The characteristic of primes

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Abstract

In this paper, we propose the axiomatic regularity of prime numbers.

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1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our goal is to axiomatize the structure of primes.

2 Results

These below are some patterns of number.

Let t_n denote the nth triangular number. Then

$$t_n = \binom{n+1}{2} \qquad n \geq 1,$$

where $\binom{n}{k}$ is the binomial coefficients.

Let F_n be the nth Fibonacci number. Then

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}},$$

where n is a positive integer.

Let B_n be the nth Bernoulli number. Then

$$B_n = (-1)^{n+1} n\zeta(1-n),$$

where $\zeta(1-n)$ is the Riemann zeta-function.

If $p(n)$ denotes the total number of partitions of n, then

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}},$$

where n is a positive integer.

Postulate 2.1 (Peano Postulates)**.** Given the number 0, the set $\mathbf{N}$, and the function σ. Then:

1. $0 \in \mathbf{N}$.
2. $\sigma : \mathbf{N} \to \mathbf{N}$ is a function from $\mathbf{N}$ to $\mathbf{N}$.
3. $0 \notin \text{range}(\sigma)$.
4. The function σ is one-to-one.
5. If $I \subset \mathbf{N}$ such that $0 \in I$ and $\sigma(n) \in I$ whenever $n \in I$, then $I = \mathbf{N}$.

We define $1 = \sigma(0)$, $2 = \sigma(1)$, $3 = \sigma(2)$, etc. Next, we propose the fundamental properties of prime numbers.

Definition 2.2. For an integer $n > 1$, where $\tau(n)$ denote the number of positive divisors of n. The function $\chi(n)$ is defined by

$$\chi(n) = \begin{cases} 0 & \text{if } \tau(n) = 2 \\ 1 & \text{if } \tau(n) > 2. \end{cases}$$

Definition 2.3. Given an integer $n > 1$, let $\Delta(n)$ denote the number of positive divisors of n besides 1 and n.

Postulate 2.4. Given a prime number p, $\sigma(n)$ denotes the sum of positive divisors of n. Then:

1. $2 \leq p$.
2. $4 \nmid p$.
3. $(-1)^{\chi(p)} = 1$.

4. $3 \leq \sigma(p)$.
5. $\Delta(p) = 0$.

By our observation, we get the estimation. Let p_n be the nth prime, where n is a positive integer. Then

$$\frac{p_{n+1}}{p_n} \leq 1.7.$$

References

[Rie59] B. Riemann. Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse. *Monatsber. Akad. Berlin*, pages 671–680, 1859.