$\zeta(5), \zeta(7)..........\zeta(331), \zeta(333)$ are irrational number

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Abstract

Using the fact that $\zeta(3)$ is an irrational number, I prove that $\zeta(5)$, $\zeta(7)$...........$\zeta(331)$ and $\zeta(333)$ are irrational numbers.

$\zeta(5)$, $\zeta(7)$...........$\zeta(331)$ and $\zeta(333)$ are confirmed that they were in perfect numerical agreement.

This is because I created an odd-number formula for ζ, and the formula was created by dividing the odd-number for ζ itself into odd and even numbers.

key words

irrational number, $\zeta(3)$, odd-number formula for ζ, $\zeta(5)$, $\zeta(7)$, $\zeta(331)$, $\zeta(333)$

1 Introduction

Write the formula I finally got in advance.

$$\zeta(2m+1) = \frac{(2^{2m+1}-4)}{(2^{2m+1}-1)}\Big[1 + \frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}} \tag{1}$$

m is a positive integer.

and

$$\zeta(2m+1) = \zeta(2m-1)\frac{(2^{2m+1}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}}} \tag{2}$$

m is a positive integer.
and

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Eq.(1) and Eq.(2) are equation these are modification of Eq.(3).

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m-1}} \tag{3}$$

m is a positive integer.

In detail

$$\zeta(3) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty}\frac{1}{(2n)^3} = \frac{1}{2^3}\sum_{n=1}^{\infty}\frac{1}{n^3} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{1}{2^3}\zeta(3) + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{4}$$

$$\zeta(3) = \frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{5}$$

do the same

$$\zeta(5) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty}\frac{1}{(2n)^5} = \frac{1}{2^5}\sum_{n=1}^{\infty}\frac{1}{n^5} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} = \frac{1}{2^5}\zeta(5) + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{6}$$

$$\zeta(5) = \frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} = \frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{7}$$

Use these for $\zeta(7), \zeta(9), \zeta(11)$ etc.

2 Discussion

Example 1

from Eq.(3)
if m=2

$$\zeta(3) = \frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{8}$$

if m=3

$$\zeta(5) = \frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} = \frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{9}$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \zeta(3)\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{10}$$

$$\zeta(5)\frac{8}{7} = \frac{32}{31}\zeta(3)\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{11}$$

$$\zeta(5)=\frac{7}{8}\frac{32}{31}\Big[\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{28}{31}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{12}$$

from Eq.(3)
if m=3

$$\zeta(5)\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{13}$$

if m=4

$$\zeta(7)\frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{14}$$

Multiply $\zeta(5)$ and $\zeta(7)$.

$$\zeta(7)\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\zeta(5)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\zeta(5)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{15}$$

$$\zeta(7)\frac{32}{31}=\zeta(5)\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{16}$$

$$\zeta(7)=\zeta(5)\frac{31}{32}\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{17}$$

$$\zeta(7)=\zeta(5)\frac{124}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}=\frac{124}{127}\zeta(5)\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}} \tag{18}$$

$$=\frac{124}{127}\Big[\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}=\frac{124}{127}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{19}$$

The new formula Eq.(1) has been followed.

Do the same for $\zeta(9),\zeta(11),\zeta(13)$ etc.

(Proof 1)
If $\zeta(5)$ is assumed to be rational number. $\zeta(5)=\frac{s}{t}$, s and t are integer.

from formula Eq.(1).
$\zeta(5)=\zeta(3)\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}$ $\frac{28\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{31\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\frac{o}{p}$ o, p are assumed to be integer.

$\zeta(5)=\zeta(3)\frac{o}{p}$ it equal $\zeta(3)=\zeta(5)\frac{p}{o}=\frac{sp}{to}$ But, $\zeta(3)\neq\frac{sp}{to}$

This is because $\zeta(3)$ is known to be an irrational number.
This contradicts.

$\zeta(5)$ is irrational number.

(Proof end)

Do the same, sequentially prove that $\zeta(7), \zeta(9), \zeta(11)$ etc. are irrational numbers.

$$\zeta(3)=\zeta(1)\frac{(2^3-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{(2^3-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}=\frac{(2^3-4)}{(2^3-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^1}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{20}$$

$$\zeta(5)=\zeta(3)\frac{(2^5-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{(2^5-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\frac{(2^5-4)}{(2^5-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{21}$$

$$\zeta(7)=\zeta(5)\frac{(2^7-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{(2^7-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}=\frac{(2^7-4)}{(2^7-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{22}$$

$$\zeta(9)=\zeta(7)\frac{(2^9-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{(2^9-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}=\frac{(2^9-4)}{(2^9-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{23}$$

$$\zeta(11)=\zeta(9)\frac{(2^{11}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{(2^{11}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}=\frac{(2^{11}-4)}{(2^{11}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{24}$$

$$\zeta(13)=\zeta(11)\frac{(2^{13}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{(2^{13}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}=\frac{(2^{13}-4)}{(2^{13}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{11}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{25}$$

$$\zeta(15)=\zeta(13)\frac{(2^{15}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{(2^{15}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}=\frac{(2^{15}-4)}{(2^{15}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{26}$$

$$\zeta(17)=\zeta(15)\frac{(2^{17}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{(2^{17}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}=\frac{(2^{17}-4)}{(2^{17}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{15}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{27}$$

$$\zeta(19)=\zeta(17)\frac{(2^{19}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{(2^{19}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}=\frac{(2^{19}-4)}{(2^{19}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{17}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{28}$$

$$\zeta(21)=\zeta(19)\frac{(2^{21}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{(2^{17}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}=\frac{(2^{21}-4)}{(2^{21}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{19}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}} \tag{29}$$

$$\zeta(23)=\zeta(21)\frac{(2^{23}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{(2^{23}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}=\frac{(2^{23}-4)}{(2^{23}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{21}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}} \tag{30}$$

$$\zeta(25)=\zeta(23)\frac{(2^{25}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{(2^{25}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}=\frac{(2^{25}-4)}{(2^{25}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{23}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}} \tag{31}$$

$$\zeta(27)=\zeta(25)\frac{(2^{27}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{(2^{27}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}=\frac{(2^{27}-4)}{(2^{27}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{25}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}} \tag{32}$$

$$\zeta(29)=\zeta(27)\frac{(2^{29}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{(2^{29}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}=\frac{(2^{29}-4)}{(2^{29}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{27}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}} \tag{33}$$

$$\zeta(31)=\zeta(29)\frac{(2^{31}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{(2^{31}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}=\frac{(2^{31}-4)}{(2^{31}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{29}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}} \tag{34}$$

$$\zeta(33)=\zeta(31)\frac{(2^{33}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{(2^{33}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}=\frac{(2^{33}-4)}{(2^{33}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{31}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}} \tag{35}$$

$$\zeta(35)=\zeta(33)\frac{(2^{35}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{(2^{35}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}=\frac{(2^{35}-4)}{(2^{35}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{33}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}} \tag{36}$$

$$\zeta(37)=\zeta(35)\frac{(2^{37}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{(2^{37}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}=\frac{(2^{37}-4)}{(2^{37}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{35}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}} \tag{37}$$

$$\zeta(39)=\zeta(37)\frac{(2^{39}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{(2^{39}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}=\frac{(2^{39}-4)}{(2^{39}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{37}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}} \tag{38}$$

$$\zeta(41)=\zeta(39)\frac{(2^{41}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{(2^{41}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}=\frac{(2^{41}-4)}{(2^{41}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{39}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}} \tag{39}$$

$$\zeta(43)=\zeta(41)\frac{(2^{43}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{(2^{43}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}=\frac{(2^{43}-4)}{(2^{43}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{41}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}} \tag{40}$$

$$\zeta(45)=\zeta(43)\frac{(2^{45}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{(2^{45}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}=\frac{(2^{45}-4)}{(2^{45}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{43}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}} \tag{41}$$

$$\zeta(47)=\zeta(45)\frac{(2^{47}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{(2^{47}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}=\frac{(2^{47}-4)}{(2^{47}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{45}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}} \tag{42}$$

$$\zeta(49)=\zeta(47)\frac{(2^{49}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{(2^{49}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}=\frac{(2^{49}-4)}{(2^{49}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{47}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}} \tag{43}$$

$$\zeta(51)=\zeta(49)\frac{(2^{51}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{(2^{51}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}=\frac{(2^{51}-4)}{(2^{51}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{49}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}} \tag{44}$$

$$\zeta(53)=\zeta(51)\frac{(2^{53}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{(2^{53}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}=\frac{(2^{53}-4)}{(2^{53}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{51}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}} \tag{45}$$

$$\zeta(55)=\zeta(53)\frac{(2^{55}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{(2^{55}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}=\frac{(2^{55}-4)}{(2^{55}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{53}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}} \tag{46}$$

$$\zeta(57)=\zeta(55)\frac{(2^{57}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{(2^{57}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}=\frac{(2^{57}-4)}{(2^{57}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{55}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}} \tag{47}$$

$$\zeta(59)=\zeta(57)\frac{(2^{59}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{(2^{59}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}=\frac{(2^{59}-4)}{(2^{59}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{57}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}} \tag{48}$$

$$\zeta(61)=\zeta(59)\frac{(2^{61}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{(2^{61}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}=\frac{(2^{61}-4)}{(2^{61}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{59}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}} \tag{49}$$

$$\zeta(63)=\zeta(61)\frac{(2^{63}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{(2^{63}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}=\frac{(2^{63}-4)}{(2^{63}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{61}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}} \tag{50}$$

$$\zeta(65)=\zeta(63)\frac{(2^{65}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{(2^{65}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}=\frac{(2^{65}-4)}{(2^{65}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{63}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}} \tag{51}$$

$$\zeta(67)=\zeta(65)\frac{(2^{67}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{(2^{67}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}=\frac{(2^{67}-4)}{(2^{67}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{65}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}} \tag{52}$$

$$\zeta(69)=\zeta(67)\frac{(2^{69}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{(2^{69}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}=\frac{(2^{69}-4)}{(2^{69}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{67}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}} \tag{53}$$

$$\zeta(71)=\zeta(69)\frac{(2^{71}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{(2^{71}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}=\frac{(2^{71}-4)}{(2^{71}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{69}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}} \tag{54}$$

$$\zeta(73)=\zeta(71)\frac{(2^{73}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{(2^{73}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}=\frac{(2^{73}-4)}{(2^{73}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{71}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}} \tag{55}$$

$$\zeta(75)=\zeta(73)\frac{(2^{75}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{(2^{75}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}=\frac{(2^{75}-4)}{(2^{75}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{73}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}} \tag{56}$$

$$\zeta(77)=\zeta(75)\frac{(2^{77}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{(2^{77}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}=\frac{(2^{77}-4)}{(2^{77}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{75}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}} \tag{57}$$

$$\zeta(79)=\zeta(77)\frac{(2^{79}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{(2^{79}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}=\frac{(2^{79}-4)}{(2^{79}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{77}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}} \tag{58}$$

$$\zeta(81)=\zeta(79)\frac{(2^{81}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{(2^{81}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}=\frac{(2^{81}-4)}{(2^{81}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{79}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}} \tag{59}$$

$$\zeta(83)=\zeta(81)\frac{(2^{83}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{(2^{83}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}=\frac{(2^{83}-4)}{(2^{83}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{81}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}} \tag{60}$$

$$\zeta(85)=\zeta(83)\frac{(2^{85}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{(2^{85}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}=\frac{(2^{85}-4)}{(2^{85}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{83}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}} \tag{61}$$

$$\zeta(87)=\zeta(85)\frac{(2^{87}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{(2^{87}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}=\frac{(2^{87}-4)}{(2^{87}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{85}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}} \tag{62}$$

$$\zeta(89)=\zeta(87)\frac{(2^{89}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{(2^{89}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}=\frac{(2^{89}-4)}{(2^{89}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{87}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}} \tag{63}$$

$$\zeta(91)=\zeta(89)\frac{(2^{91}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{(2^{91}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}=\frac{(2^{91}-4)}{(2^{91}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{89}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}} \tag{64}$$

$$\zeta(93)=\zeta(91)\frac{(2^{93}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{(2^{93}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}=\frac{(2^{93}-4)}{(2^{93}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{91}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}} \tag{65}$$

$$\zeta(95)=\zeta(93)\frac{(2^{95}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{(2^{95}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}=\frac{(2^{95}-4)}{(2^{95}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{93}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}} \tag{66}$$

$$\zeta(97)=\zeta(95)\frac{(2^{97}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{(2^{97}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}=\frac{(2^{97}-4)}{(2^{97}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{95}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}} \tag{67}$$

$$\zeta(99)=\zeta(97)\frac{(2^{99}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{(2^{99}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}=\frac{(2^{99}-4)}{(2^{99}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{97}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}} \tag{68}$$

$$\zeta(101)=\zeta(99)\frac{(2^{101}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{(2^{101}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}=\frac{(2^{101}-4)}{(2^{101}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{99}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}} \tag{69}$$

$$\zeta(103)=\zeta(101)\frac{(2^{103}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{(2^{103}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}=\frac{(2^{103}-4)}{(2^{103}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{101}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}} \tag{70}$$

$$\zeta(105)=\zeta(103)\frac{(2^{105}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{(2^{105}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}=\frac{(2^{105}-4)}{(2^{105}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{103}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}} \tag{71}$$

$$\zeta(107)=\zeta(105)\frac{(2^{107}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{(2^{107}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}=\frac{(2^{107}-4)}{(2^{107}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{105}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}} \tag{72}$$

$$\zeta(109)=\zeta(107)\frac{(2^{109}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{(2^{109}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}=\frac{(2^{109}-4)}{(2^{109}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{107}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}} \tag{73}$$

$$\zeta(111)=\zeta(109)\frac{(2^{111}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{(2^{111}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}=\frac{(2^{111}-4)}{(2^{111}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{109}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}} \tag{74}$$

$$\zeta(113)=\zeta(111)\frac{(2^{113}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{(2^{113}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}=\frac{(2^{113}-4)}{(2^{113}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{111}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}} \tag{75}$$

$$\zeta(115)=\zeta(113)\frac{(2^{115}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{(2^{115}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}=\frac{(2^{115}-4)}{(2^{115}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{113}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}} \tag{76}$$

$$\zeta(117)=\zeta(115)\frac{(2^{117}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{(2^{117}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}=\frac{(2^{117}-4)}{(2^{117}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{115}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}} \tag{77}$$

$$\zeta(119)=\zeta(117)\frac{(2^{119}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{(2^{119}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}=\frac{(2^{117}-4)}{(2^{117}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{117}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}} \tag{78}$$

$$\zeta(121)=\zeta(119)\frac{(2^{121}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{(2^{121}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}=\frac{(2^{121}-4)}{(2^{121}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{119}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}} \tag{79}$$

$$\zeta(123)=\zeta(121)\frac{(2^{123}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{(2^{123}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}=\frac{(2^{123}-4)}{(2^{123}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{121}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}} \tag{80}$$

$$\zeta(125)=\zeta(123)\frac{(2^{125}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{(2^{125}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}=\frac{(2^{125}-4)}{(2^{125}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{123}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}} \tag{81}$$

$$\zeta(127)=\zeta(125)\frac{(2^{127}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{(2^{127}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}=\frac{(2^{127}-4)}{(2^{127}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{125}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}} \tag{82}$$

$$\zeta(129)=\zeta(127)\frac{(2^{129}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{(2^{129}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}=\frac{(2^{129}-4)}{(2^{129}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{127}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}} \tag{83}$$

$$\zeta(131)=\zeta(129)\frac{(2^{131}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{(2^{131}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}=\frac{(2^{131}-4)}{(2^{131}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{129}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}} \tag{84}$$

$$\zeta(133)=\zeta(131)\frac{(2^{133}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{(2^{133}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}=\frac{(2^{133}-4)}{(2^{133}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{131}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}} \tag{85}$$

$$\zeta(135)=\zeta(133)\frac{(2^{135}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{(2^{135}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}=\frac{(2^{135}-4)}{(2^{135}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{133}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}} \tag{86}$$

$$\zeta(137)=\zeta(135)\frac{(2^{137}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{(2^{137}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}=\frac{(2^{137}-4)}{(2^{137}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{135}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}} \tag{87}$$

$$\zeta(139)=\zeta(137)\frac{(2^{139}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{(2^{139}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}=\frac{(2^{139}-4)}{(2^{139}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{137}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}} \tag{88}$$

$$\zeta(141)=\zeta(139)\frac{(2^{141}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{(2^{141}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}=\frac{(2^{141}-4)}{(2^{141}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{139}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}} \tag{89}$$

$$\zeta(143)=\zeta(141)\frac{(2^{143}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{(2^{143}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}=\frac{(2^{143}-4)}{(2^{143}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{141}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}} \tag{90}$$

$$\zeta(145)=\zeta(143)\frac{(2^{145}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{(2^{145}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}=\frac{(2^{145}-4)}{(2^{145}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{143}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}} \tag{91}$$

$$\zeta(147)=\zeta(145)\frac{(2^{147}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{(2^{147}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}=\frac{(2^{147}-4)}{(2^{147}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{145}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}} \tag{92}$$

$$\zeta(149)=\zeta(147)\frac{(2^{149}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{(2^{149}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}=\frac{(2^{149}-4)}{(2^{149}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{147}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}} \tag{93}$$

$$\zeta(151)=\zeta(149)\frac{(2^{151}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{(2^{151}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}=\frac{(2^{151}-4)}{(2^{151}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{149}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}} \tag{94}$$

$$\zeta(153)=\zeta(151)\frac{(2^{153}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{(2^{153}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}=\frac{(2^{153}-4)}{(2^{153}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{151}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}} \tag{95}$$

$$\zeta(155)=\zeta(153)\frac{(2^{155}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{(2^{155}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}=\frac{(2^{155}-4)}{(2^{155}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{153}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}} \tag{96}$$

$$\zeta(157)=\zeta(155)\frac{(2^{157}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{(2^{157}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}=\frac{(2^{157}-4)}{(2^{157}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{155}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}} \tag{97}$$

$$\zeta(159)=\zeta(157)\frac{(2^{159}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{(2^{159}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}=\frac{(2^{159}-4)}{(2^{159}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{157}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}} \tag{98}$$

$$\zeta(161)=\zeta(159)\frac{(2^{161}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}{(2^{161}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}=\frac{(2^{161}-4)}{(2^{161}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{159}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}} \tag{99}$$

$$\zeta(163)=\zeta(161)\frac{(2^{163}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}{(2^{163}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}=\frac{(2^{163}-4)}{(2^{163}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{161}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}} \tag{100}$$

$$\zeta(165)=\zeta(163)\frac{(2^{165}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}{(2^{165}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}=\frac{(2^{165}-4)}{(2^{165}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{163}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}} \tag{101}$$

$$\zeta(167)=\zeta(165)\frac{(2^{167}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}{(2^{167}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}=\frac{(2^{167}-4)}{(2^{167}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{165}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}} \tag{102}$$

$$\zeta(169)=\zeta(167)\frac{(2^{169}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}{(2^{169}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}=\frac{(2^{169}-4)}{(2^{169}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{167}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}} \tag{103}$$

$$\zeta(171)=\zeta(169)\frac{(2^{171}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}{(2^{171}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}=\frac{(2^{171}-4)}{(2^{171}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{169}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}} \tag{104}$$

$$\zeta(173)=\zeta(171)\frac{(2^{173}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}{(2^{173}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}=\frac{(2^{173}-4)}{(2^{173}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{171}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}} \tag{105}$$

$$\zeta(175)=\zeta(173)\frac{(2^{175}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}{(2^{175}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}=\frac{(2^{175}-4)}{(2^{175}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{173}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}} \tag{106}$$

$$\zeta(177)=\zeta(175)\frac{(2^{177}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}{(2^{177}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}=\frac{(2^{177}-4)}{(2^{177}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{175}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}} \tag{107}$$

$$\zeta(179)=\zeta(177)\frac{(2^{179}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}{(2^{179}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}=\frac{(2^{179}-4)}{(2^{179}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{177}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}} \tag{108}$$

$$\zeta(181)=\zeta(179)\frac{(2^{181}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}{(2^{181}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}=\frac{(2^{181}-4)}{(2^{181}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{179}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}} \tag{109}$$

$$\zeta(183)=\zeta(181)\frac{(2^{183}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}{(2^{183}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}=\frac{(2^{183}-4)}{(2^{183}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{181}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}} \tag{110}$$

$$\zeta(185)=\zeta(183)\frac{(2^{185}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{(2^{185}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}=\frac{(2^{185}-4)}{(2^{185}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{183}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}} \tag{111}$$

$$\zeta(187)=\zeta(185)\frac{(2^{187}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{(2^{187}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}=\frac{(2^{187}-4)}{(2^{187}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{185}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}} \tag{112}$$

$$\zeta(189)=\zeta(187)\frac{(2^{189}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{(2^{189}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}=\frac{(2^{189}-4)}{(2^{189}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{187}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}} \tag{113}$$

$$\zeta(191)=\zeta(189)\frac{(2^{191}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{(2^{191}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}=\frac{(2^{191}-4)}{(2^{191}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{189}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}} \tag{114}$$

$$\zeta(193)=\zeta(191)\frac{(2^{193}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{(2^{193}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}=\frac{(2^{193}-4)}{(2^{193}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{191}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}} \tag{115}$$

$$\zeta(195)=\zeta(193)\frac{(2^{195}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{(2^{195}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}=\frac{(2^{195}-4)}{(2^{195}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{193}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}} \tag{116}$$

$$\zeta(197)=\zeta(195)\frac{(2^{197}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{(2^{197}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}=\frac{(2^{197}-4)}{(2^{197}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{195}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}} \tag{117}$$

$$\zeta(199)=\zeta(197)\frac{(2^{199}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{(2^{199}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}=\frac{(2^{199}-4)}{(2^{199}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{197}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}} \tag{118}$$

$$\zeta(201)=\zeta(199)\frac{(2^{201}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}{(2^{201}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}=\frac{(2^{201}-4)}{(2^{201}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{199}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}} \tag{119}$$

$$\zeta(203)=\zeta(201)\frac{(2^{203}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}{(2^{203}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}=\frac{(2^{203}-4)}{(2^{203}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{201}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}} \tag{120}$$

$$\zeta(205)=\zeta(203)\frac{(2^{205}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}{(2^{205}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}=\frac{(2^{205}-4)}{(2^{205}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{203}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}} \tag{121}$$

$$\zeta(207)=\zeta(205)\frac{(2^{207}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}{(2^{207}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}=\frac{(2^{207}-4)}{(2^{207}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{205}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}} \tag{122}$$

$$\zeta(209)=\zeta(207)\frac{(2^{209}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}{(2^{209}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}=\frac{(2^{209}-4)}{(2^{209}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{207}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}} \tag{123}$$

$$\zeta(211)=\zeta(209)\frac{(2^{211}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}{(2^{211}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}=\frac{(2^{211}-4)}{(2^{211}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{209}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}} \tag{124}$$

$$\zeta(213)=\zeta(211)\frac{(2^{213}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}{(2^{213}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}=\frac{(2^{213}-4)}{(2^{213}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{211}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}} \tag{125}$$

$$\zeta(215)=\zeta(213)\frac{(2^{215}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}{(2^{215}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}=\frac{(2^{215}-4)}{(2^{215}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{213}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}} \tag{126}$$

$$\zeta(217)=\zeta(215)\frac{(2^{217}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}{(2^{217}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}=\frac{(2^{217}-4)}{(2^{217}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{215}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}} \tag{127}$$

$$\zeta(219)=\zeta(217)\frac{(2^{219}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}{(2^{219}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}=\frac{(2^{219}-4)}{(2^{219}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{217}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}} \tag{128}$$

$$\zeta(221)=\zeta(219)\frac{(2^{221}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}{(2^{221}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}=\frac{(2^{221}-4)}{(2^{221}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{219}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}} \tag{129}$$

$$\zeta(223)=\zeta(221)\frac{(2^{223}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}{(2^{223}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}=\frac{(2^{223}-4)}{(2^{223}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{221}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}} \tag{130}$$

$$\zeta(225)=\zeta(223)\frac{(2^{225}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}{(2^{225}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}=\frac{(2^{225}-4)}{(2^{225}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{223}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}} \tag{131}$$

$$\zeta(227)=\zeta(225)\frac{(2^{227}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}{(2^{227}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}=\frac{(2^{227}-4)}{(2^{227}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{225}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}} \tag{132}$$

$$\zeta(229)=\zeta(227)\frac{(2^{229}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}{(2^{229}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}=\frac{(2^{229}-4)}{(2^{229}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{227}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}} \tag{133}$$

$$\zeta(231)=\zeta(229)\frac{(2^{231}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}{(2^{231}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}=\frac{(2^{231}-4)}{(2^{231}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{229}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}} \tag{134}$$

$$\zeta(233)=\zeta(231)\frac{(2^{233}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}{(2^{233}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}=\frac{(2^{233}-4)}{(2^{233}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{231}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}} \tag{135}$$

$$\zeta(235)=\zeta(233)\frac{(2^{235}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}{(2^{235}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}=\frac{(2^{235}-4)}{(2^{235}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{233}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}} \tag{136}$$

$$\zeta(237)=\zeta(235)\frac{(2^{237}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}{(2^{237}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}=\frac{(2^{237}-4)}{(2^{237}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{235}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}} \tag{137}$$

$$\zeta(239)=\zeta(237)\frac{(2^{239}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}{(2^{239}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}=\frac{(2^{239}-4)}{(2^{239}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{237}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}} \tag{138}$$

$$\zeta(241)=\zeta(239)\frac{(2^{241}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}{(2^{241}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}=\frac{(2^{241}-4)}{(2^{241}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{239}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}} \tag{139}$$

$$\zeta(243)=\zeta(241)\frac{(2^{243}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}{(2^{243}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}=\frac{(2^{243}-4)}{(2^{243}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{241}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}} \tag{140}$$

$$\zeta(245)=\zeta(243)\frac{(2^{245}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}{(2^{245}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}=\frac{(2^{245}-4)}{(2^{245}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{243}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}} \tag{141}$$

$$\zeta(247)=\zeta(245)\frac{(2^{247}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}{(2^{247}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}=\frac{(2^{247}-4)}{(2^{247}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{245}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}} \tag{142}$$

$$\zeta(249)=\zeta(247)\frac{(2^{249}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}{(2^{249}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}=\frac{(2^{249}-4)}{(2^{249}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{247}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}} \tag{143}$$

$$\zeta(251)=\zeta(249)\frac{(2^{251}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}{(2^{251}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}=\frac{(2^{251}-4)}{(2^{251}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{249}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}} \tag{144}$$

$$\zeta(253)=\zeta(251)\frac{(2^{253}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}{(2^{253}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}=\frac{(2^{253}-4)}{(2^{253}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{251}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}} \tag{145}$$

$$\zeta(255)=\zeta(253)\frac{(2^{255}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}{(2^{255}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}=\frac{(2^{255}-4)}{(2^{255}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{253}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}} \tag{146}$$

$$\zeta(257)=\zeta(255)\frac{(2^{257}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}{(2^{257}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}=\frac{(2^{257}-4)}{(2^{257}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{255}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}} \tag{147}$$

$$\zeta(259)=\zeta(257)\frac{(2^{259}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}{(2^{259}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}=\frac{(2^{259}-4)}{(2^{259}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{257}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}} \tag{148}$$

$$\zeta(261)=\zeta(259)\frac{(2^{261}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}{(2^{261}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}=\frac{(2^{261}-4)}{(2^{261}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{259}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}} \tag{149}$$

$$\zeta(263)=\zeta(261)\frac{(2^{263}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}{(2^{263}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}=\frac{(2^{263}-4)}{(2^{263}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{261}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}} \tag{150}$$

$$\zeta(265)=\zeta(263)\frac{(2^{265}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}{(2^{265}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}=\frac{(2^{265}-4)}{(2^{265}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{263}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}} \tag{151}$$

$$\zeta(267)=\zeta(265)\frac{(2^{267}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}{(2^{267}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}=\frac{(2^{267}-4)}{(2^{267}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{265}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}} \tag{152}$$

$$\zeta(269)=\zeta(267)\frac{(2^{269}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}{(2^{269}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}=\frac{(2^{269}-4)}{(2^{269}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{267}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}} \tag{153}$$

$$\zeta(271)=\zeta(269)\frac{(2^{271}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}{(2^{271}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}=\frac{(2^{271}-4)}{(2^{271}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{269}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}} \tag{154}$$

$$\zeta(273)=\zeta(271)\frac{(2^{273}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}{(2^{273}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}=\frac{(2^{273}-4)}{(2^{273}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{271}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}} \tag{155}$$

$$\zeta(275)=\zeta(273)\frac{(2^{275}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}{(2^{275}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}=\frac{(2^{275}-4)}{(2^{275}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{273}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}} \tag{156}$$

$$\zeta(277)=\zeta(275)\frac{(2^{277}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}{(2^{277}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}=\frac{(2^{277}-4)}{(2^{277}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{275}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}} \tag{157}$$

$$\zeta(279)=\zeta(277)\frac{(2^{279}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}{(2^{279}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}=\frac{(2^{279}-4)}{(2^{279}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{277}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}} \tag{158}$$

$$\zeta(281)=\zeta(279)\frac{(2^{281}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}{(2^{281}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}=\frac{(2^{281}-4)}{(2^{281}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{279}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}} \tag{159}$$

$$\zeta(283)=\zeta(281)\frac{(2^{283}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}{(2^{283}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}=\frac{(2^{283}-4)}{(2^{283}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{281}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}} \tag{160}$$

$$\zeta(285)=\zeta(283)\frac{(2^{285}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}{(2^{285}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}=\frac{(2^{285}-4)}{(2^{285}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{283}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}} \tag{161}$$

$$\zeta(287)=\zeta(285)\frac{(2^{287}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}{(2^{287}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}=\frac{(2^{287}-4)}{(2^{287}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{285}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}} \tag{162}$$

$$\zeta(289)=\zeta(287)\frac{(2^{289}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}{(2^{289}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}=\frac{(2^{289}-4)}{(2^{289}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{287}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}} \tag{163}$$

$$\zeta(291)=\zeta(289)\frac{(2^{291}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}{(2^{291}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}=\frac{(2^{291}-4)}{(2^{291}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{289}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}} \tag{164}$$

$$\zeta(293)=\zeta(291)\frac{(2^{293}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}{(2^{293}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}=\frac{(2^{293}-4)}{(2^{293}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{291}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}} \tag{165}$$

$$\zeta(295)=\zeta(293)\frac{(2^{295}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}{(2^{295}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}=\frac{(2^{295}-4)}{(2^{295}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{293}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}} \tag{166}$$

$$\zeta(297)=\zeta(295)\frac{(2^{297}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}{(2^{297}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}=\frac{(2^{297}-4)}{(2^{297}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{295}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}} \tag{167}$$

$$\zeta(299)=\zeta(297)\frac{(2^{299}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}{(2^{299}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}=\frac{(2^{299}-4)}{(2^{299}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{297}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}} \tag{168}$$

$$\zeta(301)==\zeta(299)\frac{(2^{301}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}{(2^{301}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}=\frac{(2^{301}-4)}{(2^{301}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{299}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}} \tag{169}$$

$$\zeta(303)=\zeta(301)\frac{(2^{303}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}{(2^{303}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}=\frac{(2^{303}-4)}{(2^{303}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{301}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}} \tag{170}$$

$$\zeta(305)=\zeta(303)\frac{(2^{305}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}{(2^{305}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}=\frac{(2^{305}-4)}{(2^{305}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{303}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}} \tag{171}$$

$$\zeta(307)=\zeta(305)\frac{(2^{307}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}{(2^{307}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}=\frac{(2^{307}-4)}{(2^{307}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{305}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}} \tag{172}$$

$$\zeta(309)=\zeta(307)\frac{(2^{309}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}{(2^{309}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}=\frac{(2^{309}-4)}{(2^{309}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{307}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}} \tag{173}$$

$$\zeta(311)=\zeta(309)\frac{(2^{311}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}{(2^{311}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}=\frac{(2^{311}-4)}{(2^{311}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{309}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}} \tag{174}$$

$$\zeta(313)=\zeta(311)\frac{(2^{313}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}{(2^{313}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}=\frac{(2^{313}-4)}{(2^{313}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{311}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}} \tag{175}$$

$$\zeta(315)=\zeta(313)\frac{(2^{315}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}{(2^{315}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}=\frac{(2^{315}-4)}{(2^{315}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{313}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}} \tag{176}$$

$$\zeta(317)=\zeta(315)\frac{(2^{317}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}{(2^{317}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}=\frac{(2^{317}-4)}{(2^{317}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{315}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}} \tag{177}$$

$$\zeta(319)=\zeta(317)\frac{(2^{319}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}{(2^{319}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}=\frac{(2^{319}-4)}{(2^{319}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{317}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}} \tag{178}$$

$$\zeta(321)=\zeta(319)\frac{(2^{321}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}{(2^{321}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}=\frac{(2^{321}-4)}{(2^{321}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{319}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}} \tag{179}$$

$$\zeta(323)=\zeta(321)\frac{(2^{323}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}{(2^{323}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}=\frac{(2^{323}-4)}{(2^{323}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{321}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}} \tag{180}$$

$$\zeta(325)=\zeta(323)\frac{(2^{325}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}{(2^{325}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}=\frac{(2^{325}-4)}{(2^{325}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{323}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}} \tag{181}$$

$$\zeta(327)=\zeta(325)\frac{(2^{327}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}{(2^{327}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}=\frac{(2^{327}-4)}{(2^{327}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{325}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}} \tag{182}$$

$$\zeta(329)=\zeta(327)\frac{(2^{329}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}{(2^{329}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}=\frac{(2^{329}-4)}{(2^{329}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{327}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}} \tag{183}$$

$$\zeta(331)=\zeta(329)\frac{(2^{331}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}{(2^{331}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}=\frac{(2^{331}-4)}{(2^{331}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{329}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}} \tag{184}$$

$$\zeta(333)=\zeta(331)\frac{(2^{333}-4)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}}}{(2^{333}-1)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}=\frac{(2^{333}-4)}{(2^{333}-1)}\Big[1+\frac{\sum_{n=1}^{\infty}\frac{1}{(2n)^{331}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}\Big]\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}} \tag{185}$$

$\zeta(335),\zeta(337)$ etc. can also be expressed by these equations.

$\zeta(5),\zeta(7)$........$\zeta(331),\zeta(333)$ are irrational numbers.

Example 2

That $\zeta(5)$ is an irrational number is already proven at **Example 1** (proof 1).

(Proof 2)
If $\zeta(7)$ is assumed to be rational number. $\zeta(7)=\frac{s}{t}$, s and t are integer.

$\zeta(7)=\zeta(3)\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}$ $\frac{112\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{127\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\frac{o}{p}$ o, p are assumed to be integer.

$\zeta(7)=\zeta(3)\frac{o}{p}$ it equal $\zeta(3)=\zeta(7)\frac{p}{o}=\frac{sp}{to}$ But, $\zeta(3)\neq\frac{sp}{to}$
This is because $\zeta(3)$ is known to be an irrational number.
This contradicts.

$\zeta(7)$ is irrational number.

(Proof end)

Do the same for $\zeta(9),\zeta(11),\zeta(13)$ etc. prove that $\zeta(9),\zeta(11),\zeta(13)$ etc. are an irrational numbers.

and
Detailed description

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{186}$$

$$\zeta(5)=\frac{2^5}{2^5-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}=\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{187}$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\zeta(3)\frac{32}{31}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5} \tag{188}$$

$$\zeta(5) = \zeta(3)\frac{7}{8}\frac{32}{31}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{28}{31}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{189}$$

and

$$\zeta(3) = \frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{190}$$

$$\zeta(7) = \frac{2^7}{2^7-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} = \frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{191}$$

Multiply $\zeta(3)$ and $\zeta(7)$

$$\zeta(7)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \zeta(3)\frac{128}{127}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7} \tag{192}$$

$$\zeta(7)\frac{8}{7} = \zeta(3)\frac{128}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{193}$$

$$\zeta(7) = \zeta(3)\frac{128}{127}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{112}{127}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{194}$$

and

$$\zeta(3) = \frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{195}$$

$$\zeta(9) = \frac{2^9}{2^9-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} = \frac{512}{511}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{196}$$

Multiply $\zeta(3)$ and $\zeta(9)$

$$\zeta(9)\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \zeta(3)\frac{512}{511}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{197}$$

$$\zeta(9)\frac{8}{7} = \zeta(3)\frac{512}{511}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{198}$$

$$\zeta(9) = \zeta(3)\frac{512}{511}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{448}{511}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} = \zeta(3)\frac{448}{2^9-1}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{199}$$

Do the same for $\zeta(11), \zeta(13), \zeta(15)$ etc.

$$\zeta(3) = \zeta(1)\frac{4}{7}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} = \zeta(1)\frac{(2^3-4)}{(2^3-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^1}} \tag{200}$$

$$\zeta(5)=\zeta(3)\frac{2^5}{(2^5-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^2\times 7}{(2^5-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{201}$$

$$\zeta(7)=\zeta(3)\frac{2^7}{(2^7-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^4\times 7}{(2^7-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{202}$$

$$\zeta(9)=\zeta(3)\frac{2^9}{(2^9-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^6\times 7}{(2^9-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{203}$$

$$\zeta(11)=\zeta(3)\frac{2^{11}}{(2^{11}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^8\times 7}{(2^{11}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{204}$$

$$\zeta(13)=\zeta(3)\frac{2^{13}}{(2^{13}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{10}\times 7}{(2^{13}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{205}$$

$$\zeta(15)=\zeta(3)\frac{2^{15}}{(2^{15}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{12}\times 7}{(2^{15}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{206}$$

$$\zeta(17)=\zeta(3)\frac{2^{17}}{(2^{17}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{14}\times 7}{(2^{17}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{207}$$

$$\zeta(19)=\zeta(3)\frac{2^{19}}{(2^{19}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{16}\times 7}{(2^{19}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{208}$$

$$\zeta(21)=\zeta(3)\frac{2^{21}}{(2^{21}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{18}\times 7}{(2^{21}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{21}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{209}$$

$$\zeta(23)=\zeta(3)\frac{2^{23}}{(2^{23}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{20}\times 7}{(2^{23}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{23}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{210}$$

$$\zeta(25)=\zeta(3)\frac{2^{25}}{(2^{25}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{22}\times 7}{(2^{25}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{25}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{211}$$

$$\zeta(27)=\zeta(3)\frac{2^{27}}{(2^{27}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}}=\zeta(3)\frac{2^{24}\times 7}{(2^{27}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{27}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}} \tag{212}$$

$$\zeta(29)=\zeta(3)\frac{2^{29}}{(2^{29}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{26}\times 7}{(2^{29}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{29}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{213}$$

$$\zeta(31)=\zeta(3)\frac{2^{31}}{(2^{31}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{28}\times 7}{(2^{31}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{31}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{214}$$

$$\zeta(33)=\zeta(3)\frac{2^{33}}{(2^{33}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{30}\times 7}{(2^{33}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{33}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{215}$$

$$\zeta(35)=\zeta(3)\frac{2^{35}}{(2^{35}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{32}\times 7}{(2^{35}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{35}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{216}$$

$$\zeta(37)=\zeta(3)\frac{2^{37}}{(2^{37}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{34}\times 7}{(2^{37}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{37}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{217}$$

$$\zeta(39)=\zeta(3)\frac{2^{39}}{(2^{39}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{36}\times 7}{(2^{39}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{39}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{218}$$

$$\zeta(41)=\zeta(3)\frac{2^{41}}{(2^{41}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{38}\times 7}{(2^{41}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{41}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{219}$$

$$\zeta(43)=\zeta(3)\frac{2^{43}}{(2^{43}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{40}\times 7}{(2^{43}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{43}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{220}$$

$$\zeta(45)=\zeta(3)\frac{2^{45}}{(2^{45}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{42}\times 7}{(2^{45}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{45}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{221}$$

$$\zeta(47)=\zeta(3)\frac{2^{47}}{(2^{47}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{44}\times 7}{(2^{47}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{47}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{222}$$

$$\zeta(49)=\zeta(3)\frac{2^{49}}{(2^{49}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{46}\times 7}{(2^{49}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{49}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{223}$$

$$\zeta(51)=\zeta(3)\frac{2^{51}}{(2^{51}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{48}\times 7}{(2^{51}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{51}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{224}$$

$$\zeta(53)=\zeta(3)\frac{2^{53}}{(2^{53}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{50}\times 7}{(2^{53}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{53}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{225}$$

$$\zeta(55)=\zeta(3)\frac{2^{55}}{(2^{55}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{52}\times 7}{(2^{55}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{55}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{226}$$

$$\zeta(57)=\zeta(3)\frac{2^{57}}{(2^{57}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{54}\times 7}{(2^{57}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{57}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{227}$$

$$\zeta(59)=\zeta(3)\frac{2^{59}}{(2^{59}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{56}\times 7}{(2^{59}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{59}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{228}$$

$$\zeta(61)=\zeta(3)\frac{2^{61}}{(2^{61}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{58}\times 7}{(2^{61}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{61}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{229}$$

$$\zeta(63)=\zeta(3)\frac{2^{63}}{(2^{63}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{60}\times 7}{(2^{63}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{63}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{230}$$

$$\zeta(65)=\zeta(3)\frac{2^{65}}{(2^{65}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{62}\times 7}{(2^{65}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{65}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{231}$$

$$\zeta(67)=\zeta(3)\frac{2^{67}}{(2^{67}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{64}\times 7}{(2^{67}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{67}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{232}$$

$$\zeta(69)=\zeta(3)\frac{2^{69}}{(2^{69}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{66}\times 7}{(2^{69}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{69}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{233}$$

$$\zeta(71)=\zeta(3)\frac{2^{71}}{(2^{71}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{68}\times 7}{(2^{71}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{71}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{234}$$

$$\zeta(73)=\zeta(3)\frac{2^{73}}{(2^{73}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{70}\times 7}{(2^{73}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{73}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{235}$$

$$\zeta(75)=\zeta(3)\frac{2^{75}}{(2^{75}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{72}\times 7}{(2^{75}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{75}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{236}$$

$$\zeta(77)=\zeta(3)\frac{2^{77}}{(2^{77}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{74}\times 7}{(2^{77}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{77}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{237}$$

$$\zeta(79)=\zeta(3)\frac{2^{79}}{(2^{79}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{76}\times 7}{(2^{79}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{79}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{238}$$

$$\zeta(81)=\zeta(3)\frac{2^{81}}{(2^{81}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{78}\times 7}{(2^{81}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{81}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{239}$$

$$\zeta(83)=\zeta(3)\frac{2^{83}}{(2^{83}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{80}\times 7}{(2^{83}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{83}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{240}$$

$$\zeta(85)=\zeta(3)\frac{2^{85}}{(2^{85}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{82}\times 7}{(2^{85}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{85}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{241}$$

$$\zeta(87)=\zeta(3)\frac{2^{87}}{(2^{87}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{84}\times 7}{(2^{87}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{87}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{242}$$

$$\zeta(89)=\zeta(3)\frac{2^{89}}{(2^{89}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{86}\times 7}{(2^{89}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{89}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{243}$$

$$\zeta(91)=\zeta(3)\frac{2^{91}}{(2^{91}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{88}\times 7}{(2^{91}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{91}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{244}$$

$$\zeta(93)=\zeta(3)\frac{2^{93}}{(2^{93}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{90}\times 7}{(2^{93}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{93}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{245}$$

$$\zeta(95)=\zeta(3)\frac{2^{95}}{(2^{95}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{92}\times 7}{(2^{95}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{95}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{246}$$

$$\zeta(97)=\zeta(3)\frac{2^{97}}{(2^{97}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{94}\times 7}{(2^{97}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{97}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{247}$$

$$\zeta(99)=\zeta(3)\frac{2^{99}}{(2^{99}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{96}\times 7}{(2^{99}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{99}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{248}$$

$$\zeta(101)=\zeta(3)\frac{2^{101}}{(2^{101}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{98}\times 7}{(2^{101}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{101}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{249}$$

$$\zeta(103)=\zeta(3)\frac{2^{103}}{(2^{103}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{100}\times 7}{(2^{103}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{103}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{250}$$

$$\zeta(105)=\zeta(3)\frac{2^{105}}{(2^{105}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{102}\times 7}{(2^{105}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{105}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{251}$$

$$\zeta(107)=\zeta(3)\frac{2^{107}}{(2^{107}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{104}\times 7}{(2^{107}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{107}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{252}$$

$$\zeta(109)=\zeta(3)\frac{2^{109}}{(2^{109}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{106}\times 7}{(2^{109}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{109}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{253}$$

$$\zeta(111)=\zeta(3)\frac{2^{111}}{(2^{111}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{108}\times 7}{(2^{111}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{111}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{254}$$

$$\zeta(113)=\zeta(3)\frac{2^{113}}{(2^{113}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{110}\times 7}{(2^{113}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{113}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{255}$$

$$\zeta(115)=\zeta(3)\frac{2^{115}}{(2^{115}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{112}\times 7}{(2^{115}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{115}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{256}$$

$$\zeta(117)=\zeta(3)\frac{2^{117}}{(2^{117}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{114}\times 7}{(2^{117}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{117}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{257}$$

$$\zeta(119)=\zeta(3)\frac{2^{119}}{(2^{119}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{116}\times 7}{(2^{119}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{119}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{258}$$

$$\zeta(121)=\zeta(3)\frac{2^{121}}{(2^{121}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{118}\times 7}{(2^{121}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{121}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{259}$$

$$\zeta(123)=\zeta(3)\frac{2^{123}}{(2^{123}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{120}\times 7}{(2^{123}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{123}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{260}$$

$$\zeta(125)=\zeta(3)\frac{2^{125}}{(2^{125}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{122}\times 7}{(2^{125}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{125}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{261}$$

$$\zeta(127)=\zeta(3)\frac{2^{127}}{(2^{127}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{124}\times 7}{(2^{127}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{127}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{262}$$

$$\zeta(129)=\zeta(3)\frac{2^{129}}{(2^{129}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{126}\times 7}{(2^{129}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{129}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{263}$$

$$\zeta(131)=\zeta(3)\frac{2^{131}}{(2^{131}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{128}\times 7}{(2^{131}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{131}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{264}$$

$$\zeta(133)=\zeta(3)\frac{2^{133}}{(2^{133}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{130}\times 7}{(2^{133}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{133}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{265}$$

$$\zeta(135)=\zeta(3)\frac{2^{135}}{(2^{135}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{132}\times 7}{(2^{135}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{135}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{266}$$

$$\zeta(137)=\zeta(3)\frac{2^{137}}{(2^{137}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{134}\times 7}{(2^{137}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{137}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{267}$$

$$\zeta(139)=\zeta(3)\frac{2^{139}}{(2^{139}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{136}\times 7}{(2^{139}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{139}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{268}$$

$$\zeta(141)=\zeta(3)\frac{2^{141}}{(2^{141}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{138}\times 7}{(2^{141}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{141}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{269}$$

$$\zeta(143)=\zeta(3)\frac{2^{143}}{(2^{143}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{140}\times 7}{(2^{143}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{143}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{270}$$

$$\zeta(145)=\zeta(3)\frac{2^{145}}{(2^{145}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{142}\times 7}{(2^{145}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{145}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{271}$$

$$\zeta(147)=\zeta(3)\frac{2^{147}}{(2^{147}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{144}\times 7}{(2^{147}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{147}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{272}$$

$$\zeta(149)=\zeta(3)\frac{2^{149}}{(2^{149}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{146}\times 7}{(2^{149}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{149}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{273}$$

$$\zeta(151)=\zeta(3)\frac{2^{151}}{(2^{151}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{148}\times 7}{(2^{151}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{151}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{274}$$

$$\zeta(153)=\zeta(3)\frac{2^{153}}{(2^{153}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{150}\times 7}{(2^{153}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{153}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{275}$$

$$\zeta(155)=\zeta(3)\frac{2^{155}}{(2^{155}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{152}\times 7}{(2^{155}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{155}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{276}$$

$$\zeta(157)=\zeta(3)\frac{2^{157}}{(2^{157}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{154}\times 7}{(2^{157}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{157}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{277}$$

$$\zeta(159)=\zeta(3)\frac{2^{159}}{(2^{159}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{156}\times 7}{(2^{159}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{159}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{278}$$

$$\zeta(161)=\zeta(3)\frac{2^{161}}{(2^{161}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{158}\times 7}{(2^{161}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{161}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{279}$$

$$\zeta(163)=\zeta(3)\frac{2^{163}}{(2^{163}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{160}\times 7}{(2^{163}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{163}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{280}$$

$$\zeta(165)=\zeta(3)\frac{2^{165}}{(2^{165}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{162}\times 7}{(2^{165}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{165}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{281}$$

$$\zeta(167)=\zeta(3)\frac{2^{167}}{(2^{167}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{164}\times 7}{(2^{167}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{167}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{282}$$

$$\zeta(169)=\zeta(3)\frac{2^{169}}{(2^{169}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{166}\times 7}{(2^{169}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{169}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{283}$$

$$\zeta(171)=\zeta(3)\frac{2^{171}}{(2^{171}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{168}\times 7}{(2^{171}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{171}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{284}$$

$$\zeta(173)=\zeta(3)\frac{2^{173}}{(2^{173}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{170}\times 7}{(2^{173}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{173}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{285}$$

$$\zeta(175)=\zeta(3)\frac{2^{175}}{(2^{175}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{172}\times 7}{(2^{175}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{175}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{286}$$

$$\zeta(177)=\zeta(3)\frac{2^{177}}{(2^{177}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{174}\times 7}{(2^{177}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{177}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{287}$$

$$\zeta(179)=\zeta(3)\frac{2^{179}}{(2^{179}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{176}\times 7}{(2^{179}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{179}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{288}$$

$$\zeta(181)=\zeta(3)\frac{2^{181}}{(2^{181}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{178}\times 7}{(2^{181}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{181}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{289}$$

$$\zeta(183)=\zeta(3)\frac{2^{183}}{(2^{183}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{180}\times 7}{(2^{183}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{183}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{290}$$

$$\zeta(185)=\zeta(3)\frac{2^{185}}{(2^{185}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{182}\times 7}{(2^{185}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{185}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{291}$$

$$\zeta(187)=\zeta(3)\frac{2^{187}}{(2^{187}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{184}\times 7}{(2^{187}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{187}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{292}$$

$$\zeta(189)=\zeta(3)\frac{2^{189}}{(2^{189}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{186}\times 7}{(2^{189}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{189}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{293}$$

$$\zeta(191)=\zeta(3)\frac{2^{191}}{(2^{191}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{188}\times 7}{(2^{191}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{191}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{294}$$

$$\zeta(193)=\zeta(3)\frac{2^{193}}{(2^{193}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{190}\times 7}{(2^{193}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{193}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{295}$$

$$\zeta(195)=\zeta(3)\frac{2^{195}}{(2^{195}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{192}\times 7}{(2^{195}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{195}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{296}$$

$$\zeta(197)=\zeta(3)\frac{2^{197}}{(2^{197}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{194}\times 7}{(2^{197}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{197}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{297}$$

$$\zeta(199)=\zeta(3)\frac{2^{199}}{(2^{199}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{196}\times 7}{(2^{199}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{199}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{298}$$

$$\zeta(201)=\zeta(3)\frac{2^{201}}{(2^{201}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{198}\times 7}{(2^{201}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{201}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{299}$$

$$\zeta(203)=\zeta(3)\frac{2^{203}}{(2^{203}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{200}\times 7}{(2^{203}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{203}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{300}$$

$$\zeta(205)=\zeta(3)\frac{2^{205}}{(2^{205}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{202}\times 7}{(2^{205}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{205}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{301}$$

$$\zeta(207)=\zeta(3)\frac{2^{207}}{(2^{207}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{204}\times 7}{(2^{207}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{207}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{302}$$

$$\zeta(209)=\zeta(3)\frac{2^{209}}{(2^{209}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{206}\times 7}{(2^{209}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{209}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{303}$$

$$\zeta(211)=\zeta(3)\frac{2^{211}}{(2^{211}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{208}\times 7}{(2^{211}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{211}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{304}$$

$$\zeta(213)=\zeta(3)\frac{2^{213}}{(2^{213}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{210}\times 7}{(2^{213}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{213}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{305}$$

$$\zeta(215)=\zeta(3)\frac{2^{215}}{(2^{215}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{212}\times 7}{(2^{215}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{215}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{306}$$

$$\zeta(217)=\zeta(3)\frac{2^{217}}{(2^{217}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{214}\times 7}{(2^{217}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{217}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{307}$$

$$\zeta(219)=\zeta(3)\frac{2^{219}}{(2^{219}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{216}\times 7}{(2^{219}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{219}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{308}$$

$$\zeta(221) = \zeta(3)\frac{2^{221}}{(2^{221}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{218}\times 7}{(2^{221}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{309}$$

$$\zeta(223) = \zeta(3)\frac{2^{223}}{(2^{223}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{220}\times 7}{(2^{223}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{310}$$

$$\zeta(225) = \zeta(3)\frac{2^{225}}{(2^{225}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{222}\times 7}{(2^{225}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{311}$$

$$\zeta(227) = \zeta(3)\frac{2^{227}}{(2^{227}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{224}\times 7}{(2^{227}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{312}$$

$$\zeta(229) = \zeta(3)\frac{2^{229}}{(2^{229}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{226}\times 7}{(2^{229}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{313}$$

$$\zeta(231) = \zeta(3)\frac{2^{231}}{(2^{231}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{228}\times 7}{(2^{231}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{314}$$

$$\zeta(233) = \zeta(3)\frac{2^{233}}{(2^{233}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{230}\times 7}{(2^{233}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{315}$$

$$\zeta(235) = \zeta(3)\frac{2^{235}}{(2^{235}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{232}\times 7}{(2^{235}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{316}$$

$$\zeta(237) = \zeta(3)\frac{2^{237}}{(2^{237}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{234}\times 7}{(2^{237}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{317}$$

$$\zeta(239) = \zeta(3)\frac{2^{239}}{(2^{239}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{236}\times 7}{(2^{239}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{318}$$

$$\zeta(241) = \zeta(3)\frac{2^{241}}{(2^{241}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{238}\times 7}{(2^{241}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{319}$$

$$\zeta(243) = \zeta(3)\frac{2^{243}}{(2^{243}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{240}\times 7}{(2^{243}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{320}$$

$$\zeta(245)=\zeta(3)\frac{2^{245}}{(2^{245}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{242}\times 7}{(2^{245}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{321}$$

$$\zeta(247)=\zeta(3)\frac{2^{247}}{(2^{247}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{244}\times 7}{(2^{247}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{322}$$

$$\zeta(249)=\zeta(3)\frac{2^{249}}{(2^{249}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{246}\times 7}{(2^{249}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{323}$$

$$\zeta(251)=\zeta(3)\frac{2^{251}}{(2^{251}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{248}\times 7}{(2^{251}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{324}$$

$$\zeta(253)=\zeta(3)\frac{2^{253}}{(2^{253}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{250}\times 7}{(2^{253}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{325}$$

$$\zeta(255)=\zeta(3)\frac{2^{255}}{(2^{255}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{252}\times 7}{(2^{255}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{326}$$

$$\zeta(257)=\zeta(3)\frac{2^{257}}{(2^{257}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{254}\times 7}{(2^{257}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{327}$$

$$\zeta(259)=\zeta(3)\frac{2^{259}}{(2^{259}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{256}\times 7}{(2^{259}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{328}$$

$$\zeta(261)=\zeta(3)\frac{2^{261}}{(2^{261}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{258}\times 7}{(2^{261}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{329}$$

$$\zeta(263)=\zeta(3)\frac{2^{263}}{(2^{263}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{260}\times 7}{(2^{263}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{330}$$

$$\zeta(265)=\zeta(3)\frac{2^{265}}{(2^{265}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{262}\times 7}{(2^{265}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{331}$$

$$\zeta(267)=\zeta(3)\frac{2^{267}}{(2^{267}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{264}\times 7}{(2^{267}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{332}$$

$$\zeta(269)=\zeta(3)\frac{2^{269}}{(2^{269}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{266}\times 7}{(2^{269}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{333}$$

$$\zeta(271)=\zeta(3)\frac{2^{271}}{(2^{271}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{268}\times 7}{(2^{271}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{334}$$

$$\zeta(273)=\zeta(3)\frac{2^{273}}{(2^{273}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{270}\times 7}{(2^{273}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{335}$$

$$\zeta(275)=\zeta(3)\frac{2^{275}}{(2^{275}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{272}\times 7}{(2^{275}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{336}$$

$$\zeta(277)=\zeta(3)\frac{2^{277}}{(2^{277}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{274}\times 7}{(2^{277}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{337}$$

$$\zeta(279)=\zeta(3)\frac{2^{279}}{(2^{279}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{276}\times 7}{(2^{279}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{338}$$

$$\zeta(281)=\zeta(3)\frac{2^{281}}{(2^{281}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{278}\times 7}{(2^{281}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{339}$$

$$\zeta(283)=\zeta(3)\frac{2^{283}}{(2^{283}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{280}\times 7}{(2^{283}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{340}$$

$$\zeta(285)=\zeta(3)\frac{2^{285}}{(2^{285}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{282}\times 7}{(2^{285}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{341}$$

$$\zeta(287)=\zeta(3)\frac{2^{287}}{(2^{287}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{284}\times 7}{(2^{287}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{342}$$

$$\zeta(289)=\zeta(3)\frac{2^{289}}{(2^{289}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{286}\times 7}{(2^{289}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{343}$$

$$\zeta(291) = \zeta(3)\frac{2^{291}}{(2^{291}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{288}\times 7}{(2^{291}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{344}$$

$$\zeta(293) = \zeta(3)\frac{2^{293}}{(2^{293}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{290}\times 7}{(2^{293}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{345}$$

$$\zeta(295) = \zeta(3)\frac{2^{295}}{(2^{295}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{292}\times 7}{(2^{295}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{346}$$

$$\zeta(297) = \zeta(3)\frac{2^{297}}{(2^{297}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{294}\times 7}{(2^{297}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{347}$$

$$\zeta(299) = \zeta(3)\frac{2^{299}}{(2^{299}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{296}\times 7}{(2^{299}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{348}$$

$$\zeta(301) = \zeta(3)\frac{2^{301}}{(2^{301}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{298}\times 7}{(2^{301}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{349}$$

$$\zeta(303) = \zeta(3)\frac{2^{303}}{(2^{303}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{300}\times 7}{(2^{303}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{350}$$

$$\zeta(305) = \zeta(3)\frac{2^{305}}{(2^{305}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{302}\times 7}{(2^{305}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{351}$$

$$\zeta(307) = \zeta(3)\frac{2^{307}}{(2^{307}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} = \zeta(3)\frac{2^{304}\times 7}{(2^{307}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{352}$$

$$\zeta(309)=\zeta(3)\frac{2^{309}}{(2^{309}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{306}\times 7}{(2^{309}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{353}$$

$$\zeta(311)=\zeta(3)\frac{2^{311}}{(2^{311}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{308}\times 7}{(2^{311}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{354}$$

$$\zeta(313)=\zeta(3)\frac{2^{313}}{(2^{313}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{310}\times 7}{(2^{313}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{355}$$

$$\zeta(315)=\zeta(3)\frac{2^{315}}{(2^{315}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{312}\times 7}{(2^{315}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{356}$$

$$\zeta(317)=\zeta(3)\frac{2^{317}}{(2^{317}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{314}\times 7}{(2^{317}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{357}$$

$$\zeta(319)=\zeta(3)\frac{2^{319}}{(2^{319}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{316}\times 7}{(2^{319}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{358}$$

$$\zeta(321)=\zeta(3)\frac{2^{321}}{(2^{321}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{318}\times 7}{(2^{321}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{359}$$

$$\zeta(323)=\zeta(3)\frac{2^{323}}{(2^{323}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{320}\times 7}{(2^{323}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{360}$$

$$\zeta(325)=\zeta(3)\frac{2^{325}}{(2^{325}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{322}\times 7}{(2^{325}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{361}$$

$$\zeta(327)=\zeta(3)\frac{2^{327}}{(2^{327}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{324}\times 7}{(2^{327}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{362}$$

$$\zeta(329)=\zeta(3)\frac{2^{329}}{(2^{329}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{326}\times 7}{(2^{329}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{363}$$

$$\zeta(331)=\zeta(3)\frac{2^{331}}{(2^{331}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{328}\times 7}{(2^{331}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{364}$$

$$\zeta(333)=\zeta(3)\frac{2^{333}}{(2^{333}-1)}\frac{7}{8}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}}=\zeta(3)\frac{2^{330}\times 7}{(2^{333}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{365}$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(3), \zeta(5)......\zeta(331), \zeta(333)$ are irrational numbers.

And I declare a new formula.

$$\zeta(2m+1)=\zeta(3)\frac{2^{2m-2}\times 7}{(2^{2m+1}-1)}\frac{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{2m+1}}}{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{3}}} \tag{366}$$

m is a positive integer of 2 or more.

4 Postscript

$$\zeta(\pi)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{\pi}} \tag{367}$$

$$\zeta(\pi)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}}+\frac{1}{2^{\pi}}\sum_{n=1}^{\infty}\frac{1}{n^{\pi}} \tag{368}$$

$$\zeta(\pi)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}}+\frac{1}{2^{\pi}}\zeta(\pi) \tag{369}$$

$$(1-\frac{1}{2^{\pi}})\zeta(\pi)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}} \tag{370}$$

$$\left(\frac{2^{\pi}-1}{2^{\pi}}\right)\zeta(\pi)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}} \tag{371}$$

$$\zeta(\pi)=\frac{2^{\pi}}{2^{\pi}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{\pi}} \tag{372}$$

=1.17624173838258275887215045193805209116973899002165583496050834623040872376815861833572083732
557183113894566008145...

Do the same

$$\zeta(e)=\frac{2^{e}}{2^{e}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{e}} \tag{373}$$

=1.26900960433571711576556986660086110885640446257719048833581592870816524294827307650849451745
076054575828347684218...

I believe that $\zeta(\pi),\zeta(e)$ are irrational numbers.
That is, I believe that the irrational number ζ are irrational numbers.

I also believe that all even value, as well as odd values of ζ, are irrational numbers.

The figures in this paper have been fully verified by WolframAlpha.

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