

$\zeta(4), \zeta(6), \dots, \zeta(108), \zeta(110)$ are irrational number

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Abstract

$\zeta(4), \zeta(6), \dots, \zeta(108), \zeta(110)$ considered.

From these equations, it can be said that $\zeta(4), \zeta(6), \dots, \zeta(108), \zeta(110)$ are irrational numbers.

$\zeta(112), \zeta(114)$ etc. can also be expressed by these equations.

Because I use π^2 , these are to be irrational numbers.

The fact that the even value of $\zeta(2n)$ is irrational can also be explained by the fact that each even value of $\zeta(2n)$ is multiplied by π^2 .

key words

irrational number, π^2 , $\zeta(4), \zeta(6), \dots, \zeta(108), \zeta(110)$

1 introduction

(Proof)

If π is assumed to be rational number. $\pi = \frac{m}{n}$, m and n are integer.

Equal $n\pi = m$

Square on both sides.

$n^2\pi^2 = m^2$, it equal $\pi^2 = \frac{m^2}{n^2}$. it equal $\pi = \frac{m}{n}$

But, $\pi \neq \frac{m}{n}$.

This is because π is known to be an irrational number.

This contradicts.

π^2 is irrational number.

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2 Discussion

$$\zeta(4) = \frac{\pi^2}{15}\zeta(2) = \pi^2 \frac{1}{3 \times 5}\zeta(2) \quad (1)$$

$$\zeta(6) = \frac{2\pi^2}{21}\zeta(4) = \pi^2 \frac{2}{3 \times 7}\zeta(4) \quad (2)$$

$$\zeta(8) = \frac{\pi^2}{10}\zeta(6) = \pi^2 \frac{1}{2 \times 5}\zeta(6) \quad (3)$$

$$\zeta(10) = \frac{10\pi^2}{99}\zeta(8) = \pi^2 \frac{2 \times 5}{3^2 \times 11}\zeta(8) \quad (4)$$

$$\zeta(12) = \frac{691\pi^2}{6825}\zeta(10) = \pi^2 \frac{691}{3 \times 5^2 \times 7 \times 13}\zeta(10) \quad (5)$$

$$\zeta(14) = \frac{70}{691}\pi^2\zeta(12) = \pi^2 \frac{2 \times 5 \times 7}{691}\zeta(12) \quad (6)$$

$$\zeta(16) = \frac{3617\pi^2}{35700}\zeta(14) = \pi^2 \frac{3617}{2^2 \times 3 \times 5^2 \times 7 \times 17}\zeta(14) \quad (7)$$

$$\zeta(18) = \frac{438670\pi^2}{4329549}\zeta(16) = \pi^2 \frac{2 \times 5 \times 43867}{3^2 \times 7 \times 19 \times 3617}\zeta(16) \quad (8)$$

$$\zeta(20) = \frac{1222277\pi^2}{12063425}\zeta(18) = \pi^2 \frac{7 \times 283 \times 617}{5^2 \times 11 \times 43867}\zeta(18) \quad (9)$$

$$\zeta(22) = \frac{8545130\pi^2}{84337113}\zeta(20) = \pi^2 \frac{2 \times 5 \times 11 \times 131 \times 593}{3 \times 7 \times 23 \times 283 \times 617}\zeta(20) \quad (10)$$

$$\zeta(24) = \frac{236364091\pi^2}{2332820490}\zeta(22) = \pi^2 \frac{103 \times 2294797}{2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 131 \times 593}\zeta(22) \quad (11)$$

$$\zeta(26) = \frac{119743442\pi^2}{1181820455}\zeta(24) = \pi^2 \frac{2 \times 7 \times 13 \times 657931}{5 \times 103 \times 2294797}\zeta(24) \quad (12)$$

$$\zeta(28) = \frac{3392780147\pi^2}{33485398245}\zeta(26) = \pi^2 \frac{9349 \times 362903}{3^3 \times 5 \times 13 \times 29 \times 657931}\zeta(26) \quad (13)$$

$$\zeta(30) = \frac{17231682552010\pi^2}{170069890428669}\zeta(28) = \pi^2 \frac{2 \times 5 \times 1721 \times 1001259881}{3 \times 7^2 \times 11 \times 31 \times 9349 \times 362903}\zeta(28) \quad (14)$$

$$\zeta(32) = \frac{593617720173709\pi^2}{5858772067683400}\zeta(30) = \pi^2 \frac{7 \times 11 \times 37 \times 683 \times 305065927}{2^3 \times 5^2 \times 17 \times 1721 \times 1001259881}\zeta(30) \quad (15)$$

$$\zeta(34) = \frac{25776878583670\pi^2}{254407594360161}\zeta(32) = \pi^2 \frac{2 \times 5 \times 17 \times 151628697551}{3 \times 11 \times 37 \times 683 \times 305065927}\zeta(32) \quad (16)$$

$$\zeta(36) = \frac{26315271553053477373\pi^2}{259721319947216543325}\zeta(34) = \pi^2 \frac{26315271553053477373}{3^2 \times 5^2 \times 7^2 \times 13 \times 17 \times 19 \times 37 \times 151628697551}\zeta(34) \quad (17)$$

$$\zeta(38) = \frac{2666294461595818690\pi^2}{26315271553053477373}\zeta(36) = \pi^2 \frac{2 \times 5 \times 7 \times 13 \times 19 \times 154210205991661}{26315271553053477373}\zeta(36) \quad (18)$$

$$\zeta(40) = \frac{261082718496449122051\pi^2}{2576783147527959062550}\zeta(38) = \pi^2 \frac{137616929 \times 1897170067619}{2 \times 3 \times 5^2 \times 11 \times 13 \times 19 \times 41 \times 154210205991661}\zeta(38) \quad (19)$$

$$\zeta(42) = \frac{167210740830987788296010\pi^2}{1650303863616054900484371}\zeta(40) = \pi^2 \frac{2 \times 5 \times 11 \times 1520097643918070802691}{3 \times 7^2 \times 43 \times 137616929 \times 1897170067619}\zeta(40) \quad (20)$$

$$\zeta(44) = \frac{17712080641373379058651\pi^2}{174811229050578142309465}\zeta(42) = \pi^2 \frac{7 \times 59 \times 8089 \times 2947939 \times 1798482437}{5 \times 23 \times 1520097643918070802691}\zeta(42) \quad (21)$$

$$\zeta(46) = \frac{1192902223187824326555922\pi^2}{11773473032044333251414729}\zeta(44) = \pi^2 \frac{2 \times 23 \times 383799511 \times 67568238839737}{3^2 \times 11 \times 47 \times 59 \times 8089 \times 2947939 \times 1798482437}\zeta(44) \quad (22)$$

$$\zeta(48) = \frac{5609403368997817686249127547\pi^2}{55362592178146926995460340020}\zeta(46) \quad (23)$$

$$= \pi^2 \frac{653 \times 56039 \times 153289748932447906241}{2^2 \times 3 \times 5 \times 7 \times 13 \times 17 \times 23 \times 383799511 \times 67568238839737}\zeta(46) \quad (24)$$

$$\zeta(50) = \frac{43763056943311440901983013210\pi^2}{431924059412831961841182821119}\zeta(48) \quad (25)$$

$$= \pi^2 \frac{2 \times 5 \times 13 \times 17 \times 417202699 \times 47464429777438199}{7 \times 11 \times 653 \times 56039 \times 153289748932447906241}\zeta(48) \quad (26)$$

$$\zeta(52) = \frac{677909453806953040832859608491\pi^2}{6690698128833191445591633750375}\zeta(50) \quad (27)$$

$$= \pi^2 \frac{11 \times 577 \times 58741 \times 401029177 \times 4534045619429}{3 \times 5^3 \times 17 \times 53 \times 417202699 \times 47464429777438199}\zeta(50) \quad (28)$$

$$\zeta(54) = \frac{291499636348848624214181238126910\pi^2}{2876986093824544436836398645744123}\zeta(52) \quad (29)$$

$$= \pi^2 \frac{2 \times 5 \times 39409 \times 660183281 \times 1120412849144121779}{3^3 \times 7 \times 13 \times 19 \times 577 \times 58741 \times 401029177 \times 4534045619429}\zeta(52) \quad (30)$$

Below, there is no factorization.

$$\zeta(56) = \frac{47108465656951308320022899053601351\pi^2}{464941919976413555621619074812421450}\zeta(54) \quad (31)$$

$$\zeta(58) = \frac{844836133488800418620467759940360210\pi^2}{8338198421280381572644053132487439127}\zeta(56) \quad (32)$$

$$\zeta(60) = \frac{1215233140483755572040304994079820246041491\pi^2}{11993870351668116849021868789359512837003325}\zeta(58) \quad (33)$$

$$\zeta(62) = \frac{123128860195209454004949928972607895371510\pi^2}{1215233140483755572040304994079820246041491}\zeta(60) \quad (34)$$

$$\zeta(64) = \frac{106783830147866529886385444979142647942017\pi^2}{1053914159992562039874536454982322125417680}\zeta(62) \quad (35)$$

$$\zeta(66) = \frac{4551672795663219294341369142796881785111324494\pi^2}{44923209856396414057493152004610457990670957781}\zeta(64) \quad (36)$$

$$\zeta(68) = \frac{746027886367865601814551354268494800314109833\pi^2}{7363000110631678270258097142759661711209495505}\zeta(66) \quad (37)$$

$$\zeta(70) = \frac{430108956380962001086593304965102059574696617210\pi^2}{424500524884549354083934214169548805901615536889}\zeta(68) \quad (38)$$

$$\zeta(72) = \frac{64107504578369385214821049691051741018107371320286864719\pi^2}{632715709329530691786313007298754240944671673600368713350}\zeta(70) \quad (39)$$

$$\zeta(74) = \frac{590495294930633994967766974817161862377840841569574230\pi^2}{5827954961669944110438277244641067365282488301844260429}\zeta(72) \quad (40)$$

$$\zeta(76) = \frac{1297636253996598563562484002136063152861329885729779\pi^2}{1280715648345793800537377649302238195008115418657625}\zeta(74) \quad (41)$$

$$\zeta(78) = \frac{319112588904154483303983873499647748840153365671077294990\pi^2}{3149515011891462315798085483812475191383332408262804017143}\zeta(76) \quad (42)$$

$$\zeta(80) = \frac{32226490096356203528549024783133327948559805109259022437598069\pi^2}{318062708486659815053913766555833907746469261098019450689482900}\zeta(78) \quad (43)$$

$$\zeta(82) = \frac{3136016458976222714859298933300324713388466371556373885828616310\pi^2}{30951241845400393760347870516715051971166795678506915406853116841}\zeta(80) \quad (44)$$

$$\zeta(84) = \frac{2024576195935290360231131160111731009989917391198090877281083932477\pi^2}{19981766133743693383144015952236737693916692426061912909854683832105}\zeta(82) \quad (45)$$

$$\zeta(86) = \frac{3487251761286507933562768938715808177354390364156586781839820076274\pi^2}{34417795330899936123929229721899427169828595650367544913778426852109}\zeta(84) \quad (46)$$

$$\zeta(88) = \frac{119220589879456137090501038547210167576886388688366240390629080961277\pi^2}{1176660058574749737363129893002954836106225891554153814685622807054870}\zeta(86) \quad (47)$$

$$\zeta(90) = \frac{54236634834969808794669674157463893853661727684803767346651079923484436610\pi^2}{535294129867494444361083866521930650281338310232163935943183870174842782319}\zeta(88) \quad (48)$$

$$\zeta(92) = \frac{621739231052265435437634464195351363679940261685669231290131581477758813\pi^2}{6136320251123352561577507390232286338898473375197226473743391199401709275}\zeta(90) \quad (49)$$

$$\zeta(94) = \frac{12208138065797444696073016794132012039585084152026966214362151052846494470\pi^2}{120489493183300990103016788054087637718318757985907468301168418395185000971} \zeta(92) \quad (50)$$

$$\zeta(96) = \frac{211600449597266513097597728109824233673043954389060234150638733420050668349987259\pi^2}{2088412728617668579158743742906147377898831882756676556403505828316565009588052200} \zeta(94) \quad (51)$$

$$\zeta(98) = \frac{21439608012445877904164709968221709770318036079999273913579064908926344714101170\pi^2}{211600449597266513097597728109824233673043954389060234150638733420050668349987259} \zeta(96) \quad (52)$$

$$\zeta(100) = \frac{94598037819122125295227433069493721872702841533066936133385696204311395415197247711\pi^2}{933645210394025319774219795976423272143245095747810009579441990490684281748581171375} \zeta(98) \quad (53)$$

$$\zeta(102) = \frac{20731890305570575212160722707810976194358817158046995269099427948209104023060858674530\pi^2}{204615555802761157013576937729314920410656246236023782856513260889925548283071646798893} \quad (54)$$

$$\times \zeta(100) \quad (55)$$

$$\zeta(104) = \frac{172056570734370006077671036121016917130640557119069936022286483933842075273757542197567\pi^2}{1698130287756280751468801014521608141010663114490940248859871325576036611343439424159230} \quad (56)$$

$$\times \zeta(102) \quad (57)$$

$$\zeta(106) = \frac{72747806345234828816303640303186854338462597281163380077861632756563759746772404693145802\pi^2}{717992069674526035362121233733003595186163044857878843021001497455922980117390223590447091} \quad (58)$$

$$\times \zeta(104) \quad (59)$$

$$\zeta(108) = \frac{3469342247847828789552088659323852541399766785760491146870005891371501266319724897592306597338057\pi^2}{34241035518244179431189627387059297144313796366647816102702568224409569907277094216785190099328195} \quad (60)$$

$$\times \zeta(106) \quad (61)$$

$$\zeta(110) = \frac{978274212763260914091579784331393365302651756286451147428539101388233064531768422205056259610174190\pi^2}{9655179475760507521323462738898281622715550964771446861739226395686888024167794389999389260391812631} \times \zeta(108) \tag{63}$$

$\zeta(112), \zeta(114)$ etc. can also be expressed by these equations

3 Conclusion

The fact that the even value of $\zeta(2n)$ is irrational can also be explained by the fact that each even value of $\zeta(2n)$ is multiplied by π^2 .

4 Appendices

I use WolframAlpha for calculation.

References

[1] T. takami.: $\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$ are Irrational Number, viXra:1909.0315