Remarks on Birch and Swinnerton-Dyer Conj ungture

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January 19, 2020

Abstract

These short remarks show derivation of Birch and Swinnerton-Dyer conj ungture. As a consequence new one resulting constant free equality of Birch and Swinnerton-Dyer conj ungture proposed.

We know, that Euler product in general equals to the infinitesimal sum. Therefore, for big enough $N$ finite Euler product can be approximated as finite sum in the same way as follow:

$$\prod_{p<N} \frac{N_p}{p} = \prod_{p<N} P(p, 1) = \prod_{p<N} \frac{1}{1 - \frac{pN_p - p}{N_p}} \approx \sum_{n<N} \frac{1}{n} \frac{nN_n - n^2}{N_n}$$ (1)

where $N_p$ is the number of points modulo $p$ for a large number of primes $p$ on elliptic curves whose rank $r$ was known and $N_n$ is the number of points modulo $n$ for a large number of positive integers $n$ on elliptic curves for the same rank $r$. On the other hand, the finite sum can be approximated as integral by using Euler–Maclaurin formula [1]:

$$\sum_{n<N} \frac{1}{n} \frac{nN_n - n^2}{N_n} \approx \int_{x=1}^{N} \frac{N_x - x}{N_x} dx$$ (2)

Let’s deside, that $N_x = x/\left(1 - C^{r(\log x)^{r-1}}\right)$ dependance is valid$^1$. After inserting this expression into the integral we obtain:

$$\int_{x=1}^{N} \frac{N_x - x}{N_x} dx = C \int_{x=1}^{N} \frac{r(\log x)^{r-1}}{x} dx = C(\log N)^r$$ (3)

Finally, expanding $N_x$ near 0 and taking just first term, we can write for big enough $N$:

$$\prod_{p<N} \frac{N_p}{p} \approx \frac{N_N \log N}{r}$$

References


$^1$It can be proved by numeric experiment