Refutation of Tarski’s classical relevant logic

Abstract: We evaluate an equation for relevance logic and two axioms for the extension, all of which are not tautologous. This refutes Tarski’s classical relevance logic and forms a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, , , ⊓ ; \ Not And;
> Imply, greater than, ⇒, ↦, ⋄, ⊃, ⊻ ; < Not Imply, less than, ∈, ⊊, ⊂, ⊼, ⋉ ;
≡ Equivalent, ≡, :=, ⇔, ↔, ≡, ≍, = ; @ Not Equivalent, ≠, ⊪;
% possibility, for one or some, ∃, ∀, Δ, L ; # necessity, for every or all, ∀, □, D ;
(z=z) T as tautology, T, ordinal 3; (z@z) F as contradiction, Ø, Null, ⊥, zero;
(%z>z#z) N as non-contingency, A, ordinal 1; (%z<z#z) C as contingency, ∇, ordinal 2;
~( y < x) ( x ≤ y), ( x ≤ y), ( x ≤ y) ; (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.


Abstract Tarski’s classical relevant logic TR arises from his work on the foundations of the calculus of relations and on first-order logic restricted to finitely many variables. The theorems of TR are defined here as the formulas whose translations into first-order logic of binary relations can be proved using no more than four variables from the assumptions that all the relations are dense and commute under composition. Its rules are determined similarly. … The class of model structures characteristic for TR is the class of atom structures of atomic commutative dense relation algebras. An equation is true in every commutative dense relation algebra if and only if it can be established by a proof in first order logic restricted to four variables that the equation is true when formulas are interpreted as relations.

3 Extending L to L⁺ TarSKI and Givant extend L to L⁺ by adding two binary and two unary operators that act on relation symbols and produce new relation symbols. … Predicates obtained in distinct ways are distinct, so, for example,

if A+B=C+D then A=C and B=D.  (3.0.1)

LET  p, q, r, s: A, B, C or x, D or y.

((p+q)=(r+s))>((p=r)&(q=s)) ;
TTTT  TFFF  TFFT  TFFT  (3.0.2)

Table 4  Definitional axioms for extension L⁺

[xA+By ⇔ xAy ∨ xBy]  (DI)  (3.2.2.1)
((r&p)+(q&s))=((r&(p&s))+(r&(q&s))) ;
TTTT  TFFT  TFFF  TTTT  (3.2.2.2)

...
For relevance logic L, Eq. 3.0.2 is not tautologous. For extension L', definitional axioms 3.2.2.2 and 3.2.4.2 are not tautologous. This refutes relevance logic L and its extension L'.