Formula for odd values of ζ

Toshiro Takami*
mmm82889@yahoo.co.jp

Abstract

I tried to find a new expression for zeta odd-numbers.
It may be a new expression and will be published here.
The correctness of this formula was confirmed by WolframAlpha to be numerically completely correct.

key words
zeta odd-numbers, new expression, formula

1 Introduction

$$\zeta(1)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}+\sum_{n=1}^{\infty}\frac{1}{(2n)^1}=\frac{1}{2^1}\sum_{n=1}^{\infty}\frac{1}{n^1}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}=\frac{1}{2^1}\zeta(1)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1} \tag{1}$$

$$\zeta(1)=\frac{2^1}{2^1-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}=\frac{2}{1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1} \tag{2}$$

do the same

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}=\frac{1}{2^3}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{1}{2^3}\zeta(3)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{3}$$

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{4}$$

*47-8 Kuyamadai, Isahaya-shi, Nagasaki-prefecture, 854-0067 Japan

2 Discussion

do the same

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}=\frac{1}{2^5}\sum_{n=1}^{\infty}\frac{1}{n^5}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{1}{2^5}\zeta(5)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{5}$$

$$\zeta(5)=\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{6}$$

do the same

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}+\sum_{n=1}^{\infty}\frac{1}{(2n)^7}=\frac{1}{2^7}\sum_{n=1}^{\infty}\frac{1}{n^7}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{1}{2^7}\zeta(7)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{7}$$

$$\zeta(7)=\frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{8}$$

do the same

$$\zeta(9)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}+\sum_{n=1}^{\infty}\frac{1}{(2n)^9}=\frac{1}{2^9}\sum_{n=1}^{\infty}\frac{1}{n^9}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{1}{2^9}\zeta(9)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{9}$$

$$\zeta(9)=\frac{2^9}{2^9-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{512}{511}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{10}$$

do the same

$$\zeta(11)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{11}}=\frac{1}{2^{11}}\sum_{n=1}^{\infty}\frac{1}{n^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{1}{2^{11}}\zeta(11)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{11}$$

$$\zeta(11)=\frac{2^{11}}{2^{11}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{2048}{2047}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{12}$$

do the same

$$\zeta(13)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}=\frac{1}{2^{13}}\sum_{n=1}^{\infty}\frac{1}{n^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}=\frac{1}{2^{13}}\zeta(13)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{13}$$

$$\zeta(13) = \frac{2^{13}}{2^{13}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} = \frac{8192}{8191}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{14}$$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} + \sum_{n=1}^{\infty}\frac{1}{(2n)^{15}} = \frac{1}{2^{15}}\sum_{n=1}^{\infty}\frac{1}{n^{15}} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} = \frac{1}{2^{15}}\zeta(15) + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{15}$$

$$\zeta(15) = \frac{2^{15}}{2^{15}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} = \frac{32768}{32767}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{16}$$

do the same

$$\zeta(17) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} + \sum_{n=1}^{\infty}\frac{1}{(2n)^{17}} = \frac{1}{2^{17}}\sum_{n=1}^{\infty}\frac{1}{n^{17}} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} = \frac{1}{2^{17}}\zeta(17) + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{17}$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} = \frac{131072}{131071}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{18}$$

do the same

$$\zeta(19) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} + \sum_{n=1}^{\infty}\frac{1}{(2n)^{19}} = \frac{1}{2^{19}}\sum_{n=1}^{\infty}\frac{1}{n^{19}} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} = \frac{1}{2^{19}}\zeta(19) + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{19}$$

$$\zeta(19) = \frac{2^{19}}{2^{19}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} = \frac{524288}{524287}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{20}$$

3 Conclusion

make official

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}} \tag{21}$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1) = \frac{(2^{2m+1}-4)}{(2^{2m+1}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}} \tag{22}$$

m is a positive integer.

And this is just a variation of the above formula

$$\zeta(2m+1) = \zeta(2m-1)\frac{(2^{2m+1}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}} \tag{23}$$

m is a positive integer.

4 Postscript

These calculations were performed with WolframAlpha and confirmed to be numerically completely correct with WolframAlpha.

References

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