The characteristic of primes

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Abstract

In this paper, we propose the axiomatic regularity of prime numbers.

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1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our goal is to axiomatize the structure of primes.

2 Results

These below are some patterns of number.

Let $t_n$ denote the $n$th triangular number. Then

$$t_n = \binom{n + 1}{2} \quad n \geq 1,$$

where $\binom{n}{k}$ is the binomial coefficients.

Let $F_n$ be the $n$th Fibonacci number. Then

$$F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},$$

where $n$ is a positive integer.
Let $B_n$ be the $n$th Bernoulli number. Then

$$B_n = (-1)^{n+1} n \zeta(1-n),$$

where $\zeta(1-n)$ is the Riemann zeta-function.

If $p(n)$ denotes the total number of partitions of $n$, then

$$p(n) \sim \frac{e^{\sqrt{2n/3}}}{4n^{3/4}},$$

where $n$ is a positive integer.

**Postulate 2.1** (Peano Postulates). Given the number 0, the set $\mathbb{N}$, and the function $\sigma$. Then:

1. $0 \in \mathbb{N}$.
2. $\sigma : \mathbb{N} \to \mathbb{N}$ is a function from $\mathbb{N}$ to $\mathbb{N}$.
3. $0 \not\in \text{range}(\sigma)$.
4. The function $\sigma$ is one-to-one.
5. If $I \subset \mathbb{N}$ such that $0 \in I$ and $\sigma(n) \in I$ whenever $n \in I$, then $I = \mathbb{N}$.

We define $1 = \sigma(0)$, $2 = \sigma(1)$, $3 = \sigma(2)$, etc. Next, we propose the fundamental properties of prime numbers.

**Definition 2.2.** For an integer $n > 1$, where $\tau(n)$ denote the number of positive divisors of $n$. The function $\chi(n)$ is defined by

$$\chi(n) = \begin{cases} 0 \quad &\text{if } \tau(n) = 2 \\ 1 \quad &\text{if } \tau(n) > 2. \end{cases}$$

**Definition 2.3.** Given an integer $n > 1$, let $\Delta(n)$ denote the number of positive divisors of $n$ besides 1 and $n$.

**Postulate 2.4.** Given a prime number $p$, $\sigma(n)$ denotes the sum of positive divisors of $n$. Then:

1. $2 \leq p$.
2. $4 \nmid p$.
3. $(-1)^{\chi(p)} = 1$. 

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4. $3 \leq \sigma(p)$.

5. $\Delta(p) = 0$.

By our observation, we get the estimation. Let $p_n$ be the $n$th prime, where $n$ is a positive integer. Then

$$\frac{p_{n+1}}{p_n} \leq 1.7.$$ 

References