Explaining the radius and mm of a proton

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The electron-proton scattering experiment by the PRad (proton radius) team at Jefferson Lab measured the root mean square (rms) charge radius of the proton as $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm. We offer a theoretical explanation of the new measurement based on a ring current model of a proton.

1. We have the following recommended CODATA values for the magnetic moment and the mass of a proton:

$$\mu = 1.41060679736\times10^{-26}\ \text{J}\cdot\text{T}^{-1} \pm 0.00000000060\ \text{J}\cdot\text{T}^{-1}$$
$$\text{m} = 1.67262192369\times10^{-27}\ \text{kg} \pm 0.00000000051\times10^{-27}\ \text{kg}$$

We also have the following defined values for the elementary charge and the velocity of light:

$$\text{q}_e = 1.602176634\times10^{-19}\ \text{C}$$
$$c = 299\ 792\ 458\ \text{m/s}$$

Thirdly, the most recent experiment[1] measured the proton radius as:

$$r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}\ \text{fm}$$

Hence, we have two constants and a number of variables that depend on them. The two constants can be related to the variables through a number of physical laws and theorems we accept to be valid because they have not been falsified (Popper, 1959). The laws and theorems that we will use in this article are:

- The Planck-Einstein relation: $E = h\cdot f = \hbar\cdot\omega$
- The principle of relativity and the energy-mass equivalence relation: $E = m\cdot c^2$
- The force law, which states that a force acts upon a charge and changes its state of motion
- Maxwell's laws of electromagnetism
- The energy equipartition theorem

The number of laws and theorems is exceedingly larger than the number of physical constants. This reflects non-trivial *structure*—both in Nature as well as in our mind.

2. We imagine the magnetic moment of a proton to be created by a circular current of the elementary charge. It is, therefore, equal to the current times the area of the loop:

$$\mu = \text{I}\pi a^2 = \text{q}_e f \pi a^2 = \frac{\text{q}_e \omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{\text{q}_e \omega}}$$

[1] https://www.jlab.org/prad/collaboration.html

The frequency is equal to the velocity of the charge (v) divided by the circumference of the loop ($2\pi a$). However, for a reason the reader will readily understand after reading this article, we prefer to use the Planck-Einstein relation for the frequency. We believe the Planck-Einstein relation ($E = h \cdot f = \hbar \cdot \omega$) reflects a fundamental *cycle* in Nature. It, therefore, makes sense to also apply it to the ring current idea of a proton.[2] Hence, we write:

$$a = \sqrt{\frac{2\mu}{q_e \omega}} = \sqrt{\frac{2\mu\hbar}{q_e E}}$$

3. When applying this formula to an electron, we get the Compton radius of an electron ($a = \hbar/mc$). When applying the $a = \hbar/mc$ radius formula to a proton, we get a value which is about 1/4 of the *measured* proton radius. We, therefore, need to consider using the same fraction of the proton energy to calculate the frequency:

$$\omega = \frac{1}{4}\frac{E}{\hbar}$$

We should motivate the 1/4 factor, of course. We think the huge value of the proton mass and its tiny size – as compared to the mass and size of an electron – lend credibility to the assumption of another force (or another charge) inside of the proton.[3] Hence, the 1/4 factor combines (1) the energy equipartition theorem (half of the energy or mass of the electron is to be explained by the strong force) and (2) Hestenes' interpretation of Schrödinger's *Zitterbewegung* interpretation of an electron.[4] We can, finally, do an actual calculation now:

$$a = \sqrt{\frac{2\mu}{q_e \omega}} = \sqrt{\frac{4 \cdot 2\mu\hbar}{q_e E}} = 2 \cdot \sqrt{\frac{2\mu\hbar}{q_e m c^2}} \approx 2 \cdot 0.35146 \ldots \times 10^{-15} \approx 0.703 \text{ fm}$$

The gap between the 0.831 and 0.703 values suggests we are missing a $\sqrt{2}$ factor:

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[2] There is a long tradition of thinking of an electron in terms of a current ring. We may refer to Parson (1915), Schrödinger (1930) and, more recently, Hestenes (1990). It has been suggested it may also apply to protons (Consa, 2018) but, based on quick feedback from sympathetic researchers, we think this paper may be the first fully consistent theory in this regard. Alexander Burinskii, whose work on an integrated theory of the electron we admire greatly, drew our attention to earlier work of M.E. Shulman but Shulman's work seems to focus on leptons only (https://www.scirp.org/journal/paperinformation.aspx?paperid=78086). Giorgio Vassallo also sent useful references we will further examine over the coming months. We thank both for their quick feedback on our 'back-of-the-envelope' calculations.

[3] We use a model explaining mass as the equivalent mass of energy here, i.e. Wheeler's idea of "mass without mass". Energy is force over a distance and, hence, we can distinguish between electromagnetic energy (and the equivalent mass) and some new strong energy or mass, which is defined in terms of some strong force and the related strong charge. Our interpretation of Wheeler's "mass without mass" theory is explained in a previous paper (https://vixra.org/abs/2001.0453).

[4] "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field. Half of it is magnetic potential energy and half is kinetic." (email from Dr. David Hestenes to the author dated 17 March 2019)

$$a = \sqrt{\frac{\sqrt{2} \cdot 2\mu}{\mathrm{q_e}\omega}} = 2 \cdot \sqrt{\frac{2\sqrt{2}\mu\hbar}{\mathrm{q_e E}}} \approx 0.8359278\ \mathrm{fm}$$

The difference between this calculated value (which used all of the precision of the CODATA values) and the PRad result is only about 0.005 fm[5], which is well within the statistical standard error of the measurement. Hence, it is a good result.

4. We now need to motivate the insertion of the √2 factor. We think there is some *real* magnetic moment here, which we denote as μ_L:

$$\mu_L = \sqrt{2} \cdot (1.41060679736 \times 10^{-26}\ \mathrm{J \cdot T^{-1}}) \approx 1.995 \times 10^{-26}\ \mathrm{J \cdot T^{-1}}$$

The subscript L in the μ_L notation stands for (orbital) angular momentum. A magnetic dipole will *precess* when placed in a magnetic field—which is what is being done when measuring the magnetic moment of a proton. We refer to Feynman[6] for an easy and very meaningful explanation of the relation between the magnitude of the actual – or imagined?[7] – angular momentum of a precessing magnet (L) and L_z (the *measured* quantum value) as:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{j(j+1)}}{j}$$

For j = ½, we get:

$$= \frac{\sqrt{1/2(1/2+1)}}{1/2} = 2 \cdot \sqrt{\frac{3}{4}} = \sqrt{3}$$

We need a √2 factor. Hence, the spin number must be one:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{1(1+1)}}{1} = \sqrt{2}$$

We know this assumption relates to the theoretical distinction between fermions and bosons. However, we will show the j = 1 assumption makes sense.

5. Because of the apparent randomness of this √2 factor, we must try the simpler approach to calculating the magnetic moment, which calculates the frequency from the $f = c/2\pi a$ formula:

$$\mu_L = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e} \frac{c}{2\pi a} \pi a^2 = \frac{\mathrm{q_e} c}{2} a$$

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[5] 0.831 – 0.836 = 0.005. We showed a result with seven digits to show the difference between this calculation and another value we will get out of another calculation (see Section 5).

[6] See: https://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S7

[7] The difference between actual and imagined here depends on one's *interpretation* of quantum-mechanical laws. From what we present in this article, it should be obvious to the reader that we like to think this magnitude is something real. However, such metaphysical questions should not be the concern of the reader: he or she should just check our calculations so as to *verify* them. The *interpretation* of the results is a different matter.

$$\Leftrightarrow a = \frac{2\mu_L}{q_e c} = \sqrt{2} \cdot \frac{2\mu}{q_e c} = \sqrt{2} \cdot 0.587 \times 10^{-15} \approx 0.83065344 \ldots \text{fm}$$

The result differs – slightly but significantly – from the result we obtained from using the Planck-Einstein relation for the frequency calculation (see Section 3). It is a *very* small difference. To be precise, it is, again, of the order of 0.005 fm. At the same time, this result is closer to the 0.831 PRad value: the difference is 0.000346656… fm only, which is less than 5% of the standard error of the PRad point estimate (0.007 fm).

6. In our calculations, we used the CODATA value for the magnetic moment of a proton in two different formulas for the radius, and we found the result is slightly different. While the two values do not differ significantly from the experimentally measured value for the proton radius – and, thereby, may be seen as a confirmation of the relevance of the PRad experiment – the two different values suggest we may think of some *unique* or *absolute* theoretical value for the magnetic moment. Indeed, because we have two equations for the radius *a* – and both of them involve μ_L – we can just equate them:

$$a = 2 \cdot \sqrt{\frac{2\mu_L \hbar}{q_e E}} = \frac{2\mu_L}{q_e c} \Leftrightarrow \sqrt{\frac{2\mu_L \hbar \cdot q_e^2 c^2}{q_e E \cdot \mu_L^2}} = 1$$

$$\Leftrightarrow \mu_L = \frac{2q_e}{m} \hbar \approx 2.02035 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}$$

We get a value that is *almost* 2, but not quite. We think of this as a coincidence. We can now calculate an exact theoretical value for the proton radius:

$$a = \frac{2\mu_L}{q_e c} = \frac{2}{q_e c} \cdot \frac{2q_e \hbar}{m} = 4 \cdot \frac{\hbar}{mc} \approx 4 \cdot (0.21 \ldots \text{fm}) \approx 0.8413564 \ldots \text{ fm}$$

This value is not within the 0.831 ± 0.007 fm interval, but it is well within the wider $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm interval.[8]

[8] We readily admit the insertion of the √2 factor needs further examination. We have a $\mu_L = 2q_e\hbar/m_p \approx 2.02\ldots$ J/T value for the magnetic proton which, we argue, differs from the CODATA value with a √2 factor because of precession. In contrast, the formula for the magnetic moment of an electron ($\mu_e = q_e\hbar/2m_e \approx 9.274$ J/T) gives us the CODATA value (apart from the anomaly, of course) *without the need for any correction factor because of precession*. If an electron is some ring current as well, then it must precess as well. We looked on the NIST site, but couldn't find much in terms of methodology. We sent an email to the NIST Public Affairs section with a request to guide us to the necessary materials in this regard.

7. We will now come back to the question of the spin number. Quantum-mechanical spin is expressed in units of $\hbar/2$ and, according to the Copenhagen interpretation of quantum mechanics, we should *not* try to think of it as a classical property—as something that has some *physical* meaning. We obviously disagree with this point of view. We think we can just use the classical L = *I*·ω expression and substitute *I* and ω for the angular mass and the angular frequency.[9] To calculate the angular mass, one must assume some form factor: a hoop, a disk, a sphere or a shell are associated with different form factors. Our electron model[10] assumes that the effective mass of the electron is spread over a circular disk. We can, therefore, calculate the angular momentum as:

$$\mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{\hbar}{mc} = \frac{\hbar}{2}$$

Hence, we may effectively refer to an electron as a spin-1/2 particle. However, we do not think of this property as some obscure 'intrinsic' property of an equally obscure 'pointlike' particle: we think of the electron as an actual disk-like structure with some *torque* on it. Its angular momentum is, therefore, *real*.[11] Likewise, we think of the magnetic moment as being equally real[12]:

$$\mu = \mathrm{I} \cdot \pi a^2 = \frac{\mathrm{q_e}c \cdot \pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \approx 9.274 \times 10^{-24}\ \mathrm{J} \cdot \mathrm{T}^{-1}$$

We think there is a confusion in regard to spin numbers and g-factors because we cannot directly measure the angular momentum: in real-life experiments, we measure the *magnetic moment*. Having said that, it is true we can combine the two formulas to get the g-factor that is usually associated with the spin of an electron[13]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We should now apply these ideas to the proton. The idea of a current ring – and the idea of precession, of course – strongly suggests we should, once again, think of the proton as a disk-like structure. However, not all of the mass is in the electromagnetic oscillation: we think half of it remains to be explained by what is referred to as the strong force (or, what amounts to the same, the idea of a strong

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[9] The reader should not confuse the *I* and I symbols. The first (*I* in *italics*) stands for angular mass (expressed in kg·m^2), while the second (I, normal type) is the symbol for current (expressed in C/s). We could have used different symbols, but we wanted to stick to the usual conventions. The reader will, of course, also not confuse the concepts of angular mass (*I*), also known as the moment of inertia, and angular momentum (L).

[10] See: https://vixra.org/abs/1905.0521.

[11] We will not engage in philosophical discussions here. We hope the reader understands what we want him/her to understand.

[12] The CODATA value for the magnetic moment includes the anomaly and is, therefore, slightly different from the theoretical value: $\mu_e \approx 9.285$ J/T. We think the difference between the theoretical and measured value is to be explained by a form factor: the circular point charge must have some (tiny) dimension and/or must have some (very tiny) non-zero rest mass. We believe the two letters of Gregory Breit to Gregory Breit to Isaac Rabi can easily be interpreted as Breit defending the idea that an intrinsic magnetic moment "of the order of $\alpha\mu_B$" is not anomalous at all. For more details on this conversation, see: Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga* , p. 222-223.

[13] We used vector notation (**boldface**) to draw attention, once again, to our *physical* interpretation of what might be going on: the *minus* sign (–) is there because, in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions.

charge).[14] We will, therefore, use a 1/4 rather than a 1/2 factor in the angular mass formula. This yields the following result:

$$\mathrm{L_p} = I_\mathrm{p} \cdot \omega = \frac{\mathrm{m_p} r_\mathrm{p}^2}{4} \cdot \frac{c}{r_\mathrm{p}} = \frac{\mathrm{m_p} c}{4} \cdot r_\mathrm{p} = \frac{\mathrm{m_p} c}{4} \cdot \frac{4\hbar}{\mathrm{m_p} c} = \hbar$$

Hence, our 'spin number' is equal to one. Most academics will cry wolf here: we cannot possibly believe a proton is a spin-one particle, can we? We think we can. We think there is no need for the concept of a spin number and a g-factor in a *realist* interpretation of quantum mechanics. We think of the angular momentum and the magnetic moment as being *real* and, hence, whatever else is being calculated – be it a spin number or a g-factor – is not very relevant. Worse, we think it *confuses* rather than clarifies the analysis. We, therefore, think our calculation of L_p is consistent. We also think it is consistent with the use of the √2 factor – as opposed to a √3 factor – to calculate what we think of as a *real* magnetic moment of a proton (μ_p).

We should, of course, relate this to the usual conventions. We will, therefore, do some calculations involving a g-factor. Instead of the *Bohr magneton* $\mu_B = q_e\hbar/2m_e$, we should use the *nuclear magneton* $\mu_N = q_e\hbar/2m_p$. We get the following result:

$$\boldsymbol{\mu}_\mathbf{L} = \mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m_p}}\right)\mathbf{L} \Leftrightarrow \frac{2\mathrm{q_e}}{\mathrm{m_p}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\hbar \Leftrightarrow \mathrm{g} = 4$$

That is, of course, a strange number: the CODATA value is about 5.5857. However, this result depends on the use of a theoretical $\hbar/2$ value for the angular momentum. It also uses the CODATA value for the magnetic moment—as opposed to our μ_L value, which is the CODATA value corrected for precession. Hence, the CODATA calculation of the g-factor is this:

$$\mu_\mathrm{p} = \mathrm{g_p}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g_p} = \frac{4\mu_\mathrm{p}\mathrm{m_p}}{\hbar\mathrm{q_e}} = 5.58569\ldots$$

We get a slightly different value when we insert our newly found theoretical value for the magnetic moment:

$$\mu_\mathrm{p} = \mathrm{g_p}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g_p} = \frac{4\mu_\mathrm{p}\mathrm{m_p}}{\hbar\mathrm{q_e}} = \frac{4}{\sqrt{2}} \cdot \frac{\mu_\mathrm{L}\mathrm{m_p}}{\hbar\mathrm{q_e}} = \frac{4}{\sqrt{2}} \cdot \frac{2\mathrm{q_e}\hbar}{\mathrm{m_p}} \cdot \frac{\mathrm{m_p}}{\hbar\mathrm{q_e}} = \frac{8}{\sqrt{2}} = 5.65685\ldots$$

How can we explain the difference?

[14] See: https://vixra.org/abs/2001.0453.

8. The difference of about 0.071 (about 1.2%) is not surprising: the difference is of the same order of magnitude as the difference between our theoretical value for the radius – which is based on the assumption of a pointlike charge – and the actually measured radius. We think this difference confirms both the theory as well as the PRad measurement. We anticipate theorists and experimenters to argue about the next digit of the anomalous magnetic moment of a proton in pretty much the same way as they have been arguing about the anomalous magnetic moment of an electron. We think both 'anomalies' are there because of the mathematical idealization in our assumptions: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever.[15]

Why not? We can only give a philosophical answer here: something that has no dimension whatsoever probably exists in our mind only. Something real – like a charge – *must* have *some* dimension.

From what we write above, the reader will understand that we think some of the generalizations in quantum physics – most notably, the concept of bosons – are not necessary to understand Nature.

END

[15] See our paper on Consa's calculations of the anomalous magnetic moment, which also references our paper(s) on the topic: https://vixra.org/abs/2001.0264.