on $\Lambda$-generalized continuous functions*

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Abstract

In this paper, we introduce a new class of continuous functions as an application of $\Lambda$-generalized closed sets (namely $\Lambda_g$-closed set, $\Lambda$-$g$-closed set and $g\Lambda$-closed set) namely $\Lambda$-generalized continuous functions (namely $\Lambda_g$-continuous, $\Lambda$-$g$-continuous and $g\Lambda$-continuous) and study their properties in topological space.

1 Introduction and Preliminaries

Levine [7] introduced $g$-closed set. Maki [8] introduced the notion of $\Lambda$-sets in topological spaces. A subset $A$ of a topological space $(X, \tau)$ is called a $\Lambda$-set if it coincides with its kernel (the intersection of all open supersets of $A$). In [1], Arenas et al. introduced the notions of $\lambda$-open sets, and $\lambda$-closed sets and presented fundamental results for these sets. They also introduced [1] $\lambda$-continuity, which is weaker than continuity. Recently, M. Caldas, S. Jafari and T. Noiri [3] introduced $\Lambda$-generalized closed sets in topological space. The aim of this paper is to introduce a weak form of continuous functions called $\Lambda$-generalized continuous functions. Moreover, the relationships and properties of $\Lambda$-generalized continuous functions are obtained.

Throughout this paper, by $(X, \tau)$ and $(Y, \sigma)$ (or $X$ and $Y$) we always mean topological spaces. Let $A$ be a subset of $X$. We denote the interior, the closure and the complement of a set $A$ by $Int(A)$, $Cl(A)$ and $X \setminus A$ or $A^c$, respectively. A subset $A$ of a space $(X, \tau)$ is

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called $\lambda$-closed \[1\] if $A = L \cap D$, where $L$ is a $\Lambda$-set and $D$ is a closed set. The intersection of all $\lambda$-closed sets containing a subset $A$ of $X$ is called the $\lambda$-closure of $A$ and is denoted by $Cl_\lambda(A)$. The complement of a $\lambda$-closed set is called $\lambda$-open. We denote the collection of all $\lambda$-open sets by $\lambda O(X, \tau)$.

Recall that a subset $A$ of a topological space $(X, \tau)$ is called generalized closed (briefly $g$-closed) \[7\] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$. $B$ is a $g$-open set of $(X, \tau)$ if and only if $B^c$ is $g$-closed.

**Definition 1** A subset $A$ of a topological space $(X, \tau)$ is called a $\Lambda$-generalized closed, briefly $\Lambda_g$-closed \[3\], (resp. $\lambda$-$g$-closed, $g\lambda$-closed) if $Cl(A) \subseteq U$ (resp. $Cl_\lambda(A) \subseteq U$, $Cl_\lambda(A) \subseteq U$) whenever $A \subseteq U$ and $U$ is $\lambda$-open (resp. $U$ is $\lambda$-open, $U$ is open) in $(X, \tau)$.

**Remark 1.1** From the above definitions, we have the following.

1. $\Lambda_g$-closed sets and $\lambda$-closed sets are independent concepts.
2. $\lambda$-$g$-closed sets and $g$-closed sets are independent concepts.
3. $\lambda$-closed sets and $g$-closed sets are also independent concepts.

From the above definitions and remark 1.1, we have the following diagram.

\[
\begin{array}{ccc}
d\text{closed} & \Rightarrow & \text{$\Lambda_g$-closed} \\
\downarrow & & \downarrow \\
\text{$\lambda$-closed} & \Rightarrow & \text{$\Lambda$-$g$-closed} \\
& & \downarrow \\
& & \text{$g\lambda$-closed}
\end{array}
\]

**Example 1.2** (i) Let $X = \{a, b, c\}$ with a topology $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Thus $\lambda O(X, \tau) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Take $A = \{a, c\}$. Observe that $A$ is a $g$-closed set but it is not $\Lambda$-$g$-closed.

(ii) Let $X = \{a, b, c\}$ with a topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then, $A = \{b\}$ is a $\lambda$-closed set but it is not $g$-closed.

(iii) Let $X = \{a, b, c\}$ with a topology $\tau = \{\emptyset, \{a\}, X\}$. Then, $A = \{a, b\}$ is a $\Lambda_g$-closed set but it is not $\lambda$-closed.
Definition 2 A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(1) $g$-continuous [7] if $f^{-1}(V)$ is $g$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

(2) $\lambda$-continuous [1] if $f^{-1}(V)$ is $\lambda$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

2 $\Lambda$-generalized continuous functions

We introduce the following notions:

Definition 3 A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(1) $\Lambda_g$-continuous if $f^{-1}(V)$ is $\Lambda_g$-closed in $X$, for every closed set in $Y$.

(2) $\Lambda$-continuous if $f^{-1}(V)$ is $\Lambda$-closed in $X$, for every closed set in $Y$.

(3) $g\Lambda$-continuous if $f^{-1}(V)$ is $g\Lambda$-closed in $X$, for every closed set in $Y$.

Example 2.1 Let $X = \{a, b, c, d\} = Y$, $\tau = \{\phi, X, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b = f(b), f(c) = c, f(d) = d$. Then $f$ is $\Lambda_g$-continuous, $\Lambda$-continuous and $g\Lambda$-continuous.

Proposition 2.2 Every continuous function is $\Lambda_g$-continuous (resp. $\Lambda$-continuous, $g\Lambda$-continuous).

Proof. By [3], every closed set is $\Lambda_g$-closed (resp $\Lambda$-closed, $g\Lambda$-closed) and the proof follows.

Example 2.3 Let $X = Y = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\sigma = \{\phi, Y, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = b, f(c) = c, f(d) = d$. Then $f$ is $\Lambda_g$-continuous, $\Lambda$-continuous and $g\Lambda$-continuous but not continuous.

Proposition 2.4 Every $\Lambda_g$-continuous function is $g$-continuous.

Proof. It follows from the fact that every $\Lambda_g$-closed set is $g$-closed set [3].
Example 2.5 The function $f$ in Example 2.3 with $\tau = \{\phi, X, \{b\}, \{b, c\}, \{a, b, c\}\}, \sigma = \{\phi, Y, \{a\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ is $g$-continuous but not $\Lambda_g$-continuous since for the closed set $U = \{b, d\}$ in $(Y, \sigma)$, $f^{-1}(U) = \{a, b, d\}$ which is not $\Lambda_g$-closed in $(X, \tau)$.

Proposition 2.6 Every $\lambda$-continuous function and $\Lambda_g$-continuous function are $\Lambda$-$g$-continuous function.

Proof. By [3], every $\lambda$-closed set is $\Lambda$-$g$-closed set and every $\Lambda_g$-closed set is $\Lambda$-$g$-closed set, the proof follows.

Example 2.7 Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.3.

(i) Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(c) = c$, $f(b) = d = f(d)$. Then $f$ is $\Lambda$-$g$-continuous but not $\lambda$-continuous since for the closed set $U = \{c, d\}$ in $(Y, \sigma)$, $f^{-1}(U) = \{b, c, d\}$ which is not $\lambda$-closed in $(X, \tau)$.

(ii) Define a function $f : X \to Y$ by $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then $f$ is $\Lambda$-$g$-continuous but not $\Lambda_g$-continuous since for the closed set $U = \{d\}$ in $(Y, \sigma)$, $f^{-1}(U) = \{c\}$ which is not $\Lambda_g$-$g$-closed in $(X, \tau)$.

Remark 2.8 (1) $\Lambda_g$-continuous and $\lambda$-continuous are independent.

(2) $\Lambda$-$g$-continuous and $g$-continuous are independent.

(3) $\lambda$-continuous and $g$-continuous are independent.

Example 2.9 (i) The function $f$ in Example 2.7(i) is $\Lambda_g$-continuous but not $\lambda$-continuous.

(ii) Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.5. Then $f$ in Example 2.7(ii) is $\lambda$-continuous but not $\Lambda_g$-continuous.

(iii) $f$ is $\lambda$-continuous but not $g$-continuous.

(iv) $f$ is $\Lambda$-$g$-continuous but not $g$-continuous.

(v) Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.5 and the function $f$ be an identity function from $X$ to $Y$. Then $f$ is $g$-continuous but neither $\Lambda$-$g$-continuous nor $\lambda$-continuous.
We get the following diagram:

\[
\begin{array}{c}
\text{continuous} \Rightarrow \Lambda_g\text{-continuous} \Rightarrow g\text{-continuous} \\
\downarrow \quad \downarrow \quad \downarrow \\
\lambda\text{-continuous} \Rightarrow \Lambda-g\text{-continuous} \Rightarrow g\Lambda\text{-continuous}
\end{array}
\]

3 Properties of \(\Lambda\)-generalized continuous functions

**Theorem 3.1** If a function \(f : (X, \tau) \to (Y, \sigma)\) is \(\Lambda_g\)-continuous and \(X\) is \(T_1\) then \(f\) is continuous.

**Proof.** Let \(f\) be \(\Lambda_g\)-continuous and \(X\) be \(T_1\). Assume that \(V\) is closed in \(Y\). Hence \(f^{-1}(V)\) is \(\Lambda_g\)-closed set in \(X\). Since every \(\Lambda_g\)-closed is closed in a \(T_1\) space \(X\) [3], then \(f^{-1}(V)\) is closed set in \(X\). This shows that \(f\) is continuous.

**Corollary 3.2** If a function \(f : (X, \tau) \to (Y, \sigma)\) is \(\Lambda_g\)-continuous and \(X\) is \(T_1\) then \(f\) is \(\lambda\)-continuous.

**Theorem 3.3** If a function \(f : (X, \tau) \to (Y, \sigma)\) is \(\Lambda\)-\(g\)-continuous and \(X\) is \(T_0\) then \(f\) is \(\lambda\)-continuous.

**Proof.** Let \(f\) be \(\Lambda\)-\(g\)-continuous and \(X\) be \(T_0\). Let \(V\) be closed in \(Y\). \(f^{-1}(V)\) is \(\Lambda\)-\(g\)-closed in \(X\). Since \(\Lambda\)-\(g\)-closed is \(\lambda\)-closed in a \(T_0\) space \(X\) [9], then \(f^{-1}(V)\) is \(\lambda\)-closed in \(X\). This shows that \(f\) is \(\lambda\)-continuous.

**Definition 4** A function \(f : (X, \tau) \to (Y, \sigma)\) is said to be:

(i) \(\Lambda_g\)-irresolute if \(f^{-1}(V)\) is \(\Lambda_g\)-closed in \(X\) for every \(\Lambda_g\)-closed set \(V\) in \(Y\).

(ii) \(\Lambda\)-\(g\)-irresolute if \(f^{-1}(V)\) is \(\Lambda\)-\(g\)-closed in \(X\) for every \(\Lambda\)-\(g\)-closed set \(V\) in \(Y\).

(iii) \(g\Lambda\)-irresolute if \(f^{-1}(V)\) is \(g\Lambda\)-closed in \(X\) for every \(g\Lambda\)-closed set \(V\) in \(Y\).

Recall that a function \(f : (X, \tau) \to (Y, \sigma)\) is said to be \(\lambda\)-closed if \(f(F)\) is \(\lambda\)-closed in \(Y\) for every \(\lambda\)-closed set \(F\) of \(X\).
Lemma 3.4 [3]. A function \( f : (X, \tau) \to (Y, \sigma) \) is \( \lambda \)-closed if and only if for each subset \( B \) of \( Y \) and each \( U \in \lambda O(X, \tau) \) containing \( f^{-1}(B) \), there exists \( V \in \lambda O(Y, \sigma) \) such that \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Theorem 3.5 Let \( f : (X, \tau) \to (Y, \sigma) \) be a continuous \( \lambda \)-closed function. Then \( f \) is \( \Lambda_{g} \)-irresolute.

*Proof.* Let \( B \) be \( \Lambda_{g} \)-closed in \( (Y, \sigma) \) and \( U \) a \( \lambda \)-open set of \( (X, \tau) \) containing \( f^{-1}(B) \). Since \( f \) is \( \lambda \)-closed, by Lemma 3.4 there exists a \( \lambda \)-open set \( V \) of \( (Y, \sigma) \) such that \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \). Since \( B \) is \( \Lambda_{g} \)-closed in \( (Y, \sigma) \), \( Cl(B) \subseteq V \) and hence \( f^{-1}(B) \subseteq f^{-1}(Cl(B)) \subseteq f^{-1}(V) \subseteq U \). Since \( f \) is continuous, \( f^{-1}(Cl(B)) \) is closed and hence \( Cl(f^{-1}(B)) \subseteq U \). This shows that \( f^{-1}(B) \) is \( \Lambda_{g} \)-closed in \( (X, \tau) \). Therefore \( f \) is \( \Lambda_{g} \)-irresolute.

Theorem 3.6 If a function \( f : (X, \tau) \to (Y, \sigma) \) is \( \Lambda_{g} \)-irresolute and \( Y \) is \( T_{1} \) then \( f \) is \( \Lambda_{g} \)-continuous.

*Proof.* Let \( f \) be \( \Lambda_{g} \)-irresolute and \( Y \) be \( T_{1} \). Suppose \( V \) is \( \Lambda_{g} \)-closed in \( Y \). Then \( f^{-1}(V) \) is \( \Lambda_{g} \)-closed set in \( X \). Since \( Y \) is \( T_{1} \), \( V \) is closed in \( Y \). Thus \( f \) is \( \Lambda_{g} \)-continuous.

Theorem 3.7 If a function \( f : (X, \tau) \to (Y, \sigma) \) is \( \Lambda_{g} \)-irresolute and \( Y \) is \( T_{0} \) then \( f \) is \( \Lambda_{g} \)-continuous.

*Proof.* Let \( f \) be \( \Lambda_{g} \)-irresolute, \( Y \) a \( T_{0} \) space and \( V \) be \( \Lambda_{g} \)-closed in \( Y \). Then \( f^{-1}(V) \) is \( \Lambda_{g} \)-closed set in \( X \). Since \( Y \) is \( T_{0} \), \( V \) is closed in \( Y \). Thus \( f \) is \( \Lambda_{g} \)-continuous.

Theorem 3.8 If \( f : (X, \tau) \to (Y, \sigma) \) is a \( \lambda \)-irresolute bijection and \( f \) is \( \lambda \)-open, then \( f \) is \( \Lambda_{g} \)-irresolute.

*Proof.* Let \( V \) be \( \Lambda_{g} \)-closed and let \( f^{-1}(V) \subseteq U \), where \( U \in \lambda O(X, \tau) \). Clearly, \( V \subseteq f(U) \). Since \( f(U) \in \lambda O(X, \tau) \) and since \( V \) is \( \Lambda_{g} \)-closed in \( Y \), then \( Cl_{\lambda}(V) \subseteq f(U) \) and thus \( f^{-1}(Cl_{\lambda}(V)) \subseteq U \). Since \( f \) is \( \lambda \)-irresolute and \( Cl_{\lambda}(V) \) is a \( \lambda \)-closed set, then \( f^{-1}(Cl_{\lambda}(V)) \) is \( \lambda \)-closed in \( X \). Thus \( Cl_{\lambda}(f^{-1}(V)) \subseteq Cl_{\lambda}(f^{-1}(Cl_{\lambda}(V))) = f^{-1}(Cl_{\lambda}(V)) \subseteq U \). Therefore, \( Cl_{\lambda}(f^{-1}(V)) \subseteq U \). So, \( f^{-1}(V) \) is \( \Lambda_{g} \)-closed and \( f \) is a \( \Lambda_{g} \)-irresolute bijection.
Definition 5 A topological space \((X, \tau)\) is called:

1. a \(T_g\Lambda\)-space if every \(g\Lambda\)-closed is \(g\)-closed.
2. a \(T_{\Lambda g}\)-space if every \(\Lambda-g\)-closed is \(\Lambda_g\)-closed.

Recall that a function \(f : (X, \tau) \to (Y, \sigma)\) is said to be \(g\)-irresolute [2] if \(f^{-1}(V)\) is \(g\)-closed in \(X\) for every \(g\)-closed set \(V\) in \(Y\). It is clear that a function \(f : (X, \tau) \to (Y, \sigma)\) is \(ge\)-irresolute if and only if \(f^{-1}(V)\) is \(g\)-open in \(X\) for every \(g\)-open set \(V\) in \(Y\).

Theorem 3.9 If a function \(f : (X, \tau) \to (Y, \sigma)\) is \(g\)-irresolute and closed, then \(f\) is \(ge\)-irresolute.

Proof. It follows immediately from ([4], Proposition 2).

Theorem 3.10 If a function \(f : (X, \tau) \to (Y, \sigma)\) is \(g\Lambda\)-irresolute and \(X\) is a \(T_g\Lambda\)-space, then \(f\) is \(ge\)-irresolute.

Proof. Let \(f\) be \(g\Lambda\)-irresolute and \(V\) a \(g\)-closed set in \(X\). Then \(V\) is \(g\Lambda\)-closed in \(Y\). Since \(f\) is \(g\Lambda\)-irresolute, \(f^{-1}(V)\) is \(g\Lambda\)-closed in \(X\). But \(X\) is a \(T_g\Lambda\)-space. Therefore \(f^{-1}(V)\) is \(g\)-closed in \(X\) and this implies that \(f\) is \(ge\)-irresolute.

Remark 3.11 The condition that \(X\) is a \(T_g\Lambda\)-space cannot be omitted in above theorem as shown in the following example.

Example 3.12 Let \(X = \{a, b, c, d\}\), \(\tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\} \) and \(\sigma = \{\phi, Y, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}\). Note that \((X, \tau)\) is not a \(T_g\Lambda\)-space. Let \(f : (X, \tau) \to (Y, \sigma)\) be the function defined as follows \(f(a) = b\), \(f(b) = a\), \(f(c) = d\) and \(f(d) = c\). Then \(f\) is \(g\Lambda\)-irresolute but not \(ge\)-irresolute, since \(f^{-1}(\{d\}) = \{c\}\) is not \(g\)-closed in \((X, \tau)\).

Theorem 3.13 If a function function \(f : (X, \tau) \to (Y, \sigma)\) is \(\Lambda\)-irresolute and \(X\) is a \(T_{\Lambda g}\)-space then \(f\) is \(\Lambda_g\)-irresolute.

Proof. Let \(B\) be any \(\Lambda_g\)-closed set in \(Y\). Then \(B\) is \(\Lambda-g\)-closed in \(Y\). Since, \(f\) is \(\Lambda\)-irresolute, then \(f^{-1}(B)\) is \(\Lambda-g\)-closed in \(X\). But \(X\) is \(T_{\Lambda g}\)-space. Therefore \(f^{-1}(B)\) is \(\Lambda_g\)-closed in \(X\) which implies that \(f\) is \(\Lambda_g\)-irresolute.
Remark 3.14 The condition that $X$ is a $T_{\lambda}$-space can not be omitted in Theorem 3.13 as it is shown in our next example.

Example 3.15 Let $f$ be as in Example 3.12. Then $f$ is $\Lambda$-irresolute but not $\Lambda_{g}$-irresolute, where $X$ is not $T_{\lambda}$-space. $f^{-1}(\{d\}) = \{c\}$ is not $\Lambda_{g}$-closed in $(X, \tau)$.

We recall that the space $X$ is called a $\lambda$-space [1] if the set of all $\lambda$-open subsets form a topology on $X$. Clearly a space $X$ is a $\lambda$-space if and only if the intersection of two $\lambda$-open sets is $\lambda$-open. An example of a $\lambda$-space is a $T_{\frac{1}{2}}$-space, where a space $X$ is called $T_{\frac{1}{2}}$ [5] if every singleton is open or closed.

Theorem 3.16 If $f_i : (X, \tau_i) \rightarrow (Y, \sigma_i)(i \in I)$ is a family of functions, where $X$ is a $\lambda$-space and $Y$ is any topological space, then every $f_i$ is $\Lambda$-continuous.

Proof. It follows from ([9], Theorem 2.4).

Theorem 3.17 (i) If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\Lambda$-continuous then $f(Cl_{\lambda}(A)) \subseteq Cl_{\lambda}(f(A))$ for every $A$ of $X$.

(ii) If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\Lambda$-irresolute then for every subset $A$ of $X$, $f(Cl_{\lambda-g}(A)) \subseteq Cl_{\lambda}(f(A))$ (where $Cl_{\lambda-g}(A)$ is the intersection of the smallest $\Lambda$-$g$-closed set containing $A$).

Proof. (i) It follows from the fact that every $\lambda$-continuous is $\Lambda$-$g$-continuous.

(ii) If $A \subseteq X$, then consider $Cl_{\lambda}(f(A))$ which is $\lambda$-closed in $Y$. Thus by Definition 4, $f^{-1}Cl_{\lambda}(f(A))$ is $\Lambda$-$g$-closed in $X$. Furthermore, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(Cl_{\lambda}(f(A)))$. Therefore $Cl_{\lambda-g}(A) \subseteq f^{-1}(Cl_{\lambda}(f(A)))$ and consequently, $f(Cl_{\lambda-g}(A)) \subseteq f(f^{-1}(Cl_{\lambda}(f(A)))) \subseteq Cl_{\lambda}(f(A))$.

Theorem 3.18 If a map $f : X \rightarrow Y$ is $\Lambda_{g}$-irresolute, then it is $\Lambda_{g}$-continuous but not conversely.

Proof. Since every closed set is $\Lambda_{g}$-closed, it is proved that $f$ is $\Lambda_{g}$-continuous. The converse need not be true as it is seen from the following example.
Example 3.19 Let $X = Y = \{a, b, c, d\}$, $\sigma = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}\}$, $\tau = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d = f(d), f(b) = b$ and $f(c) = c$. Then $f$ is $\Lambda_g$-continuous but not $\Lambda_g$-irresolute.

Theorem 3.20 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_1$ space. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $\Lambda_g$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are $\Lambda_g$-continuous.

Proof. Let $F$ be any closed set in $Z$. Since $g$ is $\Lambda_g$-continuous, $g^{-1}(F)$ is $\Lambda_g$-closed in $Y$. But $Y$ is a $T_1$-space and so $g^{-1}(F)$ is closed in $Y$. Since $f$ is $\Lambda_g$-continuous, $f^{-1}(g^{-1}(F))$ is $\Lambda_g$-closed in $X$. Hence, $g \circ f$ is $\Lambda_g$-continuous.

Theorem 3.21 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_1$ space.

1. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $\lambda$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ is $\lambda$-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is $\Lambda_g$-continuous.

2. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $g$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ is $g$-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is $\Lambda_g$-continuous.

3. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $\Lambda_g$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ is $\Lambda_g$-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is $\Lambda_g$-continuous.

Proof. Similar to the proof of Theorem 3.20.

Theorem 3.22 Let $(X, \tau)$ and $(Z, \eta)$ be any topological spaces and $(Y, \sigma)$ be a $T_0$ space. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $\lambda$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ is $\lambda$-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ is $\Lambda_g$-continuous.

Proof. Let $V$ be any closed set in $Z$. Since $g$ is $\Lambda_g$-continuous, $g^{-1}(V)$ is $\Lambda_g$-closed in $Y$. But $Y$ is a $T_0$-space and so $g^{-1}(V)$ is $\lambda$-closed in $Y$. Since $f$ is $\lambda$-irresolute, $f^{-1}(g^{-1}(V))$ is $\lambda$-closed in $X$. Hence, $g \circ f$ is $\lambda$-continuous.

Theorem 3.23 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_0\Lambda$ space. The composition $g \circ f : (X, \tau) \to (Z, \eta)$ is $g$-continuous function where $f : (X, \tau) \to (Y, \sigma)$ is $gc$-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ is $g\Lambda$-continuous.
Proof. This follows from the definitions.

**Theorem 3.24** Let \((X, \tau)\) and \((Z, \eta)\) be topological spaces and \((Y, \sigma)\) be a \(T_{\Lambda}\) space. The composition \(g \circ f : (X, \tau) \rightarrow (Z, \eta)\) is \(\Lambda_g\)-continuous function, where \(f : (X, \tau) \rightarrow (Y, \sigma)\) is \(\Lambda_g\)-irresolute and \(g : (Y, \sigma) \rightarrow (Z, \eta)\) is \(\Lambda\)-\(g\)-continuous.

Proof. This follows from definitions.

Recall that a space \(X\) is called locally indiscrete if and only if every open set is closed if and only if every \(\lambda\)-open set of \(X\) is open in \(X\).

Finally, we get the following diagram:

\[
\begin{aligned}
\text{continuous} & \Rightarrow \Lambda_g\text{-continuous} & \Rightarrow g\text{-continuous} \\
S_1 & \updownarrow & T_{\Lambda_g} & \updownarrow & T_g & \updownarrow \\
\lambda\text{-continuous} & \Rightarrow \Lambda\text{-}g\text{-continuous} & \Rightarrow g\Lambda\text{-continuous}
\end{aligned}
\]

where \(S_1\) is a locally indiscrete space.

**References**


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