (Bel, JMP 10, 1051, 1969) [9], three decades before I independently suggested the same result.

## INTRODUCTION

I have almost 500 FB friends as I hate to decline FB friendship requests. In particular, if I find that any student has some interest in astronomy and astrophysics, I immediately honour the friendship request. And on FB messenger, I receive questions on science even from those who are not on friendship list. On Nov 22, 2019, one Chandra Prakash asked me:

"Hi, do you believe in big bang theory??? and what about ECOs your own concept??"

Even now, I have no idea about the educational status, or profession or affiliation of this boy.

When I did not respond to this oddly posed question, he insisted: "please answer".

I replied: " Sorry, for ECOs search google" to which he wrote:

"I know them, read your paper, it was nice".

One month later, on Jan 17, 2020, he wrote to me again: " Hi sir, can i get your contact number?? I would like to talk and work on ECO. I wanna understand it fully and your notion of it..".

I replied:

"You can google Eternally Collapsing Object and go through (papers)."

His response:

"I have the paper and I wanna study the mathematical details".

Before I proceed, let me remind that, in the following the symbol $M$ will denote the gravitational mass while the variable $R$ will refer to the areal radius coordinate and the surface area $4\pi R^2$ of spheres around the point mass or a spherical star. Thus this radial coordinate has a direct geometrical significance. It also has a profound physical significance as the intensity of light emitted by a spherical body decreases as $1/R^2$. In other words, $R$ defines the physically important "Luminosity Distance". Note a so-calle Schwarzschild black hole (BH) which is vacuum except for the central point mass. So the gravitational mass of a BH is essentially that of a ne utral point mass.

The concept of BH arose when, in 1916, Hilbert studied the spacetime structure around a neutral point mass by using $R$ as a radial coordinate, and the radius of the BH is fefined by its Event Horizon at $R = 2M$ (we take $G = c = 1$).

However, as per general relativity, coordinates are mere *labels*, and hence one is free to use any other suitable radial coordinate for studying any problem.

Indeed Karl Schwarzschild obtained the spacetime structure around a spherical star before Hilbert in 1915, by using a different radial coordinate $r$ which is related to $R$ in the following way [10, 11] :

$$r^3 = R^3 - (2M)^3 \tag{1}$$

If the postulate of absolute freedom choice coordinate is really correct, then choice of $r$ as the radial coordinate too would lead to the idea of BHs. However, the original Schwarzschild solution does not lead any BH [10, 11]. It is easy to see this from the above equation. When one chooses $r = 0$ as the origin of the coordinate system, obviously one cannot proceed to a *negative* range of $r$. And even at $r = 0$, one has $R = 2M$ which implies that the star cannot contract beyond the $R = 2M$ limit. On the other hand, the idea behind a BH is that And a sufficiently massive non-rotating star must collapse to a point mass ($R = 0$) surrounded by a vacuum mathermatical boundary surface (Event Horizon) having a radius of $R = 2M$.

Now let me revert to my chat with Chandra Prakash. He told me that my denial of existence of BHs is based on the original Schwarzschild Solution and hence it does not make any "sense" to him. But I clarified to him, that though I have talked about the original Schwarzschild solution, my conclusion is not at all based on it. On the other hand, I have several indepent proofs from severs directions (see later). But he would just ignore my clarifications. Getting irritated by series of his incoherent questions, on Jan 20, 2020, I shot back:

"You do not understand anything about my work but only claim to have read them. I am telling that the so-called black holes are not exact black holes, but black hole mimickers. Sorry, i have no more time. Bye."

One of the arguments offered by me and several other authors that the Event Horizon, the mathematical boundary of the black hole" is a physical singularity and not a mere "coordinate singularity" is that the physically measureable proper acceleration of a free falling particle as measured by a static observer blows up at there, at $R = 2M$.

On Jan 20, 2020, Chandra Prakash requested me for the reference where this proper acceleration was first derived. I sent him the link for the pertinent 1981 paper published in American J. Physics. He thanked me with two smileys (love and happy).

The Schwarzschild metric (actually Hilbert metric) describes both vacuum black holes as well as the exterior vacuum spacetime of a star which is not emitting any radiation at all. The idea that massive stars must eventually collapse into black holes is based on famous Oppenheimer & Snyder solution which assumes the most unrealistic assumption that a collapsing star has not pressure at all, and neither does it radiate anything [12]. On the other hand when real stars collapse their pressure and temperature keeps on increasing and so does the rate of outgoing radiation. The exterior spacetime of a real collapsing star is described by "Vaidya Metric" and not by Hilbert or Schwarzschild solution.

It appears that Chandra Prakash has learnt about this fact from my paper. On Jan 24, 2020, he wrote to me:

"Also, sir does this describe your work. Implementing the continuity condition on Vaidya metric, it demands to avoid inconsistencies that it radiates all of it's mass, and bit by bit till eternity and when the time does come, the mass becomes "0", which according to calculations would lead to infinite proper time for event horizon to form. And hence no true black holes really exist."

He seemed to finally appreciate at least one of the basic ideas in my paper. I reponded to some of this other queries and he reacted:

"*Awesome, now i understand.* (A smiley) Maths is sacred for me, helps me think."

Next, he asked me : "Sir, can you endorse my work for arxiv (electronic preprint archieve)?

Even without asking for the title of his preprint, I expressed my inability to endorse the same.

Then probably on Jan 26, 2020, my another FB friend appeared to be quite disturbed. He sent me the vixra link of the preprint by Chandra Prakash which has claimed to debunk 22 years of reaserch on ECO. Later he also spoke about a 9 min Hindi you tube video by Chandra Prakash debunking ECO research and even mentioning my work is all wrong as I do not know any general relativity.

Note even when, on Jan 24, Chandra Prakash requested me to endorse his preprint on arxiv, he never told me that his preprint was about debunking ECO research and he had already published this on vixra on Jan 22! Neither did he tell me that he had also published a youtube video on the same.

It transpires that though more than 40 research papers and articles have been published on ECO paradigm in 22 years, Chandra Prakash's preprint cites only one paper which I wrote in 1999, and an article which I wrote in 2004 (but published later).

Having made this long introduction, let me now expond why Chandra Prakash's preptint at the most containds half truths, and in reality vicious rants. The research on general relativistic gravitational collapse by me and my American colleagues spreading over almost 15 years has 3 fundamental aspects.

(1). Non formation of exact black holes and trapped surfaces during continued gravitational collapse

(2). Though the black hole solution is formally correct, it is illusive as true mathematical black holes correspond to ultimate ground state of the collapsing matter having total energy: $E = Mc^2 = 0$ or $M = 0$. Hence the so-called black holes cannot be true black holes, and must be only Black Hole Mimickers (BHM).

(3). The only exact solution for formation of black holes assume that the collapsing star material has no pressure at all ($p = 0$). My papers claimed that no real star, no real matter can be strictly pressureless, and this $p = 0$ condition

is never true. Mathematically, it is true when there is no matter at all, or density of the matter is zero ($\rho = 0$). If so the gravitational mass of the mathematical black holes imagined to be born in pressureless collapse too is zero ($M = 0$).

(4). Most likely nature of the Black Hole Mimickers: Eternally Collapsing Objects (ECOs).

## HAS ANY RESEARCH PAPER CRITICIZED THE BASIC IDEA OF ECOS? POINT (4)

The black hole mimickers should be ultra-compact ultra-hot balls of plasma. Since are Horizonless Compact Objects (HCO), they keep on radiating and contracting at infinitesimally slow rate, and should keep on contracting for ever. Hence they are Eternally Collapsing Objects (ECOs). During gravitational contraction, the magnetic field strength of a star increases as $B \sim 1/R^2$, and this is the reason, they magnetic field of a new born neutron star could be trillion times stronger than that of Sun: $B \sim 10^{12}$ G. Since ECOs are much more compact that neutron stars, in view of general relativistic effects, the magnetic field of the ECOs could be tens of thousand times that of neutron stars: $B \sim 10^{16-17}$ G. Even supermassive ECOs should possess very strong intrinsic magnetic fields. So ECOs should also be magnetospheric ECOs or MECOs.

The basic idea ECO/MECO mentioned above is *independent of the aspects*, (1), (2) & (3). And this *basic idea of ECO/MECO was developed in as many as 5 peer reviewed papers* (2006-2010) in well respected journals, and as of now nobody has questioned any of them [3–7]. As far as Chandra Prakash is concerned he may not be even aware of these 5 papers. Thus contrary to the bragging by Chandra Pakash his preprint **his preprint does not at all debunk the ECO paradigm**.

One of the fundamental precitions of the ECO/MECO paradigm is that so-called astrophysical black holes should possess strong intrinsic magnetic field when true neutral black holes have no intrinsic magnetic field. The accretion disk and plasma around true black holes can indeed create some ambiend magnetic field, but this is often insufficient to explain most of the astrophysical phenomenon [13–15]. On the other hand, the idea that the so-called black holes are MECOs naturally explain most of the observations. And indeed, from such a considerations, there have been lot of observational evidences in favour of ECO/MECO paradigm. Most of such evidences have been published in reputable peer reviewed journals [16–21].

In general there are several theoretical studies on general relativistic continued gravitational collapse which support the idea that massive star should collapse into ultra-hot ball of plasma rather than a true black hole [22–27].

## WHY OPPENHEIMER SNYDER BLACK HOLES CORRESPOND TO $M = 0$. POINT (3)

When one talks of mathematics, the physical variabe pressure ($p$) must be continuous. So if pressure of a collapsing star is zero, it must be so at the instant the collapse began from a static state. And then it is trivially seen that the density of the star at the instant the collapse began too is zero ($\rho = 0$). Then the initial mass of the startoo was zero. And since for assumed radiationless collapse, the gravitational mass of the star remains fixed, the mass of the eventual Oppenheimer- Snyder black hole [12] too must be zero ($M = 0$)[2]. Later, I presented several indepenent proofs that indeed the mass of the ideal Oppenheimer Snyder black hole is zero [28–30]. It seems Chandra Prakash is not even aware of these papers.

## NON FORMATION OF EXACT BLACK HOLES. POINT(1)

A spherically symmetric spacetime may be represented by series of areal radius coordinates ($R$) which as if are the fixed milestones on the bank of a flowing river. On the other hand the flowing river may be represented by series of floating flowers which are at fixed locations with respect to the river. And such flowers fixed in the river represent *comoving radial coordinate r*. The time recorded by the floating flowers is *comoving time t*. The river is flowing with respect to the fixed milestones on its bank ($R$), and a given flower encounters ever changing milestones ($R$). It is in this sense, that areal radius of the collapsing star or any section of it is constantly changing:

$$R = R(r,t) \qquad (2)$$

Then, in general,

$$dR = (\partial R/\partial r)dr + (\partial R/\partial t)dt \tag{3}$$

Here $d$ denotes total differentiation while $\partial$ denotes partial differentiation, i.e., differentiation with respect to only one variable. When we treat the 3 variables $R, r, t$ without their physical significance, $dR$ denotes passage of a spherical stellar interior from one milestone to a neighboring one, and does not reflect the view of any particular milestone. For a such general mathematical description, of course,

$$\frac{dr}{dt} \neq \frac{\partial R/\partial t}{\partial R/\partial r} \tag{4}$$

But suppose we are are focussing our attention on the view of a given milestone sitting at a *fixed location*: $R = fixed$. For this observer $dR = 0$. Therefore from the view point of a fixed observer on the river bank

$$(\partial R/\partial r)dr + (\partial R/\partial t)dt = 0 \tag{5}$$

So from the view point of a fixed observer

$$\frac{dr}{dt} = -\frac{\partial R/\partial t}{\partial R/\partial r} \tag{6}$$

and

$$\left(\frac{dr}{dt}\right)^2 = \left(\frac{\partial R/\partial t}{\partial R/\partial r}\right)^2 \tag{7}$$

And my initial proof [2] for non-occurrence of exact black holes (more fundamentally, non-occurence trapped surfaces) is based on the foregoing equation, contrary to the claim by Chandra Prakash [1]. However it is true that my initial proofs for non-occurrence of trapped surfaces [2, 31] were confusing as I failed to state that motion of the stellar material was being studied by adjoining $R = fixed$ observers. Part of my confusion arose from the fact that Misner, Thorne & Wheeler defined two quantities related to gravitational as total derivatives [32]:

$$\Gamma = \frac{1}{2\pi} \frac{d(Circumference\ of\ the\ star)}{d(Proper\ Radius)} \tag{8}$$

and

$$U = \frac{1}{2\pi} \frac{d(Circumference\ of\ the\ star)}{d(Proper\ Time)} \tag{9}$$

when these two quantities actually involved partial derivatives and not total derivatives. I resolved my confusion later and offered transparent proofs for non-occurrence of trapped surfaces [33–35]:

$$\frac{2M(r,t)}{R} < 1 \tag{10}$$

As per both special and general relativity, material particles must move on *timelike worldlines* while massless particles must always move along *null or lightlike worldlines*. Physically this means that appropriately defined proper velocity of material particles is always less that the speed of light in vacuum. The foregoing constraint was obtained to ensure that worldlines of stellar material must be lightlike.

However in principle stellar material might attempt to be lightlike only asympotically, i.e., only after infinite comoving proper time:

$$\frac{2M(r,t)}{R} \to 1; \ proper\ time \to \infty \tag{11}$$

In other words, continued gravitational collapse must result in horizonless compact objects which however might approach the mathematical black hole state $2M/R = 1$ after infinite comoving proper time. Hence continued gravitational collapse must result into static compact objects like White Dwarfs and Neutron Stars orquasi-static Eternally Collapsing Objects.

If we rule out existence of negitive gravitational masses, from Eq. (10), it follows that if the star would indeed collapse into any point singularity ($R = 0$), the eventual gravitational mass $M = 0$:

$$M \to 0, as\ R \to 0 \tag{12}$$

Bottom Line: My proof for non-occurence of exact black holes is correct despite their intial confusions. There is also an independent proof for non-occurrence of exact black holes by Leiter & Robertson [36].

## WHY MATHEMATICAL BLACK HOLES CORRESPOND TO $M = 0$? POINT (2)

I have already mentioned of several independent proofs that that mathematical so-called Schwarzschild black holes correspond to $M = 0$ [1, 22–24]. And the condition for non-occurrence of exact black holes too lead to $M = 0$ (equations (11-12)). In 2009, I attempted to offer yet another independent proof that mathematical black holes correspond to $M = 0$ [37]. But it is likely that this particular proof is incorrect [38], a fact already acknowledged by me [39]. To this extent Chandra Prakash may be corrrect, yet his claim is only half truth because there are many independent proofs that mathematical black holes correspond to zero gravitational mass.

Having made this admission, let me offer some relevant discussions. My proof was based on the 170 year old rule of multivariate calculus first obtained by Carl Jacobi (1804 -51 CE) which says that coordinate transformation from say from $x, y$ to $u(x, y), v(x, y)$, obeys the rule

$$dxdy = dydx = Jdudv = Jdvdu \tag{13}$$

where the "Jacobian"

$$J = \det \begin{vmatrix} x_u & y_v \\ x_v & y_u \end{vmatrix}$$

The simplest example of the Jacobi identity is that an element of area $dxdy$ in Cartesian coordinates may be transformed in polat coordinates as $dxdy = rdrd\theta$, where the Jacobian turns out to be $r$. We know that this is obviously correct. But there may be deeper subtle truth lurking here. In this 2-D case, there are is obtainec as some appropriate vector cross products. And for arbitrary higher dimensional cases, the volume is obtained from wedge product of one forms whose concept was developed by Elie Cartan in 1899, *more than half a century afterJacobi developed multivariate calculus.* Both the formalisms appear to be equivalent. Yet not so exactly. And centainly in terms of wedge products of one forms, my one proof [38] gets completely invalidated [39].

## EVEN MORE INDEPENDENT PROOFS THAT MATHEMATICAL BLACK HOLES HAVE $M = 0$

For a material particle test moving radially inward towards a black hole, the coordinate speed is given by [1, 31]

$$\left(\frac{dR}{dT}\right)^2 = \frac{(1 - 2M/R)^2}{E^2}\left[E^2 - (1 - 2M/R)\right] \tag{14}$$

When the test particle approaces the event horizon $R \to 2M$, this equation reduces to

$$\left(\frac{dR}{dT}\right)^2 \to (1 - 2M/R)^2 \tag{15}$$

And this is exactly what gives the coordinate speed of a radially moving photon. This suggests that material particle would behave like a photon at $R = 2M$. Note the above equation tells that:

$$(1 - 2M/R)dT^2 \rightarrow (1 - 2M/R)^{-1}dR^2; \ R \rightarrow 2M \tag{16}$$

Further since, for radial motion of the material particle ($d\theta = d\phi = 0$), by feeding the foregoing equation into the vacuum Schwarzschild equation, we find that [1, 31]

$$ds^2 = (1 - 2M/R)dT^2 - \frac{dR^2}{1 - 2M/R} \rightarrow 0 \ as \ R \rightarrow 2M \tag{17}$$

This is a confirmation, that the worldline of a material particle would become null or light like if it would hit the event horizon. But since this is not allowed, and worldline of the material particle must remain timelike, one must have $\tau_{EH} = \infty$ [1, 31].

For a test particle starting its journey from $R = R_b$, the comoving proper time to reach the event horizon is

$$\tau_{EH} = \frac{2}{3} \frac{\pi R_b^{3/2}}{\sqrt{2M}} - \pi M \tag{18}$$

and which can be infinite only if mass of the black hole $M = 0$.

## SIMPLEST PROOF THAT SCHWARZSCHILD BLACKS HAVE $M = 0$

By definition, the energy momentum tensor (pressure, density) of the point singularity of the black hole must be infinite. A specific form fott the sdame was irst suggested by Tangherlini [40]. Then it follows that the Ricci Scalar, $Ric$, an important quantity related to energy momentum tensor, of the point should blow up at the singularity. In addition $Ric$ should be also proportional to the gravitational mass of the point singularity ($M$). Narlikar & Padmabhan [41] too arrived at a similar conclusion. Thus on physical grounds, one expects

$$Ric \sim M\infty \ at \ singularity \ (R = 0) \tag{19}$$

However, as is well known, the vacuum Schwarzschild - Hilbert solution is obtained from the condition

$$Ric = 0 \tag{20}$$

everywhere including at $R = 0$. Then compatibility of these two foregoing equations directly demands that $M = 0$ for a point mass, so that the Schwarzschild Singularity ($R = 2M$) is a point rather than than a sphere.

### Gravitational Mass of a Neutral Point

I revisit here the Arnowitt, Desar and Misner prescription about the gravitational mass $M$ and bare mass $M_b$ of a tiny static sphere of radius $\epsilon$ ($c = 1, G = G$) [42]:

$$M = G^{-1} \left( -\epsilon + [\epsilon^2 + 2M_b G\epsilon]^{1/2} \right) \tag{21}$$

In particular, this paper suggests that for a point charge, one has $M = 2|e|$ [42]. And thus for a neutral point having $|e| = 0$, one has $M = 0$. Since the entire gravitational mass of a Schwarzschild black hole arises from the point mass, the mass of the same should be zero.

## SCHARZSCHILD SINGULARITY: SPHERE OR POINT?

Earlier "Event Horizon" was known as "Schwarzschild Singularity". And in 1968, Janis, Newman, and Winicour [43] concluded that

"A spherically symmetric solution of the Einstein equations is presented that coincides with the exterior Schwarzschild solution" as long as $R > 2M$. But as soon as one would attempt to have a solution for $R = 2M$, one would obtain a *point singularity instead of Schwarzschild surface*". Since this behaviour persists for arbitrary weak coupling of the massless scalar field, they wondered whether there could ever be a Schwarzschild surface with finite radius [43]:

"It is clear that if our truncated Schwarzschild metric is to be considered as the physical solution corresponding to a, spherically symmetric point mass, *then the entire question of gravitational collapse beyond the Schwarzschild radius becomes meaningless.* This point of view also obviates all discussion of the topological questions of the Schwarzschild interior, which for many people has always been disturbing.[43]"

And immediately afterwards, Bel offered explicit proof that the Schwarzschild surface behaves like a point singularity [8]:

"A new point of view is presented for which the Schwarzschild singularity becomes a real point singularity on which the sources of Schwarzschild's exterior solution are localized."

## EVIDENCE THAT EVENT HORIZON IS A PHYSICAL SINGULARITY

At the event horizon, $R = 2M$ weird mathematical anomalies may happen. But the black hole hole paradigm claims that such weird behaviour happens only because of the failure of the Schwarzschild coordinates $(R, T)$, on and within the even horizon. As per black hole paradigm, event horizon is such a normal region of spacetime that
- No physically measureable quantity can blow up there
- A free falling observer diving through the event horizon cannot even be aware of it.

But it turned out later that both these pillars of the black hole paradigm are fallible.

To see this, note that the proper acceleration of a free falling test particle as measured by an adjoining fixed observer is[9]:

$$a = \frac{M}{R^2 \sqrt{1 - 2M/R}} \tag{22}$$

Clearly,

$$a \to \infty \ as \ R \to 2M \tag{23}$$

And this is precisely the reason that no observer can stay at rest at the event horizon. So it is a physical effect and not a coordinate effect.
- In 1982, Karlhede et al., investigated the properties of local geometry of Scharzschild space time in terms of the Riemann curvature tensor and its higher derivatives [44]. They found that there is indeed a measurable quantity, now called Karlhede Invariant which changes sign at $R = 2M$:

$$\mathcal{I} = \frac{-720M^2(R - 2M)}{R^9} \tag{24}$$

By noting such a change of sign of I, a free falling observer can very well detect the physical peculiarity of the event horizon, contrary to the assertion of the black hole paradigm.

## DISCUSSIONS & CONCLUSIONS

ECO/MECO paradigm has 4 aspects:
(1). Non formation of exact black holes and trapped surfaces during continued gravitational collapse

(2). Though the black hole solution is formally correct, it is illusive as true mathematical black holes correspond to ultimate ground state of the collapsing matter having total energy: $E = Mc^2 = 0$ or $M = 0$. Hence the so-called black holes cannot be true black holes, and must be only Black Hole Mimickers (BHM).

(3). The only exact solution for formation of black holes assume that the collapsing star material has no pressure at all ($p = 0$). My papers claimed that no real star, no real matter can be strictly pressureless, and this $p = 0$ condition is never true. Mathematically, it is true when there is no matter at all, or density of the matter is zero ($\rho = 0$). If so the gravitational mass of the mathematical black holes imagined to be born in pressureless collapse too is zero ($M = 0$).

(4). Most likely nature of the Black Hole Mimickers: Eternally Collapsing Objects (ECOs).

and is based on 20 odd peer reviewed research papers by Mitra, Leiter, Robertson, Glendenning, Schild and others. There are also many supporting papers by other authors. Out of these 20 off papers, one paper related to the point (2) is invalidated. But this does not at all invalidate many other independent parallel proofs that that true black holes have $M = 0$. It may be noted that the occurrence of $M = 0$ does not imply absence of matter, on the other hand it indicates a state where the bare mass comprising rest masses and internal energies are neutralized by negative gravitational energy. And of course, it does not affect at all the papers and proofs related to aspects (1), (3) and (4). In fact there are significant observational evidences that the so-called astrophysical black holes are MECOs having strong intrinsic magnetic fields.

Therefore the claim by Chandra Prakash that his preprint has debunked the ECO paradigm is rant of an immature researcher. Incidentally Chandra Prakash's abstract alleges that My paper says: " why $R = 0$ (inside black hole) should also be treated as another coordinate singularity". But my paper [2] never made any such statement!

Our offered proofs apart [1, 33–36], there are deeper physical reason why trapped surfaces and exact black holes are inhibited in physical gravitational collapse:

(i) pressure and its gradient actually decrease the active gravitation mass of the star [44, 45], and of course

(ii) the outward radiation pressure of the ultra-hot star halts collapse immediately before formation of any event horizon [3–7].

Historically starting from Einstein, many authors have questioned either the black hole paradigm or at least some aspect of it [47–58].

Recall in 1988, Narlikar and Padmanabhan wrote that [41]

"...the discussion of physical behavior of black holes, classical or quantum, is only of academic interest. It is suggested that problems related to the source could be avoided if the event horizon did not form and that the universe only contained quasi-black holes."

These two authors however had no idea how continued gravitational collapse may give rise to quasi - black holes instead of true mathematical black holes. In contrast, long ago, McCrea [47] had a worthwhile insight how this might happen. He considered the possibility that if a body would contract to the limit $R \to 2M$, then it may radiate out even 100% of the original mass-energy so that the final mass may approach $M \to 0$ and exact $M = 0$ not forming ever:

"Thus if we tried to make a body of the Schwarzschild critical radius we could in prin ciple get as near to doing so as we wish, but we could never quite finish it. So the problem of the properties of a body having exactly the Schwarzschild radius is entirely academic."

And my research papers have shown that McCrea's insights were largely correct. Note, contrary to the myth, BH paradigm got never established because the physical nature of BH interiors have never been understood. This becomes clear from the following admission by three physicists, made in 2008, who believe in the BH paradigm [59]:

"*The solutions that do away with the interior singularity and the event horizon, although interesting in themselves, sweep the inherent conceptual difficulties of black holes under the rug. In concluding, we note that the interior structure of realistic black holes have not been satisfactorily determined, and are still open to considerable debate.*

And let me conclude this discussion by citing Paul Dirac who was not nly one of the greatest theoretical physicists ever but one of the best experts on general relativity too [60]:

"The mathematicians can go beyond this Schwarzschild radius, and get inside, but I would maintain that this inside region is not physical space, because to send a signal inside and get it out again would take an infinite time, so I feel that the space inside the Schwarzschild radius must belong to a different universe and should not be taken into account in any physical theory. So from the physical point of view, the possibility of having a point singularity in the Einstein field is ruled out. Each particle *must have a finite size no smaller than the Schwarzschild radius.*"

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