

Formula of ζ odd-numbers

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Abstract

I tried to find a new expression for zeta odd-numbers.

It may be a new expression and will be published here.

The correctness of this formula was confirmed by WolframAlpha to be numerically completely correct.

key words

zeta odd-numbers, new expression, formula

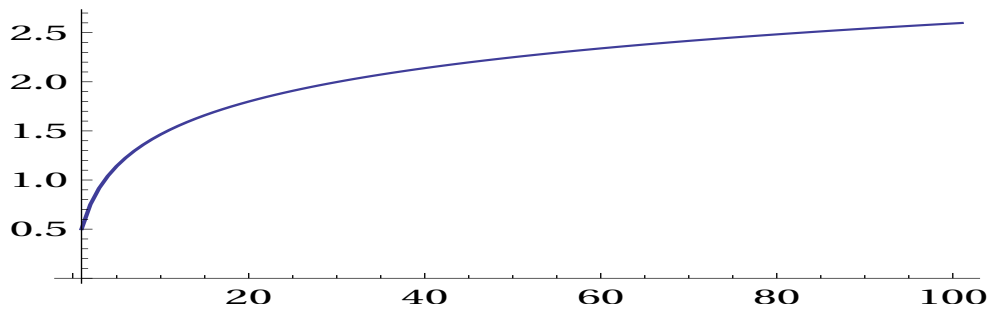
1 Introduction

$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1} = \frac{1}{2^1} \sum_{n=1}^{\infty} \frac{1}{n^1} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = \frac{1}{2^1} \zeta(1) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (1)$$

$$\zeta(1) = \frac{2^1}{2^1 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = \frac{2}{1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (2)$$

If $n=\infty = 10^{2199}$

$\zeta(1) = 2 \times 762.4039328581106341345499735883063378145334773970706137050\dots$



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10^{2199} seemed to be the limit of the calculator.

do the same

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{2^3} \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{2^3} \zeta(3) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (3)$$

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (4)$$

If $\infty=10$

$\zeta(3) \approx 1.2013\dots$

If $\infty=20$

$\zeta(3) \approx 1.2018\dots$

$\infty=40$ seem to be the limit of the calculator.

If $\infty=40$

$\zeta(3) \approx 1.202012\dots$

$\zeta(3) = 1.202056\dots$

2 Discussion

do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{2^5} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{2^5} \zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (5)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (6)$$

If $\infty=10$

$\zeta(5) \approx 1.036926955\dots$

$\infty=20$ seem to be the limit of the calculator.

If $\infty=20$

$\zeta(5) \approx 1.036927704\dots$

$\zeta(5) = 1.036927755\dots$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{2^7} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{2^7} \zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (7)$$

$$\zeta(7) = \frac{2^7}{2^7 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (8)$$

$\infty=10$ seem to be the limit of the calculator.

If $\infty=10$

$$\zeta(7) \approx 1.0083492760\dots$$

$$\zeta(7) = 1.0083492773\dots$$

do the same

$$\zeta(9) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} + \sum_{n=1}^{\infty} \frac{1}{(2n)^9} = \frac{1}{2^9} \sum_{n=1}^{\infty} \frac{1}{n^9} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{1}{2^9} \zeta(9) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (9)$$

$$\zeta(9) = \frac{2^9}{2^9 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{512}{511} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (10)$$

$\infty=10$ seem to be the limit of the calculator.

If $\infty=10$

$$\zeta(9) \approx 1.0020083928237\dots$$

$$\zeta(9) = 1.0020083928260\dots$$

do the same

$$\zeta(11) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{11}} = \frac{1}{2^{11}} \sum_{n=1}^{\infty} \frac{1}{n^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{1}{2^{11}} \zeta(11) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (11)$$

$$\zeta(11) = \frac{2^{11}}{2^{11} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{2048}{2047} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (12)$$

$\infty=10$ seem to be the limit of the calculator.

If $\infty=10$

$$\zeta(11) \approx 1.0004941886041147\dots$$

$$\zeta(11) = 1.0004941886041194\dots$$

do the same

$$\zeta(13) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{13}} = \frac{1}{2^{13}} \sum_{n=1}^{\infty} \frac{1}{n^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{1}{2^{13}} \zeta(13) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (13)$$

$$\zeta(13) = \frac{2^{13}}{2^{13} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{8192}{8191} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (14)$$

$\infty=10$ seem to be the limit of the calculator.

If $\infty=10$

$$\zeta(13) \approx 1.000122713347578479\dots$$

$$\zeta(13) = 1.000122713347578489\dots$$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{15}} = \frac{1}{2^{15}} \sum_{n=1}^{\infty} \frac{1}{n^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{1}{2^{15}} \zeta(15) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (15)$$

$$\zeta(15) = \frac{2^{15}}{2^{15}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{32768}{32767} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (16)$$

$\infty=10$ seem to be the limit of the calculator.

If $\infty=10$

$$\zeta(15) \approx 1.000030588236307020473\dots$$

$$\zeta(15) = 1.000030588236307020493\dots$$

do the same

$$\zeta(17) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{17}} = \frac{1}{2^{17}} \sum_{n=1}^{\infty} \frac{1}{n^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{1}{2^{17}} \zeta(17) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (17)$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{131072}{131071} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (18)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$$\zeta(17) \approx 1.00000763719763789963\dots$$

$$\zeta(17) = 1.00000763719763789976\dots$$

do the same

$$\zeta(19) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{19}} = \frac{1}{2^{19}} \sum_{n=1}^{\infty} \frac{1}{n^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{1}{2^{19}} \zeta(19) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (19)$$

$$\zeta(19) = \frac{2^{19}}{2^{19}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{524288}{524287} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (20)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$$\zeta(19) \approx 1.00000190821271655393819\dots$$

$$\zeta(19) = 1.00000190821271655393892\dots$$

do the same

$$\zeta(21) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{21}} = \frac{1}{2^{21}} \sum_{n=1}^{\infty} \frac{1}{n^{21}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} = \frac{1}{2^{21}} \zeta(21) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \quad (21)$$

$$\zeta(21) = \frac{2^{21}}{2^{21}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} = \frac{2097152}{2097151} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \quad (22)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$$\zeta(21) \approx 1.000000476932986787806458\dots$$

$$\zeta(21) = 1.000000476932986787806463\dots$$

do the same

$$\zeta(23) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{23}} = \frac{1}{2^{23}} \sum_{n=1}^{\infty} \frac{1}{n^{23}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} = \frac{1}{2^{23}} \zeta(23) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \quad (23)$$

$$\zeta(23) = \frac{2^{23}}{2^{23}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} = \frac{8388608}{8388607} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \quad (24)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$$\zeta(23) \approx 1.000000119209\dots$$

$$\zeta(23) = 1.000000119219\dots$$

do the same

$$\zeta(25) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{25}} = \frac{1}{2^{25}} \sum_{n=1}^{\infty} \frac{1}{n^{25}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} = \frac{1}{2^{25}} \zeta(25) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \quad (25)$$

$$\zeta(25) = \frac{2^{25}}{2^{25}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} = \frac{33554432}{33554431} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \quad (26)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$$\zeta(25) \approx 1.00000002980350351465228018591\dots$$

$\zeta(25) = 1.00000002980350351465228018606\dots$

do the same

$$\zeta(27) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{27}} = \frac{1}{2^{27}} \sum_{n=1}^{\infty} \frac{1}{n^{27}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} = \frac{1}{2^{27}} \zeta(27) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \quad (27)$$

$$\zeta(27) = \frac{2^{27}}{2^{27}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} = \frac{134217728}{134217727} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \quad (28)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$\zeta(27) \approx 1.00000000745071178983542949198014\dots$

$\zeta(27) = 1.00000000745071178983542949198100\dots$

do the same

$$\zeta(29) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{29}} = \frac{1}{2^{29}} \sum_{n=1}^{\infty} \frac{1}{n^{29}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} = \frac{1}{2^{29}} \zeta(29) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \quad (29)$$

$$\zeta(29) = \frac{2^{29}}{2^{29}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} = \frac{536870912}{536870911} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \quad (30)$$

$\infty=6$ seem to be the limit of the calculator.

If $\infty=6$

$\zeta(29) \approx 1.0000000018626597235130490064039049\dots$

$\zeta(29) = 1.0000000018626597235130490064039099\dots$

do the same

$$\zeta(31) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{31}} = \frac{1}{2^{31}} \sum_{n=1}^{\infty} \frac{1}{n^{31}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} = \frac{1}{2^{31}} \zeta(31) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \quad (31)$$

$$\zeta(31) = \frac{2^{31}}{2^{31}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} = \frac{2147483648}{2147483647} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \quad (32)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$\zeta(31) \approx 1.0000000004656629065033784072962\dots$

$\zeta(31) = 1.0000000004656629065033784072989\dots$

do the same

$$\zeta(33) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{33}} = \frac{1}{2^{33}} \sum_{n=1}^{\infty} \frac{1}{n^{33}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} = \frac{1}{2^{33}} \zeta(33) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} \quad (33)$$

$$\zeta(33) = \frac{2^{33}}{2^{33}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} = \frac{8589934592}{8589934591} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} \quad (34)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(33) \approx 1.000000000116415501727005197759264\dots$$

$$\zeta(33) = 1.000000000116415501727005197759297\dots$$

do the same

$$\zeta(35) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{35}} = \frac{1}{2^{35}} \sum_{n=1}^{\infty} \frac{1}{n^{35}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} = \frac{1}{2^{35}} \zeta(35) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} \quad (35)$$

$$\zeta(35) = \frac{2^{35}}{2^{35}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} = \frac{34359738368}{34359738367} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} \quad (36)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(35) \approx 1.0000000000291038504449709968692938\dots$$

$$\zeta(35) = 1.0000000000291038504449709968692942\dots$$

do the same

$$\zeta(37) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{37}} = \frac{1}{2^{37}} \sum_{n=1}^{\infty} \frac{1}{n^{37}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} = \frac{1}{2^{37}} \zeta(37) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} \quad (37)$$

$$\zeta(37) = \frac{2^{37}}{2^{37}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} = \frac{137438953472}{137438953471} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} \quad (38)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(37) \approx 1.0000000000072759598350574810145208640\dots$$

$$\zeta(37) = 1.0000000000072759598350574810145208690\dots$$

do the same

$$\zeta(39) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{39}} = \frac{1}{2^{39}} \sum_{n=1}^{\infty} \frac{1}{n^{39}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} = \frac{1}{2^{39}} \zeta(39) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} \quad (39)$$

$$\zeta(39) = \frac{2^{39}}{2^{39}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} = \frac{549755813888}{549755813887} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} \quad (40)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(39) \approx 1.00000000000181898965030706594758483203\dots$$

$$\zeta(39) = 1.00000000000181898965030706594758483210\dots$$

do the same

$$\zeta(41) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{41}} = \frac{1}{2^{41}} \sum_{n=1}^{\infty} \frac{1}{n^{41}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} = \frac{1}{2^{41}} \zeta(41) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} \quad (41)$$

$$\zeta(41) = \frac{2^{41}}{2^{41}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} = \frac{2199023255552}{2199023255551} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} \quad (42)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(41) \approx 1.00000000000045474737830421540267991120219\dots$$

$$\zeta(41) = 1.00000000000045474737830421540267991120294\dots$$

do the same

$$\zeta(43) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{43}} = \frac{1}{2^{43}} \sum_{n=1}^{\infty} \frac{1}{n^{43}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} = \frac{1}{2^{43}} \zeta(43) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} \quad (43)$$

$$\zeta(43) = \frac{2^{43}}{2^{43}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} = \frac{8796093022208}{8796093022207} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} \quad (44)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$$\zeta(43) \approx 1.00000000000011368684076802278493491048379\dots$$

$$\zeta(43) = 1.00000000000011368684076802278493491048380\dots$$

do the same

$$\zeta(45) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{45}} = \frac{1}{2^{45}} \sum_{n=1}^{\infty} \frac{1}{n^{45}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} = \frac{1}{2^{45}} \zeta(45) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} \quad (45)$$

$$\zeta(45) = \frac{2^{45}}{2^{45}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} = \frac{35184372088832}{35184372088831} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} \quad (46)$$

$\infty=4$ seem to be the limit of the calculator.

If $\infty=4$

$\zeta(45) \approx 1.00000000000002842170976889301855455073704931\dots$

$\zeta(45) = 1.00000000000002842170976889301855455073704942\dots$

3 Conclusion

make official

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \quad (47)$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1) = \frac{(2^{2m+1}-4)}{(2^{2m+1}-1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{2m+1}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}} \quad (48)$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1) = \zeta(2m-1) \frac{(2^{2m+1}-4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1}-1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \quad (49)$$

m is a positive integer.

4 Postscript

Using WolframAlpha, it was confirmed numerically completely correct.

References

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