The Pascal Triangle of Maximum Deng Entropy

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Abstract

Pascal-Triangle (known as Yang Hui Triangle) is an important structure in mathematics, which has been used many fields. Entropy plays an essential role in physics. In various, information entropy is used to measure the uncertainty of information. Hence, setting the connection between Pascal-Triangle and information uncertainty is a question worth exploring. Deng proposed the Deng entropy that it can measure non-specificity and discord of basic probability assignment (BPA) in Dempster-Shafer (D-S) evidence theory. D-S evidence theory and power set are very closely related. Hence, by analysing the maximum Deng entropy, the paper find that there is an potential rule of BPA with changes of frame of discernment. Finally, the paper set the relation between the maximum Deng entropy and Pascal-Triangle.

Keywords: Pascal Triangle, Dempster-Shafer evidence theory, Maximum Deng entropy, Entropy, Basic Probability Assignment

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1. Introduction

Pascal-Triangle (also named Yanghui Triangle) is an important tool in mathematics and is a graphical representation of binomial coefficients, which directly reflects some algebraic properties of combinatorial numbers from graphics, and is a discrete combination of numbers and forms [1, 2, 3]. The most striking feature of Pascal-Triangle is that each number equals the sum of the two above it [4]. Pascal-Triangle contains several properties of binomial coefficients, including the symmetry, increasing and decreasing of binomial coefficients, maximum and the sum of binomial coefficients. Pascal-Triangle has attracted many people to study and has been used in many fields [5, 6]. However, how to apply Pascal-Triangle into the information theory is also an open issue.

In information theory, entropy plays an essential role. Entropy is a measure of the degree of system chaos, which derived from physics. Shannon firstly introduce entropy into information theory. Shannon entropy can measure the information uncertainty and has been used in many fields. Based on Shannon entropy, there are various entropies, such Renyi entropy, Tsallis entropy [7] and so on. Tsallis proposed Tsallis entropy as generalization of Boltzmann-Gibbs statistics according to multi-fractal concepts and structures are quickly acquiring importance in many active areas of research [7], besides, Tsallis analyzed the connection between Tsallis entropy and Pascal-triangle, further expanding the application of entropy. Deng proposed Deng entropy to measure the uncertainty of basic probability assignment (BPA) for Dempster-Shafer evidence theory (D-S evidence theory) [8]. Deng entropy has attracted many people attention since proposed [9, 10, 11, 12, 13, 14, 15], which has been used in many fields, such as quantum information [16], pattern recognition [17], information fusion [18, 19, 20] and so on [21, 22]. Recently, Kang and Deng proposed the maximum Deng entropy, which BPA satisfies some conditions with the changes of frame of discernment.

Based on the maximum Deng entropy, the paper analyses the distribution of BPA with different frame of discernment and proposed the connection between BPA and Pascal-Triangle. Besides, the paper also discusses the connection between *Bel* and *Pl* and Pascal-Triangle.

The paper is organized as follows. The preliminaries Dempster-Shafer evidence theory, Deng entropy and the maximum Deng entropy are briefly introduced in Section 1. Section 2 introduced the examples of maximum Deng entropy and discussed the some properties of BPA, besides, analysed the Pascal-Triangle of BPA. Finally, this paper is concluded in Section 3.

2. Preliminaries

In this section, the preliminaries of D-S theory [23, 24] and Deng entropy [8] and the maximum Dneg entropy will be briefly introduced.

2.1. Dempster-Shafer evidence theory

D-S evidence theory assigns the probability into the power set of events [23, 24], so as to better grasp the unknown and imprecise of the problem [25, 26, 27, 28]. D-S evidence theory needs weaker conditions than the Bayesian theory of probability [29, 30]. D-S evidence theory has been used to many applications, such as data fusion [31, 32, 33, 34], conflict management [35, 36, 37, 38, 39], evidential reasoning [40, 41], target classification [42, 43] and so on [44, 45, 46, 47, 48]. Some preliminaries in D-S theory are introduced as follows. For additional details about D-S theory, refer to [23, 24].

Definition 2.1. *(Frame of discernment)*

Let Θ be the set of mutually exclusive and collectively exhaustive events A_i, namely

$$\Theta = \{A_1, A_2, \cdots, A_n\} \tag{1}$$

The power set of Θ composed of 2^N elements of is indicated by 2^Θ, namely:

$$2^\Theta = \{\phi, \{A_1\}, \{A_2\}, \cdots, \{A_1, A_2\}, \cdots, \Theta\} \tag{2}$$

Definition 2.2. *(Mass Function)*

For a frame of discernment $\Theta = \{A_1, A_2, \cdots, A_n\}$, the mass function m *is defined as a mapping of m from 0 to 1, namely:*

$$m : 2^\Theta \to [0, 1] \tag{3}$$

which satifies

$$m(\phi) = 0 \tag{4}$$

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{5}$$

In D-S theory, a mass function is also called a basic probability assignment (BPA) or a piece of evidence or belief structure. The $m(A)$ measures the belief exactly assigned to A and represents how strongly the piece of evidence supports A. If $m(A) > 0$, A is called a focal element, and the union of all focal elements is called the core of a mass function.

Definition 2.3. *(Belief function)*

The belief function (Bel) is a mapping from set 2^{Θ} to [0,1] and satisfied:

$$Bel\left(A\right)=\sum_{B\subseteq A}m\left(B\right) \tag{6}$$

Definition 2.4. *(Plausibility function)*
The plausibility function (Pl): $2^{\Theta}\rightarrow[0,1]$, and satisfied:

$$Pl\left(A\right)=\sum_{B\cap A\neq\phi}m\left(B\right)=1-Bel(\bar{A}) \tag{7}$$

As can be seen from the above, $\forall A\subseteq\Theta$, $Bel(A)<Pl(A)$, $Bel(A)$, $Pl(A)$ are respectively the lower and upper limits of A, namely [Bel(A), Pl(A)], which indicates uncertain interval for A. According to Shafers explanation, the difference between the belief and the plausibility of a proposition A expresses the ignorance of the assessment for the proposition A. From the above, it has already shown that D-S theory has more advantages than probability.

2.2. Deng entropy

Deng proposed *Deng Entropy*. *Deng Entropy* is an generalization of Shannon entropy, for more details about *Deng Entropy* refer to [8].

Definition 2.5. *(Deng entropy)*

Given a BPA, Deng entropy can be defined as:

$$H_D=-\sum_{A\subseteq\Theta}m(A)log_2\frac{m(A))}{2^{|A|}-1} \tag{8}$$

Through a simple transformation, Deng Entropy can be rewrite as follows:

$$H_D = \sum_{A \subseteq \Theta} m(A) log_2(2^{|A|} - 1) - \sum_{A \subseteq \Theta} m(A) log_2 m(A) \tag{9}$$

where m is a BPA defined on the frame of discernment Θ, and A is the focal element of m, $|A|$ is the cardinality of A. Besides, the term $\sum m(A) \times log_2(2^{|A|} - 1)$ could be interpreted as a measure of total nonspecificity in the mass function m, and the term $-m(A) \times log_2 m(A)$ is the measure of discord of the mass function among various focal elements.

2.3. The Maximum Deng Entropy

Given a BPA, the maximum Deng entropy is as follows [49]:

$$H_{M-D} = -\sum m(A) \times log_2 \frac{m(A)}{2^{|A|} - 1} \tag{10}$$

if and only if

$$m(A) = \frac{2^{|A|} - 1}{\sum 2^{|A|} - 1} \tag{11}$$

The more details about maximum Deng entropy refer to [49].

3. The Pascal Triangle of Maximum Deng Entropy

In this section, the paper mainly discuss the distribution of maximum Deng entropy and set the Pascal-Triangle of maximum Deng entropy.

3.1. The Example of Maximum Deng entropy

The specific BPA of having maximum Deng entropy was showed as follows. In D-S evidence theory, the empty set ϕ plays an important role, hence, in the next discussion, the empty set ϕ can be considered.

Example 3.1. *Given the event space* $\Theta = \{A\}$*, there is only one phenomenon, as follows:*

$$m(A) = 1, m(\phi) = 0$$

$$H_D = 0$$

When there is a element only in frame of discernment, namely, the event is certain. In this cases, the information uncertainty is 0.

Example 3.2. *Given the event space* $\Theta = \{A, B\}$*, the BPA and maximum Deng entropy are shown below:*

$$m(A) = \frac{1}{5}, m(B) = \frac{1}{5}, m(A, B) = \frac{3}{5}, m(\phi) = 0$$

$$H_{M-D} = 2.3219$$

Example 3.3. *Given the event space* $\Theta = \{A, B, C\}$*, the BPA and maximum Deng entropy are as follows:*

$$m(A) = \frac{1}{19}, m(B) = \frac{1}{19}, m(C) = \frac{1}{19}$$

$$m(A, B) = \frac{3}{19}, m(B, C) = \frac{3}{19}, m(A, C) = \frac{3}{19}, m(A, B, C) = \frac{7}{19}$$

$$m(\phi) = 0$$

$$H_{M-D} = 4.2474$$

From the above examples, there is a certain rule with different frame of discernment. Next, more details can be discussed.

3.2. *Distribution of maximum Deng entropy*

Table 1 presents the BPAs of maximum Deng entropy in different frame of discernment based on *Example*. $1-3$. In *Table* 1, the ϕ is the empty set, $|X|$ represents the cardinality of focal elements. For example, $\Theta = \{A, B\}$, $(\frac{1}{5}, \frac{1}{5})$ means the $m(A) = \frac{1}{5}$ and $m(B) = \frac{1}{5}$, $(\frac{3}{5})$ means the $m(A, B) = \frac{3}{5}$.

Frame of Discrement	ϕ	$\|X\| = 1$	$\|X\| = 2$	$\|X\| = 3$	$\|X\| = 4$
$\Theta = \{A\}$	0	1			
$\Theta = \{A, B\}$	0	(1/5,1/5)	3/5		
$\Theta = \{A, B, C\}$	0	(1/19,1/19,1/19)	(3/19,3/19,3/19)	7/19	
$\Theta = \{A, B, C, D\}$	0	(1/65,1/65, 1/65,1/65)	(3/65,3/65,3/65, 3/65,3/65,3/65)	(7/65,7/65, 7/65,7/65)	15/65

Table 1: The BPA of Maximum Deng entropy

Besides, the BPA can be rewrite the *Fig* .1. In *Fig* .1, the first column, N corresponds to the number of frame of discernments. The second column represents the BPA with maximum Deng entropy. For example, in $(2, 3/5)$, 2 represents that the cardinality of focal element, $3/5$ represents the BPA whose the cardinality of focal element is 2, $(1, 0)$ represent that there is 1 empty set and the BPA of empty set is 0.

N=0 (1,0)
N=1 (1,1) (1,0)
N=2 (1,1/5) (2,3/5) (1,0)
N=3 (1,1/19) (2,3/19) (3,7/19) (1,0)
N=4 (1,1/65) (2,3/65) (3,7/65) (4,15/65) (1,0)

Figure 1: The BPA of Maximum Deng entropy

Taking (A, B, C) as an example, dividing 1 equally into 19 small parts, and $m(A)$, $m(B)$ and $m(C)$ have 1 $(2^{|1|} - 1)$ small part, namely $(\frac{1}{19})$,

$m(A,B)$, $m(A,C)$ and $m(B,C)$ has 3 ($2^{|2|}-1$) small parts, namely ($\frac{3}{19}$), $m(A,B,C)$ has 7 ($2^{|3|}-1$) small parts, namely ($\frac{7}{19}$). There is 1 empty set ϕ that it does not have any parts, that is to say, $m(\phi)$ is 0. It can be seen as *Fig* .2. Besides, when the cardinality of focal element is n, the subsystem can have 2^n-1 small parts, which is consistent with the fact that D-S theory have power-laws.

N=0	(1,0)
N=1	(1,0) (1,1)
N=2	(1,0) (2,1/5) (1,3/5)
N=3	(1,0) (3,1/19) (3,5/19) (1,7/19)
N=4	(1,0) (4,1/65) (6,3/65) (4,7/65) (1,15/65)

Figure 2: The analysis of BPA with Maximum Deng entropy

In D-S theory, another important definition is *Bel* and *Pl*. Next, it can be considered that the *Bel* and *Pl* of BPA have the maximum Deng entropy. Besides, the *Bel* of BPA are shown as *Fig* .3.

N=0	(1,0)
N=1	(1,0) (1,1)
N=2	(1,0) (2,1/5) (1,1)
N=3	(1,0) (3,1/19) (3,5/19) (1,1)
N=4	(1,0) (4,1/65) (6,5/65) (4,19/65) (1,1)

Figure 3: The *Bel* of having Maximum Deng entropy

From the *Eq* $.6-7$ and the BPA of maximum Deng entropy, it can know focal element having the same cardinality has the same *Bel* and *Pl*. More importantly, molecule in *Bel* that cardinality of focal element is N is the same with denominator that frame of discernments is N. That is to say, no matter what small parts the 1 can be divided, the focal elements having 2 cardinality always has 5 small parts, which is the same with the frame of discernment which is 2 divide 1 into 5 small parts.

N=0	(1,0)				
N=1	(1,0)	(1,1)			
N=2	(1,0)	(2,4/5)	(1,1)		
N=3	(1,0)	(3,14/19)	(3,18/19)	(1,1)	
N=4	(1,0)	(4,46/65)	(6,60/65)	(4,64/65)	(1,1)

Figure 4: The *Pl* of having Maximum Deng entropy

N=0	(1,1)				
N=1	(1,1)	(1,0)			
N=2	(1,1)	(2,1/5)	(1,0)		
N=3	(1,1)	(3,5/19)	(3,1/19)	(1,0)	
N=4	(1,1)	(4,19/65)	(6,5/65)	(4,1/65)	(1,0)

Figure 5: The 1-*Pl* of having Maximum Deng entropy

Considering the Pl, it is shown as Fig .4. Take $N = 3$ as example, when the $|m| = 1$, the corresponding Pl can have $(2^1 - 1) + (2^2 - 1) * 2 + (2^3 - 1)$ small parts. But it can be easily found some phenomenons from another sides as Fig .5. Fig .5 shows the $1 - Pl\,(A) = Bel\,(\bar{A})$. It can be seen that the Fig. 4 and Fig. 5 are just the opposite.

3.3. Pascal-Triangle of maximum Deng entropy

Pascal-Triangle plays an essential role in mathematics, the paper can explore Pascal-Triangle and the maximum Deng entropy. From the Fig. 3 $-$

5, it can be seen that the left numbers within the parentheses correspond to Pascal Triangle. Hence, the paper set the connection between the maximum Deng entropy and Pascal Triangle, as *Tab*. 2. In *Tab*. 2, N is the number of frame of Discernment, ϕ represents the frame of discernment is empty set. the $|m|$ represents the cardinality of focal element. For example, when $N = 3$, (1) suppose the system consists 2^3 subsystems; (2) dividing evenly the system into some parts; (3) empty set ϕ does not occupy any information; (4) there is 3 subsystems that have $2^1 - 1$ parts; (5) there is 3 subsystems that have $2^2 - 1$ parts; (6) there is 1 subsystem that have $2^3 - 1$ parts.

$2^{\|m\|} - 1$	ϕ	$2^{\|1\|} - 1$	$2^{\|2\|} - 1$	$2^{\|3\|} - 1$	$2^{\|4\|} - 1$	$2^{\|5\|} - 1$	$2^{\|6\|} - 1$	$2^{\|7\|} - 1$	$2^{\|8\|} - 1$	$2^{\|9\|} - 1$
N=0	1									
N=1	1	1								
N=2	1	2	1							
N=3	1	3	3	1						
N=4	1	4	6	4	1					
N=5	1	5	10	10	5	1				
N=6	1	6	15	20	15	6	1			
N=7	1	7	21	35	35	21	7	1		
N=8	1	8	28	56	70	56	28	8	1	
N=9	1	9	36	84	126	126	84	36	9	1

Table 2: The BPA of Maximum Deng entropy

By analysing the *Tab*. 2, if when the frame of Discernment is N, the system can be divided into $\sum_{m=1}^{m=N}(2^{|m|} - 1) \times C_N^m$ parts, the cardinality of focal elements which is $|m|$ has $2^{|m|} - 1$ parts.

Besides, *Bel* and *Pl* also have same distribution with BPAs, due to the *Bel* and $1 - Pl$ is same, hence, the paper only analyse the *Bel*, as *Tab*. 3. In *Fig* .7, the first column represents the frame of discernment. The first row represents that the number of *Bel* occupies when the system can be evenly divided into $\sum_{m=1}^{m=N}(2^{|m|} - 1) \times C_N^m$. For example, when $N = 3$, we can evenly divided the system into 19 parts, and there are 3 subsystems occupying 5 parts.

	ϕ	1	5	19	65
N=0	1				
N=1	1	1			
N=2	1	2	1		
N=3	1	3	3	1	
N=4	1	4	6	4	1

Table 3: The *Bel* of Maximum Deng entropy

4. Conclusion

Dempster-Shafer (D-S) evidence theory is an useful tool to handle imprecise and unknown information. Deng entropy plays an important role to measure uncertainty in D-S evidence theory. Pascal Triangle (known as Yng Hui Triangle) is an essential tool in mathematics. Hence, studying the connection between Pascal Triangle and information entropy is significant for physics and mathematics. By analysing the maximum Deng entropy, the paper discusses the BPA of the maximum Deng entropy which changes with N according to evolutionary rules. D-S evidence theory has an important relation with power set. By analysing, it can be seen that (1) the system consists 2^N subsystems; (2) the system is divided evenly into $\sum_{m=1}^{m=N}(2^{|m|}-1)\times C_N^m$; (3) there is C_N^m subsystems have 2^m-1 parts; (4) empty set ϕ has no part. Finally, the paper set the connection between Pascal-Triangle and BPAs.

The paper only discusses the connection between the maximum Deng entropy and Pascal Triangle. Next, we will explore the meaning of BPA having maximum Deng entropy.

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