# Numbers: Part 5 , Two Formulas

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ABSTRACT. This note presents two formulas.
keywords: number Pi, integrals, series.

## I. Introduction. The number Pi

Recall that: $\pi = 4\int_0^1 \frac{1}{1+x^2}\,dx = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592\ldots$

## II. Two Formulas

$$\frac{4\pi}{3\sqrt{3}} = \int_0^1 \sqrt{\frac{2(1-x^2)}{x\left(x+\sqrt{4-3x^2}\right)}}\,dx + \frac{4}{3}\int_1^{\sqrt{3}} \sin^{-1}\left(\sqrt{\frac{9}{8}\left(1-\frac{1}{x^2}\right)}\right)dx$$

$$\frac{4\pi}{3\sqrt{3}} = \int_0^1 \sqrt{\frac{2(1-x^2)}{x\left(x+\sqrt{4-3x^2}\right)}}\,dx +$$

$$+\frac{4}{3\sqrt{3}}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{(3/16)^n}{(2n+1)(2n+3)}F\left(\frac{3}{2},\, n+\frac{3}{2},\, n+\frac{5}{2},\, \frac{2}{3}\right)$$

Remark: $F(a, b, c, x) = {}_2F_1(a, b; c; x)$ is the Gauss hypergeometric function.

## III. References.

[1] Boros, G., and Moll, V.H. : Irresistible Integrals, Cambridge University Press, 2004.

[2] Gradshteyn, I.S., and Ryzhik, I.M. : Table of Integrals, Series and Products. 7th ed., edited by Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.