Proof of the Riemann hypothesis

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Abstract

Up to now, I have tried to expand this equation and prove Riemann hypothesis with the equation of cos, sin, but the proof was impossible.

However, I realized that a simple formula before expansion can prove it.

The real value is zero only when the real part of s is 1/2. Non-trivial zeros must always have a real value of zero.

The real part of s being 1/2 is the minimum requirement for s to be a non-trivial zeros.

This is true whether the imaginary value increases or decreases to the limit.

key words
Riemann hypothesis, non-trivial zeros, 1/2, minimum requirement, to the limit

1 introduction

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{s\pi}{2}\right)\Gamma(1-s)\zeta(1-s) \tag{1}$$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i14.1347\} = -0.950558 - 0.310547i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i21.022\} = -0.904282 + 0.426936i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i25.0109\} = -0.784761 - 0.619798i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i30.4249\} = -0.475849 + 0.879527i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i32.9351\} = -0.410261 - 0.911968i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i37.58618\} = -0.832147 + 0.554555i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i40.91872\} = -0.917431 + 0.397894i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i43.32707\} = -0.275249 - 0.961373i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i48.00515\} = 0.130432 + 0.991457i$

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$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i49.77383\} = -0.579292 - 0.81512i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i52.97032\} = -0.867736 - 0.497025i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i56.44625\} = -0.752855 + 0.658186i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i315.4756\} = -0.286121 - 0.958193i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i1393.4334\} = 0.973556 - 0.228449i$
$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i74920.8275\} = -0.827399 - 0.561615i$

From the above calculation, in Euler's formula Eq.(1), $\zeta(s)$=0 (s is non-trivial zeros) is not from $2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)$ but $\zeta(1-s) = 0$.

$$\zeta(s) = \zeta(1-s) = 0 \tag{2}$$

At the non-trivial zeros, $\zeta(s) = \zeta(1-s) = 0$ holds. in this case. Eq.(10)=0, Eq(11)=0, and $\eta(s) = \eta(1-s) = 0$ holds.

$$\eta(1-s) = (1 - \frac{2}{2^{1-s}})\zeta(1-s) = \frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s) = \frac{2-2^{s+1}}{2}\zeta(1-s) = (1-2^s)\zeta(1-s) \tag{3}$$

$$\eta(s) + \frac{2}{2^s}\zeta(s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{4}$$

$$\eta(s) = \frac{2^s-2}{2^s}\frac{2^{1-s}}{2^{1-s}-2}\eta(1-s) = \frac{2^s-2}{2^s}\zeta(1-s) = (1-\frac{2}{2^s})\zeta(1-s) = 0 \tag{5}$$

$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i25.0109\} = 0.0000600703 - 0.0000774542i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i30.4249\} = 2.30973 \times 10^{-7} - 0.0000678699i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i32.9351\} = -9.25931 \times 10^{-6} - 0.000117068i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i37.5862\} = -0.0000437932 - 0.0000195875i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i40.9187\} = -0.0000173311 + 0.0000661198i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i74919.0752\} = -0.0000827382 - 0.000177009i$
$\{\frac{2^s-2}{2^s}\zeta(1-s)\}, \{s = 1/2 + i74920.8275\} = -0.0000166426 + 8.31396 \times 10^{-6}i$

From the above calculation, $\eta(s)$=0 (s is non-trivial zeros).

$$\eta(1-s) = \frac{2^{1-s}-2}{2^{1-s}}\frac{2^s}{2^s-2}\eta(s) = \frac{2^{1-s}-2}{2^{1-s}}\zeta(s) = (1-\frac{2}{2^{1-s}})\zeta(s) = 0 \tag{6}$$

$$\zeta(s) = \frac{2^s}{2^s-2}\eta(s) = (\frac{2^s-2+2}{2^s-2})\eta(s) = (1+\frac{2}{2^s-2})\eta(s) \tag{7}$$

$$= (1+\frac{2}{2^s}\frac{2^s}{2^s-2})\eta(s) = \eta(s) + \frac{2}{2^s}\zeta(s) = \eta(s) + \frac{2}{2^s}[\eta(s) + \frac{2}{2^s}\zeta(s)] \tag{8}$$

$$= \eta(s) + \frac{2}{2^s}[\eta(s) + \frac{2}{2^s}(\eta(s) + \frac{2}{2^s}\zeta(s))] = \eta(s) + \frac{2}{2^s}\eta(s) + (\frac{2}{2^s})^2\eta(s) + (\frac{2}{2^s})^3\zeta(s) \tag{9}$$

$$= \eta(s)[1 + \frac{2}{2^s} + (\frac{2}{2^s})^2] + (\frac{2}{2^s})^3\zeta(s) \neq \eta(s)[\frac{1-(\frac{2}{2^s})^k}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^{k+1}\zeta(s) \tag{10}$$

when k=2 (If the formula is a geometric sequence and is the same up to the k-th term, = holds.)

$$\neq \eta(s)[\frac{1-(\frac{2}{2^s})^2}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^3\zeta(s) = (1-\frac{2}{2^s})\zeta(s)[\frac{1-(\frac{2}{2^s})^2}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^3\zeta(s) \tag{11}$$

$$= \zeta(s)[1 - (\frac{2}{2^s})^2 + (\frac{2}{2^s})^3] \tag{12}$$

$$\zeta(1-s) = \frac{2^{1-s}}{2^{1-s}-2}\eta(1-s) = (\frac{2^{1-s}-2+2}{2^{1-s}-2})\eta(1-s) = (1+\frac{2}{2^{1-s}-2})\eta(1-s) \tag{13}$$

$$= (1+\frac{2}{2^{1-s}}\frac{2^{1-s}}{2^{1-s}-2})\eta(1-s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{14}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}[\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)] = \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2\zeta(1-s) \tag{15}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2[\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)] \tag{16}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2\eta(1-s) + (\frac{2}{2^{1-s}})^3\zeta(1-s) \tag{17}$$

when $\frac{2}{2^{1-s}} = 2^s$

$$= \eta(1-s) + 2^s\eta(1-s) + (2^s)^2\eta(1-s) + (2^s)^3\zeta(1-s) \tag{18}$$

$$= \eta(1-s)[1 + 2^s + (2^s)^2] + (2^s)^3\zeta(1-s) \tag{19}$$

$$\neq \eta(1-s)[\frac{1-(2^s)^k}{1-2^s}] + (2^s)^{k+1}\zeta(1-s) \tag{20}$$

when k=2

$$\neq \eta(1-s)[\frac{1-2^{2s}}{1-2^s}] + 2^{3s}\zeta(1-s) \tag{21}$$

$$= \zeta(1-s)(1-2^s)[\frac{1-2^{2s}}{1-2^s}] + 2^{3s}\zeta(1-s) \tag{22}$$

$$= \zeta(1-s)[1-2^{2s}] + 2^{3s}\zeta(1-s) \tag{23}$$

$$= \zeta(1-s)[1-2^{2s}+2^{3s}] \tag{24}$$

from Eq.(12) and Eq.(24)

$$\zeta(s)[1-(\frac{2}{2^s})^2+(\frac{2}{2^s})^3] = \zeta(1-s)[1-2^{2s}+2^{3s}] \tag{25}$$

2 Discussion

Define $0 < \Re(s) < 1$
from Eq.(25)
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i14.1347\} = -0.000160889i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i15.1347\} = -0.280343i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i16.1347\} = -4.17572i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i16.1347\} = 4.17572i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i17.1347\} = 4.82094i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i17.1347\} = -4.82094i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i21.022\} = -0.0000820167i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i21.022\} = 0.0000820167i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i74879.422\} = 0.00056128i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i74879.8804\} = -0.00111728i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i74891.93\} = 0.0000554776i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74892.5452\} = -0.000641199i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74895.7013\} = 0.00117245i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74896.2133\} = 0.000808722i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74896.6987\} = -0.00106666i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74897.0517\} = 0.000224195i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74898.1134\} = -0.000935263i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74898.9041\} = 0.000102353i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=0.49999+i74911.8951\} = 0.000232008+0.000914211i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74911.9\} = 2.7105110^{-20}+0.000914218i$
$\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}],\{s=0.51+i74911.8951\} = -0.232499-0.00586949i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i74912.4918\} = -0.0000277175i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74916.2765\} = 0.000952275i$

$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74918.7\}=0.000152616i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74919.1\}=-8.4703310^{-22}+0.0000171143i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74920.2598\}=0.000360484i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i74920.8275\}=0.0000329616i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i99999.422\}=-9.48598i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i999999.422\}=-0.270142i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i9999999.422\}=-6.86408i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i99999999.422\}=0.0172762i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i999999999.422\}=-0.0048036i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2+i9999999999.422\}$=no result
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i99999999.422\}=0.0172762i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i999999999.422\}=-0.0048036i$
$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\},\{s=1/2-i9999999999.422\}$=no result

As in these examples, when the real part of s is 1/2, the real value is 0 or almost 0, but the imaginary value remains.
Even if s is a non-trivial zero, the imaginary value is close to 0 but not 0.

However, even if the real part of s is 1/2, a real value close to 0 but not 0 is frequently generated.

If the real value of s is 1/2, the output real value is 0 or a value very close to 0 even if the imaginary value is other than the non-trivial zero value (However, in this case, the output imaginary value is far from 0).

That is, the minimum requirement for the non-trivial zeros is that the real part of s is 1/2.

That is, the lowest condition in which a non-trivial zero exists is a real part value of 1/2.

That is, a non-trivial zero can have a real part only 1/2.

$$\Re(s)=\frac{1}{2} \tag{26}$$

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

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