The Goldbach Conjecture

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1 Introduction

The Goldbach Conjecture states that every even number can be represented as the sum of two primes. The following is a proof by induction of this conjecture. Mathematical induction can be used to prove that a statement, f(n), holds for all natural numbers n. If I were to show the base case(s) and the inductive step to be true then I have proven the conjecture as shown below.

2 Proof by Induction

PROPOSITION: Every even number 2n can be represented as the sum of two primes; p, q. n ∈ N; p, q ∈ P. (n is natural; p, q are primes). Let f(n) denote the nth Goldbach number/even number then equations can be formed as follows:

\[ f(n) = 2n = p + q. \]
\[ f(n + 1) = 2n + 2 = p + q + 2 \]

BASE CASES:
The following equations are the sums of two primes; f(n), f(n + 1):

\[ f(3) = 6 = 3 + 3 \]
\[ f(4) = 8 = (3 + 3) + 2 = 3 + 3 \]

INDUCTIVE STEP:

If we show that f(n + 2) is the sum of two primes based on f(n), f(n + 1) or any true mathematical axiom then the proposition is true for any (f(n + 2) = 2n + 4) ≥ 10. We have:

\[ f(n) = 2n = p + q \]
\[ f(n + 1) = 2n + 2 = p + q + 2 \]
\[ f(n + 2) = 2n + 4 = p + q + 4 \]

\[ \Rightarrow f(n + 2) = (f(n + 1) - p) + (p + 2) \]
\[ \Rightarrow f(n + 2) = (f(n + 1) - p) + (f(n + 1) - q) \]
We know that $f(n + 1) - p = q + 2 = 3 + 2 = 5$ ($n = 3$, $q = 3$, $p = 3$) is prime and $f(n + 1) - q = p + 2 = 3 + 2 = 5$ ($n = 3$, $p = 3$, $q = 3$) is prime due to the base cases listed above. This shows $f(n + 2)$ can be represented as the sum of two primes. It is also true that every even number $2n \geq 4$ can be represented in form $f(n + 2) = 2n + 4$ therefore, by induction every even number of form $f(n + 2) = 2n + 4$ which is greater than the base cases $f(n)$, $f(n + 1)$ can be represented as the sum of two primes. The remaining trivial case $f(2) = 4 = 2 + 2$ is the sum of two primes.

This paper wholly proves all numbers greater than or equal to four can be represented as the sum of two primes.

QED