

$\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$

are irrational number

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Abstract

Using the fact that $\zeta(3)$ is an irrational number, I prove that $\zeta(5), \zeta(7), \dots, \zeta(331)$ and $\zeta(333)$ are irrational numbers.

$\zeta(5), \zeta(7), \dots, \zeta(331)$ and $\zeta(333)$ are confirmed that they were in perfect numerical agreement.

This is because I created an odd-number formula for ζ , and the formula was created by dividing the odd-number for ζ itself into odd and even numbers.

key words

irrational number, $\zeta(3)$, odd-number formula for ζ , $\zeta(5)$, $\zeta(7)$, $\zeta(331)$, $\zeta(333)$

1 Introduction

Write the formula I finally got in advance.

$$\zeta(2m+1) = \zeta(2m-1) \frac{(2^{2m+1} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \quad (1)$$

m is a positive integer greater than or equal to 1.

In detail

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$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{2^3} \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{2^3} \zeta(3) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (2)$$

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (3)$$

do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{2^5} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{2^5} \zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (4)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (5)$$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{2^7} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{2^7} \zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (6)$$

$$\zeta(7) = \frac{2^7}{2^7-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (7)$$

Use these for $\zeta(9), \zeta(11), \zeta(13)$ etc.

and

Detailed description

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (8)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (9)$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (10)$$

$$\zeta(5) \frac{8}{7} = \zeta(3) \frac{32 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (11)$$

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (12)$$

2 Discussion

Example 1

I use my previous paper[5],

$$\zeta(2m - 1) = \frac{2^{2m-1}}{2^{2m-1} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^{2m-1}} \quad (13)$$

m is a positive integer greater than or equal to 2.

from Eq.(13)

if m=3

$$\zeta(5) = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^5} \quad (14)$$

if m=4

$$\zeta(7) = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^7} \quad (15)$$

Multiply $\zeta(5)$ and $\zeta(7)$.

$$\zeta(7) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^5} = \zeta(5) \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^7} \quad (16)$$

$$\zeta(7) \frac{32}{31} = \zeta(5) \frac{128}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (17)$$

$$\zeta(7) = \zeta(5) \frac{31}{32} \frac{128}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (18)$$

$$\zeta(7) = \zeta(5) \frac{124}{127} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (19)$$

The new formula, Eq.(1), has been followed.

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

(Proof 1)

If $\zeta(5)$ is assumed to be rational number.

$$\zeta(5) = \frac{s}{t}, \quad s \text{ and } t \text{ are integer.}$$

from formula Eq.(1).

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \qquad \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{o}{p} \qquad \text{o, p are assumed to be integer.}$$

$$\zeta(5) = \zeta(3) \frac{o}{p} \qquad \text{it equal} \qquad \zeta(3) = \zeta(5) \frac{p}{o} = \frac{sp}{to} \qquad \text{But, } \zeta(3) \neq \frac{sp}{to}$$

This is because $\zeta(3)$ is known to be an irrational number.

This contradicts.

$\zeta(5)$ is irrational number.

(Proof end)

Do the same, sequentially prove that $\zeta(7), \zeta(9), \zeta(11)$ etc. are irrational numbers.

Assuming that this formula Eq.(1) holds even when $\zeta(3)$

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \frac{4}{7} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \qquad (20)$$

If $\infty = 10000$

The calculator gives the following results:

$$\zeta(3) \approx \zeta(1) \frac{4 \sum_{n=1}^{10000} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} = \infty \qquad (21)$$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{\sum_{n=1}^{10000} \frac{1}{(2n)^1}}{\sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{10000} \frac{1}{(2n-1)^3} = \text{can not calculate.} \qquad (22)$$

$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1} = \infty \qquad \frac{\sum_{n=1}^{10000} \frac{1}{(2n)^1}}{\sum_{n=1}^{10000} \frac{1}{(2n-1)^1}} = \text{can not calculate.} \qquad (23)$$

But

$$\sum_{n=1}^{1000000000000} \frac{1}{(2n)^1} = 14.104118390415290534411204731480719794461021933075937155602... \qquad (24)$$

$$\sum_{n=1}^{1000000000000} \frac{1}{(2n-1)^1} = 14.79726557097498584382843691543889636253652206743618459722... \qquad (25)$$

$$\sum_{n=1}^{1000000000000} \frac{1}{(2n-1)^3} = 1.051799790264644999724770828822518741919363005797952146568... \qquad (26)$$

when assuming $\infty = 1000000000000$

$$\zeta(3) \approx \frac{4}{7} \left[1 + \frac{14.10411839041529}{14.79726557097498} \right] \times 1.0517997902646449997$$

$$= 1.173902973323675078443392566878600895507141148997041472187\dots$$

$$\zeta(3) = 1.202056903159594285399738161511449990764986292340498881792\dots$$

The formula Eq.(1) seems to hold.

I thought $\zeta(1) = \infty$, but I was surprised to realize that it wasn't.

This may imply $\frac{\infty}{\infty} = 1$

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (27)$$

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (28)$$

$$\zeta(7) = \zeta(5) \frac{124 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \quad (29)$$

$$\zeta(9) = \zeta(7) \frac{508 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{511 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}} \quad (30)$$

$$\zeta(11) = \zeta(9) \frac{2044 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{2047 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}} \quad (31)$$

$$\zeta(13) = \zeta(11) \frac{8188 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{8191 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}} \quad (32)$$

$$\zeta(15) = \zeta(13) \frac{32764 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{32767 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}} \quad (33)$$

$$\zeta(17) = \zeta(15) \frac{131068 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{131071 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}} \quad (34)$$

$$\zeta(19) = \zeta(17) \frac{524284 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{524287 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}} \quad (35)$$

$$\zeta(21) = \zeta(19) \frac{2097148 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{2097151 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}} \quad (36)$$

$$\zeta(23) = \zeta(21) \frac{8388604 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{8388607 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}} \quad (37)$$

$$\zeta(25) = \zeta(23) \frac{33554428 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{33554431 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}} \quad (38)$$

$$\zeta(27) = \zeta(25) \frac{134217724 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{134217727 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}} \quad (39)$$

$$\zeta(29) = \zeta(27) \frac{536870908 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{536870911 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}} \quad (40)$$

$$\zeta(31) = \zeta(29) \frac{2147483644 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{2147483647 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}} \quad (41)$$

$$\zeta(33) = \zeta(31) \frac{8589934588 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{8589934591 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}} \quad (42)$$

$$\zeta(35) = \zeta(33) \frac{34359738364 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{34359738367 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}} \quad (43)$$

$$\zeta(37) = \zeta(35) \frac{137438953468 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{137438953471 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}} \quad (44)$$

$$\zeta(39) = \zeta(37) \frac{549755813884 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{549755813887 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}} \quad (45)$$

$$\zeta(41) = \zeta(39) \frac{2199023255548 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{2199023255551 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}} \quad (46)$$

$$\zeta(43) = \zeta(41) \frac{8796093022204 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{8796093022207 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}} \quad (47)$$

$$\zeta(45) = \zeta(43) \frac{35184372088828 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{35184372088831 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}} \quad (48)$$

$$\zeta(47) = \zeta(45) \frac{140737488355324 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{140737488355327 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}} \quad (49)$$

$$\zeta(49) = \zeta(47) \frac{562949953421308 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{562949953421311 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}} \quad (50)$$

$$\zeta(51) = \zeta(49) \frac{2251799813685244 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{2251799813685247 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}} \quad (51)$$

$$\zeta(53) = \zeta(51) \frac{9007199254740988 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{9007199254740991 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}} \quad (52)$$

$$\zeta(55) = \zeta(53) \frac{36028797018963964 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{36028797018963967 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}} \quad (53)$$

$$\zeta(57) = \zeta(55) \frac{144115188075855868 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{144115188075855871 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}} \quad (54)$$

$$\zeta(59) = \zeta(57) \frac{576460752303423484 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{576460752303423487 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}} \quad (55)$$

$$\zeta(61) = \zeta(59) \frac{2305843009213693948 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{2305843009213693951 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}} \quad (56)$$

$$\zeta(63) = \zeta(61) \frac{9223372036854775804 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{9223372036854775807 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}} \quad (57)$$

$$\zeta(65) = \zeta(63) \frac{36893488147419103228 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{36893488147419103231 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}} \quad (58)$$

$$\zeta(67) = \zeta(65) \frac{147573952589676412924 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{147573952589676412927 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}} \quad (59)$$

$$\zeta(69) = \zeta(67) \frac{590295810358705651708 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{590295810358705651711 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}} \quad (60)$$

$$\zeta(71) = \zeta(69) \frac{2361183241434822606844 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{2361183241434822606847 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}} \quad (61)$$

$$\zeta(73) = \zeta(71) \frac{9444732965739290427388 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{9444732965739290427391 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}} \quad (62)$$

$$\zeta(75) = \zeta(73) \frac{37778931862957161709564 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{37778931862957161709567 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}} \quad (63)$$

$$\zeta(77) = \zeta(75) \frac{151115727451828646838271 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{151115727451828646838271 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}} \quad (64)$$

$$\zeta(79) = \zeta(77) \frac{604462909807314587353084 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{604462909807314587353087 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}} \quad (65)$$

$$\zeta(81) = \zeta(79) \frac{2417851639229258349412348 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{2417851639229258349412351 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}} \quad (66)$$

$$\zeta(83) = \zeta(81) \frac{9671406556917033397649404 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{9671406556917033397649407 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}} \quad (67)$$

$$\zeta(85) = \zeta(83) \frac{38685626227668133590597628 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{38685626227668133590597631 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}} \quad (68)$$

$$\zeta(87) = \zeta(85) \frac{154742504910672534362390524 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{154742504910672534362390527 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}} \quad (69)$$

$$\zeta(89) = \zeta(87) \frac{618970019642690137449562108 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{618970019642690137449562111 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}} \quad (70)$$

$$\zeta(91) = \zeta(89) \frac{2475880078570760549798248444 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{2475880078570760549798248447 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}} \quad (71)$$

$$\zeta(93) = \zeta(91) \frac{9903520314283042199192993788 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{9903520314283042199192993791 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}} \quad (72)$$

$$\zeta(95) = \zeta(93) \frac{39614081257132168796771975164 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{39614081257132168796771975167 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}} \quad (73)$$

$$\zeta(97) = \zeta(95) \frac{158456325028528675187087900668 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{158456325028528675187087900671 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}} \quad (74)$$

$$\zeta(99) = \zeta(97) \frac{633825300114114700748351602684 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{633825300114114700748351602687 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}} \quad (75)$$

$$\zeta(101) = \zeta(99) \frac{2535301200456458802993406410748 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{2535301200456458802993406410751 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}} \quad (76)$$

$$\zeta(103) = \zeta(101) \frac{10141204801825835211973625643004 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{10141204801825835211973625643007 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}} \quad (77)$$

$$\zeta(105) = \zeta(103) \frac{40564819207303340847894502572028 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{40564819207303340847894502572031 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}} \quad (78)$$

$$\zeta(107) = \zeta(105) \frac{162259276829213363391578010288124 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{162259276829213363391578010288127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}} \quad (79)$$

$$\zeta(109) = \zeta(107) \frac{649037107316853453566312041152508 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{649037107316853453566312041152511 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}} \quad (80)$$

$$\zeta(111) = \zeta(109) \frac{2596148429267413814265248164610044 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{111}}}{2596148429267413814265248164610047 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}} \quad (81)$$

$$\zeta(113) = \zeta(111) \frac{10384593717069655257060992658440188 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{113}}}{10384593717069655257060992658440191 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{111}}} \quad (82)$$

$$\zeta(115) = \zeta(113) \frac{41538374868278621028243970633760764 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{115}}}{41538374868278621028243970633760767 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{113}}} \quad (83)$$

$$\zeta(117) = \zeta(115) \frac{166153499473114484112975882535043068 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{117}}}{166153499473114484112975882535043071 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{115}}} \quad (84)$$

$$\zeta(119) = \zeta(117) \frac{664613997892457936451903530140172284 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{119}}}{664613997892457936451903530140172287 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{117}}} \quad (85)$$

$$\zeta(121) = \zeta(119) \frac{2658455991569831745807614120560689148 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{121}}}{2658455991569831745807614120560689151 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{119}}} \quad (86)$$

$$\zeta(123) = \zeta(121) \frac{10633823966279326983230456482242756604 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{123}}}{10633823966279326983230456482242756607 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{121}}} \quad (87)$$

$$\zeta(125) = \zeta(123) \frac{42535295865117307932921825928971026428 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}}{42535295865117307932921825928971026431 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{123}}} \quad (88)$$

$$\zeta(127) = \zeta(125) \frac{170141183460469231731687303715884105724 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}}{170141183460469231731687303715884105727 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}} \quad (89)$$

$$\zeta(129) = \zeta(127) \frac{680564733841876926926749214863536422908 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}}{680564733841876926926749214863536422911 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}} \quad (90)$$

$$\zeta(131) = \zeta(129) \frac{2722258935367507707706996859454145691644 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}}{2722258935367507707706996859454145691647 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}} \quad (91)$$

$$\zeta(133) = \zeta(131) \frac{10889035741470030830827987437816582766588 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}}{10889035741470030830827987437816582766591 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}} \quad (92)$$

$$\zeta(135) = \zeta(133) \frac{43556142965880123323311949751266331066364 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{43556142965880123323311949751266331066367 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}} \quad (93)$$

$$\zeta(137) = \zeta(135) \frac{174224571863520493293247799005065324265468 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{174224571863520493293247799005065324265471 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}} \quad (94)$$

$$\zeta(139) = \zeta(137) \frac{696898287454081973172991196020261297061884 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{696898287454081973172991196020261297061887 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}} \quad (95)$$

$$\zeta(141) = \zeta(139) \frac{2787593149816327892691964784081045188247548 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{2787593149816327892691964784081045188247551 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}} \quad (96)$$

$$\zeta(143) = \zeta(141) \frac{11150372599265311570767859136324180752990204 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{11150372599265311570767859136324180752990207 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}} \quad (97)$$

$$\zeta(145) = \zeta(143) \frac{44601490397061246283071436545296723011960828 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{44601490397061246283071436545296723011960831 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}} \quad (98)$$

$$\zeta(147) = \zeta(145) \frac{178405961588244985132285746181186892047843324 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{178405961588244985132285746181186892047843327 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}} \quad (99)$$

$$\zeta(149) = \zeta(147) \frac{713623846352979940529142984724747568191373308 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{713623846352979940529142984724747568191373311 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}} \quad (100)$$

$$\zeta(151) = \zeta(149) \frac{2854495385411919762116571938898990272765493244 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{2854495385411919762116571938898990272765493247 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}} \quad (101)$$

$$\zeta(153) = \zeta(151) \frac{11417981541647679048466287755595961091061972988 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{11417981541647679048466287755595961091061972991 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}} \quad (102)$$

$$\zeta(155) = \zeta(153) \frac{45671926166590716193865151022383844364247891964 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{45671926166590716193865151022383844364247891967 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}} \quad (103)$$

$$\zeta(157) = \zeta(155) \frac{182687704666362864775460604089535377456991567868 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{182687704666362864775460604089535377456991567871 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}} \quad (104)$$

$$\zeta(159) = \zeta(157) \frac{730750818665451459101842416358141509827966271484 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}}{730750818665451459101842416358141509827966271487 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}} \quad (105)$$

$$\zeta(161) = \zeta(159) \frac{2923003274661805836407369665432566039311865085948 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}}{2923003274661805836407369665432566039311865085951 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}} \quad (106)$$

$$\zeta(163) = \zeta(161) \frac{11692013098647223345629478661730264157247460343804 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}}{11692013098647223345629478661730264157247460343807 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}} \quad (107)$$

$$\zeta(165) = \zeta(163) \frac{46768052394588893382517914646921056628989841375228 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}}{46768052394588893382517914646921056628989841375231 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}} \quad (108)$$

$$\zeta(167) = \zeta(165) \frac{187072209578355573530071658587684226515959365500924 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}}{187072209578355573530071658587684226515959365500927 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}} \quad (109)$$

$$\zeta(169) = \zeta(167) \frac{748288838313422294120286634350736906063837462003708 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}}{748288838313422294120286634350736906063837462003711 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}} \quad (110)$$

$$\zeta(171) = \zeta(169) \frac{2993155353253689176481146537402947624255349848014844 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}}{2993155353253689176481146537402947624255349848014847 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}} \quad (111)$$

$$\zeta(173) = \zeta(171) \frac{11972621413014756705924586149611790497021399392059388 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}}{11972621413014756705924586149611790497021399392059391 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}} \quad (112)$$

$$\zeta(175) = \zeta(173) \frac{47890485652059026823698344598447161988085597568237564 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}}{47890485652059026823698344598447161988085597568237567 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}} \quad (113)$$

$$\zeta(177) = \zeta(175) \frac{191561942608236107294793378393788647952342390272950268 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}}{191561942608236107294793378393788647952342390272950271 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}} \quad (114)$$

$$\zeta(179) = \zeta(177) \frac{766247770432944429179173513575154591809369561091801084 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}}{766247770432944429179173513575154591809369561091801087 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}} \quad (115)$$

$$\zeta(181) = \zeta(179) \frac{3064991081731777716716694054300618367237478244367204348 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}}{3064991081731777716716694054300618367237478244367204351 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}} \quad (116)$$

$$\zeta(183) = \zeta(181) \frac{12259964326927110866866776217202473468949912977468817404 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{12259964326927110866866776217202473468949912977468817407 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}} \quad (117)$$

$$\zeta(185) = \zeta(183) \frac{49039857307708443467467104868809893875799651909875269628 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{49039857307708443467467104868809893875799651909875269631 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}} \quad (118)$$

$$\zeta(187) = \zeta(185) \frac{196159429230833773869868419475239575503198607639501078524 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{196159429230833773869868419475239575503198607639501078527 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}} \quad (119)$$

$$\zeta(189) = \zeta(187) \frac{784637716923335095479473677900958302012794430558004314108 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{784637716923335095479473677900958302012794430558004314111 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}} \quad (120)$$

$$\zeta(191) = \zeta(189) \frac{3138550867693340381917894711603833208051177722232017256444 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{3138550867693340381917894711603833208051177722232017256447 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}} \quad (121)$$

$$\zeta(193) = \zeta(191) \frac{12554203470773361527671578846415332832204710888928069025788 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{12554203470773361527671578846415332832204710888928069025791 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}} \quad (122)$$

$$\zeta(195) = \zeta(193) \frac{50216813883093446110686315385661331328818843555712276103164 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{50216813883093446110686315385661331328818843555712276103167 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}} \quad (123)$$

$$\zeta(197) = \zeta(195) \frac{200867255532373784442745261542645325315275374222849104412668 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{200867255532373784442745261542645325315275374222849104412671 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}} \quad (124)$$

$$\zeta(199) = \zeta(197) \frac{803469022129495137770981046170581301261101496891396417650684 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{803469022129495137770981046170581301261101496891396417650687 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}} \quad (125)$$

$$\zeta(201) = \zeta(199) \frac{3213876088517980551083924184682325205044405987565585670602748 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{3213876088517980551083924184682325205044405987565585670602751 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}} \quad (126)$$

$$\zeta(203) = \zeta(201) \frac{12855504354071922204335696738729300820177623950262342682411004 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{12855504354071922204335696738729300820177623950262342682411007 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}} \quad (127)$$

$$\zeta(205) = \zeta(203) \frac{51422017416287688817342786954917203280710495801049370729644028 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{51422017416287688817342786954917203280710495801049370729644031 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}} \quad (128)$$

$$\zeta(207) = \zeta(205) \frac{205688069665150755269371147819668813122841983204197482918576124 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{205688069665150755269371147819668813122841983204197482918576127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}} \quad (129)$$

$$\zeta(209) = \zeta(207) \frac{822752278660603021077484591278675252491367932816789931674304508 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{822752278660603021077484591278675252491367932816789931674304511 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}} \quad (130)$$

$$\zeta(211) = \zeta(209) \frac{3291009114642412084309938365114701009965471731267159726697218044}{3291009114642412084309938365114701009965471731267159726697218047} \quad (131)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}$$
 (132)

$$\zeta(213) = \zeta(211) \frac{13164036458569648337239753460458804039861886925068638906788872188}{13164036458569648337239753460458804039861886925068638906788872191}$$
 (133)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}$$
 (134)

$$\zeta(215) = \zeta(213) \frac{52656145834278593348959013841835216159447547700274555627155488764}{52656145834278593348959013841835216159447547700274555627155488767}$$
 (135)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}$$
 (136)

$$\zeta(217) = \zeta(215) \frac{210624583337114373395836055367340864637790190801098222508621955068}{210624583337114373395836055367340864637790190801098222508621955071}$$
 (137)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}$$
 (138)

$$\zeta(219) = \zeta(217) \frac{842498333348457493583344221469363458551160763204392890034487820284}{842498333348457493583344221469363458551160763204392890034487820287}$$
 (139)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}$$
 (140)

$$\zeta(221) = \zeta(219) \frac{3369993333393829974333376885877453834204643052817571560137951281148}{3369993333393829974333376885877453834204643052817571560137951281151}$$
 (141)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}$$
 (142)

$$\zeta(223) = \zeta(221) \frac{13479973333575319897333507543509815336818572211270286240551805124604}{13479973333575319897333507543509815336818572211270286240551805124607}$$
 (143)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}$$
 (144)

$$\zeta(225) = \zeta(223) \frac{53919893334301279589334030174039261347274288845081144962207220498428}{53919893334301279589334030174039261347274288845081144962207220498431}$$
 (145)

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}$$
 (146)

$$\zeta(227) = \zeta(225) \frac{215679573337205118357336120696157045389097155380324579848828881993724}{215679573337205118357336120696157045389097155380324579848828881993727} \quad (147)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}} \quad (148)$$

$$\zeta(229) = \zeta(227) \frac{862718293348820473429344482784628181556388621521298319395315527974908}{862718293348820473429344482784628181556388621521298319395315527974911} \quad (149)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}} \quad (150)$$

$$\zeta(231) = \zeta(229) \frac{3450873173395281893717377931138512726225554486085193277581262111899644}{3450873173395281893717377931138512726225554486085193277581262111899647} \quad (151)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}} \quad (152)$$

$$\zeta(233) = \zeta(231) \frac{13803492693581127574869511724554050904902217944340773110325048447598588}{13803492693581127574869511724554050904902217944340773110325048447598591} \quad (153)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}} \quad (154)$$

$$\zeta(235) = \zeta(233) \frac{55213970774324510299478046898216203619608871777363092441300193790394364}{55213970774324510299478046898216203619608871777363092441300193790394367} \quad (155)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}} \quad (156)$$

$$\zeta(237) = \zeta(235) \frac{220855883097298041197912187592864814478435487109452369765200775161577468}{220855883097298041197912187592864814478435487109452369765200775161577471} \quad (157)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}} \quad (158)$$

$$\zeta(239) = \zeta(237) \frac{883423532389192164791648750371459257913741948437809479060803100646309884}{883423532389192164791648750371459257913741948437809479060803100646309887} \quad (159)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}} \quad (160)$$

$$\zeta(241) = \zeta(239) \frac{3533694129556768659166595001485837031654967793751237916243212402585239548}{3533694129556768659166595001485837031654967793751237916243212402585239551} \quad (161)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}} \quad (162)$$

$$\zeta(243) = \zeta(241) \frac{14134776518227074636666380005943348126619871175004951664972849610340958204}{14134776518227074636666380005943348126619871175004951664972849610340958207} \quad (163)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}} \quad (164)$$

$$\zeta(245) = \zeta(243) \frac{56539106072908298546665520023773392506479484700019806659891398441363832828}{56539106072908298546665520023773392506479484700019806659891398441363832831} \quad (165)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}} \quad (166)$$

$$\zeta(247) = \zeta(245) \frac{226156424291633194186662080095093570025917938800079226639565593765455331324}{226156424291633194186662080095093570025917938800079226639565593765455331327} \quad (167)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}} \quad (168)$$

$$\zeta(249) = \zeta(247) \frac{904625697166532776746648320380374280103671755200316906558262375061821325308}{904625697166532776746648320380374280103671755200316906558262375061821325311} \quad (169)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}} \quad (170)$$

$$\zeta(251) = \zeta(249) \frac{3618502788666131106986593281521497120414687020801267626233049500247285301244}{3618502788666131106986593281521497120414687020801267626233049500247285301247} \quad (171)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}} \quad (172)$$

$$\zeta(253) = \zeta(251) \frac{14474011154664524427946373126085988481658748083205070504932198000989141204988}{14474011154664524427946373126085988481658748083205070504932198000989141204991} \quad (173)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}} \quad (174)$$

$$\zeta(255) = \zeta(253) \frac{57896044618658097711785492504343953926634992332820282019728792003956564819964}{57896044618658097711785492504343953926634992332820282019728792003956564819967} \quad (175)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}} \quad (176)$$

$$\zeta(257) = \zeta(255) \frac{231584178474632390847141970017375815706539969331281128078915168015826259279868}{231584178474632390847141970017375815706539969331281128078915168015826259279871} \quad (177)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}} \quad (178)$$

$$\zeta(259) = \zeta(257) \frac{926336713898529563388567880069503262826159877325124512315660672063305037119484}{926336713898529563388567880069503262826159877325124512315660672063305037119487} \quad (179)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}} \quad (180)$$

$$\zeta(261) = \zeta(259) \frac{3705346855594118253554271520278013051304639509300498049262642688253220148477948}{3705346855594118253554271520278013051304639509300498049262642688253220148477951} \quad (181)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}} \quad (182)$$

$$\zeta(263) = \zeta(261) \frac{14821387422376473014217086081112052205218558037201992197050570753012880593911804}{14821387422376473014217086081112052205218558037201992197050570753012880593911807} \quad (183)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}} \quad (184)$$

$$\zeta(265) = \zeta(263) \frac{59285549689505892056868344324448208820874232148807968788202283012051522375647228}{59285549689505892056868344324448208820874232148807968788202283012051522375647231} \quad (185)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}} \quad (186)$$

$$\zeta(267) = \zeta(265) \frac{237142198758023568227473377297792835283496928595231875152809132048206089502588924}{237142198758023568227473377297792835283496928595231875152809132048206089502588927} \quad (187)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}} \quad (188)$$

$$\zeta(269) = \zeta(267) \frac{948568795032094272909893509191171341133987714380927500611236528192824358010355708}{948568795032094272909893509191171341133987714380927500611236528192824358010355711} \quad (189)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}} \quad (190)$$

$$\zeta(271) = \zeta(269) \frac{3794275180128377091639574036764685364535950857523710002444946112771297432041422844}{3794275180128377091639574036764685364535950857523710002444946112771297432041422847} \quad (191)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}} \quad (192)$$

$$\zeta(273) = \zeta(271) \frac{15177100720513508366558296147058741458143803430094840009779784451085189728165691388}{15177100720513508366558296147058741458143803430094840009779784451085189728165691391} \quad (193)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}} \quad (194)$$

$$\zeta(275) = \zeta(273) \frac{60708402882054033466233184588234965832575213720379360039119137804340758912662765564}{60708402882054033466233184588234965832575213720379360039119137804340758912662765567} \quad (195)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}} \quad (196)$$

$$\zeta(277) = \zeta(275) \frac{242833611528216133864932738352939863330300854881517440156476551217363035650651062268}{242833611528216133864932738352939863330300854881517440156476551217363035650651062271} \quad (197)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}} \quad (198)$$

$$\zeta(279) = \zeta(277) \frac{971334446112864535459730953411759453321203419526069760625906204869452142602604249084}{971334446112864535459730953411759453321203419526069760625906204869452142602604249087} \quad (199)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}} \quad (200)$$

$$\zeta(281) = \zeta(279) \frac{3885337784451458141838923813647037813284813678104279042503624819477808570410416996348}{3885337784451458141838923813647037813284813678104279042503624819477808570410416996351} \quad (201)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}} \quad (202)$$

$$\zeta(283) = \zeta(281) \frac{15541351137805832567355695254588151253139254712417116170014499277911234281641667985404}{15541351137805832567355695254588151253139254712417116170014499277911234281641667985407} \quad (203)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}} \quad (204)$$

$$\zeta(285) =$$

$$\zeta(283) \frac{62165404551223330269422781018352605012557018849668464680057997111644937126566671941628}{62165404551223330269422781018352605012557018849668464680057997111644937126566671941631} \quad (205)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}} \quad (206)$$

$$\zeta(287) =$$

$$\zeta(285) \frac{248661618204893321077691124073410420050228075398673858720231988446579748506266687766524}{248661618204893321077691124073410420050228075398673858720231988446579748506266687766527} \quad (207)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}} \quad (208)$$

$$\zeta(289) =$$

$$\zeta(287) \frac{994646472819573284310764496293641680200912301594695434880927953786318994025066751066108}{994646472819573284310764496293641680200912301594695434880927953786318994025066751066111} \quad (209)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}} \quad (210)$$

$$\zeta(291) =$$

$$\zeta(289) \frac{3978585891278293137243057985174566720803649206378781739523711815145275976100267004264444}{3978585891278293137243057985174566720803649206378781739523711815145275976100267004264447} \quad (211)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}} \quad (212)$$

$$\zeta(293)=$$

$$\zeta(291) \frac{15914343565113172548972231940698266883214596825515126958094847260581103904401068017057788}{15914343565113172548972231940698266883214596825515126958094847260581103904401068017057791} \quad (213)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}} \quad (214)$$

$$\zeta(295)=$$

$$\zeta(293) \frac{63657374260452690195888927762793067532858387302060507832379389042324415617604272068231164}{63657374260452690195888927762793067532858387302060507832379389042324415617604272068231167} \quad (215)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}} \quad (216)$$

$$\zeta(297)=$$

$$\zeta(295) \frac{254629497041810760783555711051172270131433549208242031329517556169297662470417088272924668}{254629497041810760783555711051172270131433549208242031329517556169297662470417088272924671} \quad (217)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}} \quad (218)$$

$$\zeta(299)=$$

$$\zeta(297) \frac{1018517988167243043134222844204689080525734196832968125318070224677190649881668353091698684}{1018517988167243043134222844204689080525734196832968125318070224677190649881668353091698687} \quad (219)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}} \quad (220)$$

$$\zeta(301)=$$

$$\zeta(299) \frac{4074071952668972172536891376818756322102936787331872501272280898708762599526673412366794748}{4074071952668972172536891376818756322102936787331872501272280898708762599526673412366794751} \quad (221)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}} \quad (222)$$

$$\zeta(303)=\zeta(301) \times$$

$$\frac{4074071952668972172536891376818756322102936787331872501272280898708762599526673412366794748}{4074071952668972172536891376818756322102936787331872501272280898708762599526673412366794751} \quad (223)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}} \quad (224)$$

$$\zeta(305) = \zeta(303) \times$$

$$\frac{65185151242703554760590262029100101153646988597309960020356494379340201592426774597868716028}{65185151242703554760590262029100101153646988597309960020356494379340201592426774597868716031} \quad (225)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}} \quad (226)$$

$$\zeta(307) = \zeta(305) \times$$

$$\frac{260740604970814219042361048116400404614587954389239840081425977517360806369707098391474864124}{260740604970814219042361048116400404614587954389239840081425977517360806369707098391474864127} \quad (227)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}} \quad (228)$$

$$\zeta(309) = \zeta(307) \times$$

$$\frac{1042962419883256876169444192465601618458351817556959360325703910069443225478828393565899456508}{1042962419883256876169444192465601618458351817556959360325703910069443225478828393565899456511} \quad (229)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}} \quad (230)$$

$$\zeta(311) = \zeta(309) \times$$

$$\frac{4171849679533027504677776769862406473833407270227837441302815640277772901915313574263597826047}{4171849679533027504677776769862406473833407270227837441302815640277772901915313574263597826047} \quad (231)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}} \quad (232)$$

$$\zeta(313) = \zeta(311) \times$$

$$\frac{16687398718132110018711107079449625895333629080911349765211262561111091607661254297054391304188}{16687398718132110018711107079449625895333629080911349765211262561111091607661254297054391304191} \quad (233)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}} \quad (234)$$

$$\zeta(315)=\zeta(313)\times$$

$$\frac{66749594872528440074844428317798503581334516323645399060845050244444366430645017188217565216764}{66749594872528440074844428317798503581334516323645399060845050244444366430645017188217565216767} \quad (235)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}} \quad (236)$$

$$\zeta(317)=\zeta(315)\times$$

$$\frac{266998379490113760299377713271194014325338065294581596243380200977777465722580068752870260867068}{266998379490113760299377713271194014325338065294581596243380200977777465722580068752870260867071} \quad (237)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}} \quad (238)$$

$$\zeta(319)=\zeta(317)\times$$

$$\frac{1067993517960455041197510853084776057301352261178326384973520803911109862890320275011481043468284}{1067993517960455041197510853084776057301352261178326384973520803911109862890320275011481043468287} \quad (239)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}} \quad (240)$$

$$\zeta(321)=\zeta(319)\times$$

$$\frac{4271974071841820164790043412339104229205409044713305539894083215644439451561281100045924173873148}{4271974071841820164790043412339104229205409044713305539894083215644439451561281100045924173873151} \quad (241)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}} \quad (242)$$

$$\zeta(323)=\zeta(321)\times$$

$$\frac{17087896287367280659160173649356416916821636178853222159576332862577757806245124400183696695492604}{17087896287367280659160173649356416916821636178853222159576332862577757806245124400183696695492607} \quad (243)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}} \quad (244)$$

$$\zeta(325)=\zeta(323)\times$$

$$\frac{68351585149469122636640694597425667667286544715412888638305331450311031224980497600734786781970428}{68351585149469122636640694597425667667286544715412888638305331450311031224980497600734786781970431} \quad (245)$$

$$\zeta(325) = \zeta(323) \times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}} \quad (246)$$

$$\zeta(327) = \zeta(325) \times$$

$$\frac{273406340597876490546562778389702670669146178861651554553221325801244124899921990402939147127881724}{273406340597876490546562778389702670669146178861651554553221325801244124899921990402939147127881727} \quad (247)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}} \quad (248)$$

$$\zeta(329) = \zeta(327) \times$$

$$\frac{1093625362391505962186251113558810682676584715446606218212885303204976499599687961611756588511526911}{1093625362391505962186251113558810682676584715446606218212885303204976499599687961611756588511526911} \quad (249)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}} \quad (250)$$

$$\zeta(331) = \zeta(329) \times$$

$$\frac{4374501449566023848745004454235242730706338861786424872851541212819905998398751846447026354046107644}{4374501449566023848745004454235242730706338861786424872851541212819905998398751846447026354046107647} \quad (251)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}} \quad (252)$$

$$\zeta(333) = \zeta(331) \times$$

$$\frac{17498005798264095394980017816940970922825355447145699491406164851279623993595007385788105416184430588}{17498005798264095394980017816940970922825355447145699491406164851279623993595007385788105416184430591} \quad (253)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}} \quad (254)$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations.

$\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

Example 2

That $\zeta(5)$ is an irrational number is already proven at **Example 1** (proof 1).

(Proof 2)

If $\zeta(7)$ is assumed to be rational number. $\zeta(7) = \frac{s}{t}$, s and t are integer.

$$\zeta(7) = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{o}{p} \quad o, p \text{ are assumed to be integer.}$$

$$\zeta(7) = \zeta(3) \frac{o}{p} \quad \text{it equal} \quad \zeta(3) = \zeta(7) \frac{p}{o} = \frac{sp}{to} \quad \text{But, } \zeta(3) \neq \frac{sp}{to}$$

This is because $\zeta(3)$ is known to be an irrational number.

This contradicts.

$\zeta(7)$ is irrational number.

(Proof end)

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc. prove that $\zeta(9), \zeta(11), \zeta(13)$ etc. are an irrational numbers.

and

Detailed description

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (255)$$

$$\zeta(7) = \frac{2^7}{2^7 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (256)$$

Multiply $\zeta(3)$ and $\zeta(7)$

$$\zeta(7) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (257)$$

$$\zeta(7) \frac{8}{7} = \zeta(3) \frac{128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (258)$$

$$\zeta(7) = \zeta(3) \frac{128}{127} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (259)$$

Do the same for $\zeta(9), \zeta(11), \zeta(13)$ etc.

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \zeta(1) \frac{4}{(2^3 - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (260)$$

$$\zeta(5) = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{31 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{28 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{(2^5 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (261)$$

$$\zeta(7) = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{127 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{(2^7 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (262)$$

$$\zeta(9) = \zeta(3) \frac{2^6 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{73 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (263)$$

$$\zeta(11) = \zeta(3) \frac{1792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{2047 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{(2^{11} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (264)$$

$$\zeta(13) = \zeta(3) \frac{7168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{8191 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{(2^{13} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (265)$$

$$\zeta(15) = \zeta(3) \frac{2^{12} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{4681 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (266)$$

$$\zeta(17) = \zeta(3) \frac{114688 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{131072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{114688 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{(2^{17} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (267)$$

$$\zeta(19) = \zeta(3) \frac{458752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{524287 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{458752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{(2^{19} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (268)$$

$$\zeta(21) = \zeta(3) \frac{2^{18} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{299593 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (269)$$

$$\zeta(23) = \zeta(3) \frac{7340032 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{8388607 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7340032 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{(2^{23} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (270)$$

$$\zeta(25) = \zeta(3) \frac{29360128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{33554431 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{29360128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{(2^{25} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (271)$$

$$\zeta(27) = \zeta(3) \frac{2^{24} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{19173961 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (272)$$

$$\zeta(29) = \zeta(3) \frac{469762048 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{536870911 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{469762048 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{(2^{29} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (273)$$

$$\zeta(31) = \zeta(3) \frac{1879048192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{2147483647 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1879048192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{(2^{31} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (274)$$

$$\zeta(33) = \zeta(3) \frac{2^{30} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{1227133513 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (275)$$

$$\zeta(35) = \zeta(3) \frac{30064771072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{34359738367 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{30064771072 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{(2^{35} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (276)$$

$$\zeta(37) = \zeta(3) \frac{120259084288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{137438953471 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{120259084288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{(2^{37} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (277)$$

$$\zeta(39) = \zeta(3) \frac{2^{36} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{78536544841 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (278)$$

$$\zeta(41) = \zeta(3) \frac{1924145348608 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{2199023255551 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{1924145348608 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{(2^{41} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (279)$$

$$\zeta(43) = \zeta(3) \frac{7696581394432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{8796093022207 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7696581394432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{(2^{43} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (280)$$

$$\zeta(45) = \zeta(3) \frac{2^{42} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{5026338869833 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (281)$$

$$\zeta(47) = \zeta(3) \frac{123145302310912 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{140737488355327 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{123145302310912 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{(2^{47} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (282)$$

$$\zeta(49) = \zeta(3) \frac{492581209243648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{562949953421311 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{492581209243648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{(2^{49} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (283)$$

$$\zeta(51) = \zeta(3) \frac{2^{48} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{321685687669321 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (284)$$

$$\zeta(53) = \zeta(3) \frac{7881299347898368 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{9007199254740991 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{7881299347898368 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{(2^{53} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (285)$$

$$\zeta(55) = \zeta(3) \frac{31525197391593472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{36028797018963967 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{31525197391593472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{(2^{55} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (286)$$

$$\zeta(57) = \zeta(3) \frac{2^{54} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{20587884010836553 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (287)$$

$$\zeta(59) = \zeta(3) \frac{504403158265495552 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{576460752303423487 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{504403158265495552 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{(2^{59} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (288)$$

$$\zeta(61) = \zeta(3) \frac{2017612633061982208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{2305843009213693951 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2017612633061982208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{(2^{61} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (289)$$

$$\zeta(63) = \zeta(3) \frac{2^{60} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{1317624576693539401 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (290)$$

$$\zeta(65) = \zeta(3) \frac{32281802128991715328 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{36893488147419103231 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{32281802128991715328 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{(2^{65} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (291)$$

$$\zeta(67) = \zeta(3) \frac{129127208515966861312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{147573952589676412927 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{129127208515966861312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{(2^{67} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (292)$$

$$\zeta(69) = \zeta(3) \frac{2^{66} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{84327972908386521673 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (293)$$

$$\zeta(71) = \zeta(3) \frac{2066035336255469780992 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{2361183241434822606847 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2066035336255469780992 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{(2^{71} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (294)$$

$$\zeta(73) = \zeta(3) \frac{8264141345021879123968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{9444732965739290427391 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{8264141345021879123968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{(2^{73} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (295)$$

$$\zeta(75) = \zeta(3) \frac{2^{72} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{5396990266136737387081 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (296)$$

$$\zeta(77) = \zeta(3) \frac{132226261520350065983488 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{151115727451828646838271 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{132226261520350065983488 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{(2^{77} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (297)$$

$$\zeta(79) = \zeta(3) \frac{528905046081400263933952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{604462909807314587353087 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{528905046081400263933952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{(2^{79} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (298)$$

$$\zeta(81) = \zeta(3) \frac{2^{78} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{345407377032751192773193 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (299)$$

$$\zeta(83) = \zeta(3) \frac{8462480737302404222943232 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{9671406556917033397649407 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{8462480737302404222943232 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{(2^{83} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (300)$$

$$\zeta(85) = \zeta(3) \frac{33849922949209616891772928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{38685626227668133590597631 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{33849922949209616891772928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{(2^{85} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (301)$$

$$\zeta(87) = \zeta(3) \frac{2^{84} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{22106072130096076337484361 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (302)$$

$$\zeta(89) = \zeta(3) \frac{541598767187353870268366848 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{(2^{89} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (303)$$

$$\zeta(91) = \zeta(3) \frac{2166395068749415481073467392 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{(2^{91} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (304)$$

$$\zeta(93) = \zeta(3) \frac{2^{90} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{1414788616326148885598999113 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (305)$$

$$\zeta(95) = \zeta(3) \frac{34662321099990647697175478272 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{(2^{95} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (306)$$

$$\zeta(97) = \zeta(3) \frac{138649284399962590788701913088 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{(2^{97} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (307)$$

$$\zeta(99) = \zeta(3) \frac{2^{96} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{90546471444873528678335943241 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (308)$$

$$\zeta(101) = \zeta(3) \frac{2218388550399401452619230609408 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{(2^{101} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (309)$$

$$\zeta(103) = \zeta(3) \frac{8873554201597605810476922437632 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{(2^{103} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (310)$$

$$\zeta(105) = \zeta(3) \frac{2^{102} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{5794974172471905835413500367433 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (311)$$

$$\zeta(107) = \zeta(3) \frac{141976867225561692967630759002112 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{(2^{107} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (312)$$

$$\zeta(109) = \zeta(3) \frac{567907468902246771870523036008448 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{(2^{109} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (313)$$

$$\zeta(111) = \zeta(3) \frac{2^{108} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{111}}}{370878347038201973466464023515721 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (314)$$

$$\zeta(113) = \zeta(3) \frac{9086519502435948349928368576135168 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{113}}}{(2^{113} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (315)$$

$$\zeta(115) = \zeta(3) \frac{36346078009743793399713474304540672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{115}}}{(2^{115} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (316)$$

$$\zeta(117) = \zeta(3) \frac{2^{114} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{117}}}{23736214210444926301853697505006153 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (317)$$

$$\zeta(119) = \zeta(3) \frac{581537248155900694395415588872650752 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{119}}}{(2^{119} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (318)$$

$$\zeta(121) = \zeta(3) \frac{2326148992623602777581662355490603008 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{121}}}{(2^{121} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (319)$$

$$\zeta(123) = \zeta(3) \frac{2^{120} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{123}}}{1519117709468475283318636640320393801 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (320)$$

$$\zeta(125) = \zeta(3) \frac{37218383881977644441306597687849648128 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{125}}}{(2^{125} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (321)$$

$$\zeta(127) = \zeta(3) \frac{148873535527910577765226390751398592512 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{127}}}{(2^{127} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (322)$$

$$\zeta(129) = \zeta(3) \frac{2^{126} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{129}}}{97223533405982418132392744980505203273 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (323)$$

$$\zeta(131) = \zeta(3) \frac{2381976568446569244243622252022377480192 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{131}}}{(2^{131} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (324)$$

$$\zeta(133) = \zeta(3) \frac{9527906273786276976974489008089509920768 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}}{(2^{133} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (325)$$

$$\zeta(135) = \zeta(3) \frac{2^{132} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{6222306137982874760473135678752333009481 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (326)$$

$$\zeta(137) = \zeta(3) \frac{152446500380580431631591824129432158732288 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{(2^{137} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (327)$$

$$\zeta(139) = \zeta(3) \frac{609786001522321726526367296517728634929152 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{(2^{139} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (328)$$

$$\zeta(141) = \zeta(3) \frac{2^{138} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{398227592830903984670280683440149312606793 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (329)$$

$$\zeta(143) = \zeta(3) \frac{9756576024357147624421876744283658158866432 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{(2^{143} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (330)$$

$$\zeta(145) = \zeta(3) \frac{39026304097428590497687506977134632635465728 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{(2^{145} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (331)$$

$$\zeta(147) = \zeta(3) \frac{2^{144} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{25486565941177855018897963740169556006834761 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (332)$$

$$\zeta(149) = \zeta(3) \frac{624420865558857447963000111634154122167451648 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{(2^{149} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (333)$$

$$\zeta(151) = \zeta(3) \frac{2497683462235429791852000446536616488669806592 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{(2^{151} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (334)$$

$$\zeta(153) = \zeta(3) \frac{2^{150} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{1631140220235382721209469679370851584437424713 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (335)$$

$$\zeta(155) = \zeta(3) \frac{39962935395766876669632007144585863818716905472 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{(2^{155} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (336)$$

$$\zeta(157) = \zeta(3) \frac{159851741583067506678528028578343455274867621888 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{(2^{157} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (337)$$

$$\zeta(159) = \zeta(3) \frac{2^{156} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}}{104392974095064494157406059479734501403995181641 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (338)$$

$$\zeta(161) = \zeta(3) \frac{2557627865329080106856448457253495284397881950208 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}}{(2^{161} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (339)$$

$$\zeta(163) = \zeta(3) \frac{10230511461316320427425793829013981137591527800832 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}}{(2^{163} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (340)$$

$$\zeta(165) = \zeta(3) \frac{2^{162} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}}{6681150342084127626073987806703008089855691625033 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (341)$$

$$\zeta(167) = \zeta(3) \frac{163688183381061126838812701264223698201464444813312 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}}{(2^{167} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (342)$$

$$\zeta(169) = \zeta(3) \frac{654752733524244507355250805056894792805857779253248 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}}{(2^{169} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (343)$$

$$\zeta(171) = \zeta(3) \frac{2^{168} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}}{427593621893384168068735219628992517750764264002121 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (344)$$

$$\zeta(173) = \zeta(3) \frac{10476043736387912117684012880910316684893724468051968 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}}{(2^{173} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (345)$$

$$\zeta(175) = \zeta(3) \frac{41904174945551648470736051523641266739574897872207872 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}}{(2^{175} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (346)$$

$$\zeta(177) = \zeta(3) \frac{2^{174} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}}{27365991801176586756399054056255521136048912896135753 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (347)$$

$$\zeta(179) = \zeta(3) \frac{670466799128826375531776824378260267833198365955325952 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}}{(2^{179} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (348)$$

$$\zeta(181) = \zeta(3) \frac{2681867196515305502127107297513041071332793463821303808 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}}{(2^{181} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (349)$$

$$\zeta(183) = \zeta(3) \frac{2^{180} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{1751423475275301552409539459600353352707130425352688201 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (350)$$

$$\zeta(185) = \zeta(3) \frac{42909875144244888034033716760208657141324695421140860928 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{(2^{185} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (351)$$

$$\zeta(187) = \zeta(3) \frac{171639500576979552136134867040834628565298781684563443712 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{(2^{187} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (352)$$

$$\zeta(189) = \zeta(3) \frac{2^{186} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{112091102417619299354210525414422614573256347222572044873 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (353)$$

$$\zeta(191) = \zeta(3) \frac{2746232009231672834178157872653354057044780506953015099392 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{(2^{191} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (354)$$

$$\zeta(193) = \zeta(3) \frac{10984928036926691336712631490613416228179122027812060397568 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{(2^{193} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (355)$$

$$\zeta(195) = \zeta(3) \frac{2^{192} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{7173830554727635158669473626523047332688406222244610871881 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (356)$$

$$\zeta(197) = \zeta(3) \frac{175758848590827061387402103849814659650865952444992966361088 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{(2^{197} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (357)$$

$$\zeta(199) = \zeta(3) \frac{703035394363308245549608415399258638603463809779971865444352 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{(2^{199} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (358)$$

$$\zeta(201) = \zeta(3) \frac{2^{198} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{459125155502568650154846312097475029292057998223655095800393 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (359)$$

$$\zeta(203) = \zeta(3) \frac{11248566309812931928793734646388138217655420956479549847109632 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{(2^{203} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (360)$$

$$\zeta(205) = \zeta(3) \frac{44994265239251727715174938585552552870621683825918199388438528 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{(2^{205} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (361)$$

$$\zeta(207) = \zeta(3) \frac{2^{204} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{29384009952164393609910163974238401874691711886313926131225161 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (362)$$

$$\zeta(209) = \zeta(3) \frac{719908243828027643442799017368840845929946941214691190215016448 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (363)$$

$$\zeta(211) = \zeta(3) \frac{2879632975312110573771196069475363383719787764858764760860065792 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (364)$$

$$\zeta(213) = \zeta(3) \frac{2^{210} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{1880576636938521191034250494351257719980269560724091272398410313 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (365)$$

$$\zeta(215) = \zeta(3) \frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (366)$$

$$\zeta(217) = \zeta(3) \frac{46074127604993769180339137111605814139516604237740236173761052672 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (367)$$

$$\zeta(219) = \zeta(3) \frac{2^{216} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{120356904764065356226192031638480494078737251886341841433498260041 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (368)$$

$$\zeta(221) = \zeta(3) \frac{2948744166719601227541704775142772104929062671215375115120707371008}{(2^{221} - 1)} \quad (369)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (370)$$

$$\zeta(223) = \zeta(3) \frac{11794976666878404910166819100571088419716250684861500460482829484032}{(2^{223} - 1)} \quad (371)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (372)$$

$$\zeta(225) = \zeta(3) \frac{2^{222}}{7702841904900182798476290024862751621039184120725877851743888642633} \quad (373)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (374)$$

$$\zeta(227) = \zeta(3) \frac{188719626670054478562669105609137414715460010957784007367725271744512}{(2^{227} - 1)} \quad (375)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (376)$$

$$\zeta(229) = \zeta(3) \frac{754878506680217914250676422436549658861840043831136029470901086978048}{(2^{229} - 1)} \quad (377)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (378)$$

$$\zeta(231) = \zeta(3) \frac{2^{228}}{492981881913611699102482561591216103746507783726456182511608873128521} \quad (379)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (380)$$

$$\zeta(233) = \zeta(3) \frac{12078056106883486628010822758984794541789440701298176471534417391648768}{(2^{233} - 1)} \quad (381)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (382)$$

$$\zeta(235) = \zeta(3) \frac{48312224427533946512043291035939178167157762805192705886137669566595072}{(2^{235} - 1)} \quad (383)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (384)$$

$$\zeta(237) = \zeta(3) \frac{2^{234}}{31550840442471148742558883941837830639776498158493195680742967880225353} \quad (385)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (386)$$

$$\zeta(239) = \zeta(3) \frac{772995590840543144192692656575026850674524204883083294178202713065521152}{(2^{239} - 1)} \quad (387)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (388)$$

$$\zeta(241) = \zeta(3) \frac{3091982363362172576770770626300107402698096819532333176712810852262084608}{(2^{241} - 1)} \quad (389)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (390)$$

$$\zeta(243) = \zeta(3) \frac{2^{240}}{2019253788318153519523768572277621160945695882143564523567549944334422601} \quad (391)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (392)$$

$$\zeta(245) = \zeta(3) \frac{49471717813794761228332330020801718443169549112517330827404973636193353728}{(2^{245} - 1)} \quad (393)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (394)$$

$$\zeta(247) = \zeta(3) \frac{197886871255179044913329320083206873772678196450069323309619894544773414912}{(2^{247} - 1)} \quad (395)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (396)$$

$$\zeta(249) = \zeta(3) \frac{2^{246}}{129232242452361825249521188625767754300524536457188129508323196437403046473} \quad (397)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (398)$$

$$\zeta(251) = \zeta(3) \frac{3166189940082864718613269121331309980362851143201109172953918312716374638592}{(2^{251} - 1)} \quad (399)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (400)$$

$$\zeta(253) = \zeta(3) \frac{12664759760331458874453076485325239921451404572804436691815673250865498554368}{(2^{253} - 1)} \quad (401)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (402)$$

$$\zeta(255) = \zeta(3) \frac{2^{252}}{8270863516951156815969356072049136275233570333260040288532684571993794974281} \quad (403)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{255}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (404)$$

$$\zeta(257) = \zeta(3) \frac{202636156165303341991249223765203838743222473164870987069050772013847976869888}{(2^{257} - 1)} \quad (405)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{257}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (406)$$

$$\zeta(259) = \zeta(3) \frac{810544624661213367964996895060815354972889892659483948276203088055391907479552}{(2^{259} - 1)} \quad (407)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{259}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (408)$$

$$\zeta(261) = \zeta(3) \frac{2^{258}}{529335265084874036222038788611144721614948501328642578466091812607602878353993} \quad (409)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{261}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (410)$$

$$\zeta(263) = \zeta(3) \frac{12968713994579413887439950320973045679566238282551743172419249408886270519672832}{(2^{263} - 1)} \quad (411)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{263}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (412)$$

$$\zeta(265) = \zeta(3) \frac{51874855978317655549759801283892182718264953130206972689676997635545082078691328}{(2^{265} - 1)} \quad (413)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{265}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (414)$$

$$\zeta(267) = \zeta(3) \frac{2^{264}}{33877456965431938318210482471113262183356704085033125021829876006886584214655561} \quad (415)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{267}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (416)$$

$$\zeta(269) = \zeta(3) \frac{829997695653082488796156820542274923492239250083311563034831962168721313259061248}{(2^{269} - 1)} \quad (417)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{269}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (418)$$

$$\zeta(271) = \zeta(3) \frac{3319990782612329955184627282169099693968957000333246252139327848674885253036244992}{(2^{271} - 1)} \quad (419)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{271}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (420)$$

$$\zeta(273) = \zeta(3) \frac{2^{270}}{2168157245787644052365470878151248779734829061442120001397112064440741389737955913} \quad (421)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{273}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (422)$$

$$\zeta(275) = \zeta(3) \frac{53119852521797279282954036514705595103503312005331940034229245578798164048579919872}{(2^{275} - 1)} \quad (423)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{275}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (424)$$

$$\zeta(277) = \zeta(3) \frac{212479410087189117131816146058822380414013248021327760136916982315192656194319679488}{(2^{277} - 1)} \quad (425)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (426)$$

$$\zeta(279) = \zeta(3) \frac{2^{276}}{138762063730409219351390136201679921903029059932295680089415172124207448943229178441} \quad (427)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (428)$$

$$\zeta(281) = \zeta(3) \frac{3399670561395025874109058336941158086624211968341244162190671717043082499109114871808}{(2^{281} - 1)} \quad (429)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (430)$$

$$\zeta(283) = \zeta(3) \frac{13598682245580103496436233347764632346496847873364976648762686868172329996436459487232}{(2^{283} - 1)} \quad (431)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (432)$$

$$\zeta(285) = \zeta(3) \frac{2^{282}}{8880772078746190038488968716907515001793859835666923525722571015949276732366667420233} \quad (433)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (434)$$

$$\zeta(287)=$$

$$\zeta(3) \frac{217578915929281655942979733564234117543949565973839626380202989890757279942983351795712}{(2^{287} - 1)} \quad (435)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (436)$$

$$\zeta(289)=$$

$$\zeta(3) \frac{870315663717126623771918934256936470175798263895358505520811959563029119771933407182848}{(2^{289} - 1)} \quad (437)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (438)$$

$$\zeta(291)=$$

$$\zeta(3) \frac{2^{288}}{568369413039756162463293997882080960114807029482683105646244545020753710871466714894921} \quad (439)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (440)$$

$$\zeta(293)=$$

$$\zeta(3) \frac{13925050619474025980350702948110983522812772222325736088332991353008465916350934514925568}{(2^{293} - 1)} \quad (441)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (442)$$

$$\zeta(295)=$$

$$\zeta(3) \frac{55700202477896103921402811792443934091251088889302944353331965412033863665403738059702272}{(2^{295} - 1)} \quad (443)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (444)$$

$$\zeta(297) = \zeta(3) \frac{2^{294}}{36375642434544394397650815864453181447347649886891718761359650881328237495773869753274953} \quad (445)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (446)$$

$$\zeta(299) = \zeta(3) \frac{891203239646337662742444988679102945460017422228847109653311446592541818646459808955236352}{(2^{299} - 1)} \quad (447)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (448)$$

$$\zeta(301) = \zeta(3) \frac{3564812958585350650969779954716411781840069688915388438613245786370167274585839235820945408}{(2^{301} - 1)} \quad (449)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (450)$$

$$\zeta(303) = \zeta(3) \frac{2^{300}}{2328041115810841241449652215325003612630249592761070000727017656405007199729527664209597001} \quad (451)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (452)$$

$$\zeta(305) = \zeta(3) \frac{57037007337365610415516479275462588509441115022646215017811932581922676393373427773135126528}{(2^{305} - 1)} \quad (453)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (454)$$

$$\zeta(307) = \zeta(3) \frac{228148029349462441662065917101850354037764460090584860071247730327690705573493711092540506112}{(2^{307} - 1)} \quad (455)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (456)$$

$\zeta(309)=$

$$\zeta(3) \frac{2^{306}}{148994631411893839452777741780800231208335973936708480046529130009920460782689770509414208073} \quad (457)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (458)$$

$\zeta(311)=$

$$\zeta(3) \frac{3650368469591399066593054673629605664604231361449357761139963685243051289175899377480648097792}{(2^{311} - 1)} \quad (459)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (460)$$

$\zeta(313)=$

$$\zeta(3) \frac{14601473878365596266372218694518422658416925445797431044559854740972205156703597509922592391168}{(2^{313} - 1)} \quad (461)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (462)$$

$\zeta(315)=$

$$\zeta(3) \frac{2^{312}}{9535656410361205724977775473971214797333502331949342722977864320634909490092145312602509316681} \quad (463)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (464)$$

$\zeta(317)=\zeta(3) \times$

$$\frac{233623582053849540261955499112294762534670807132758896712957675855555282507257560158761478258688}{(2^{317} - 1)} \quad (465)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (466)$$

$$\zeta(319) = \zeta(3) \times \frac{934494328215398161047821996449179050138683228531035586851830703422221130029030240635045913034752}{(2^{319} - 1)} \quad (467)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (468)$$

$$\zeta(321) = \zeta(3) \times \frac{2^{318}}{610282010263117166398577630334157747029344149244757934270583316520634207365897300006560596267593} \quad (469)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (470)$$

$$\zeta(323) = \zeta(3) \times \frac{14951909251446370576765151943186864802218931656496569389629291254755538080464483850160734608556032}{(2^{323} - 1)} \quad (471)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (472)$$

$$\zeta(325) = \zeta(3) \times \frac{59807637005785482307060607772747459208875726625986277558517165019022152321857935400642938434224128}{(2^{325} - 1)} \quad (473)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (474)$$

$$\zeta(327) = \zeta(3) \times \frac{2^{324}}{39058048656839498649508968341386095809878025551664507793317332257320589271417427200419878161125961} \quad (475)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (476)$$

$$\zeta(329) = \zeta(3) \times \frac{956922192092567716912969724363959347342011626015780440936274640304354437149726966410287014947586048}{(2^{329} - 1)} \quad (477)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (478)$$

$$\zeta(331) = \zeta(3) \times$$

$$\frac{3827688768370270867651878897455837389368046504063121763745098561217417748598907865641148059790344192}{(2^{331} - 1)} \quad (479)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (480)$$

$$\zeta(333) = \zeta(3) \times$$

$$\frac{2^{330}}{2499715114037727913568573973848710131832193635306528498772309264468517713370715340826872202312061513} \quad (481)$$

$$\times \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (482)$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(3), \zeta(5), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

4 Postscript

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{\pi}} \quad (483)$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \frac{1}{2^{\pi}} \sum_{n=1}^{\infty} \frac{1}{n^{\pi}} \quad (484)$$

$$\zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} + \frac{1}{2^{\pi}} \zeta(\pi) \quad (485)$$

$$\left(1 - \frac{1}{2^{\pi}}\right) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (486)$$

$$\left(\frac{2^{\pi}-1}{2^{\pi}}\right) \zeta(\pi) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (487)$$

$$\zeta(\pi) = \frac{2^{\pi}}{2^{\pi}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{\pi}} \quad (488)$$

=1.17624173838258275887215045193805209116973899002165583496050834623040872376815861833572083732
557183113894566008145...

Do the same

$$\zeta(e) = \frac{2^e}{2^e-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^e} \quad (489)$$

=1.26900960433571711576556986660086110885640446257719048833581592870816524294827307650849451745
076054575828347684218...

I believe that $\zeta(\pi), \zeta(e)$ are irrational numbers.

That is, I believe that the irrational number ζ are irrational numbers.

I also believe that all even value, as well as odd values of ζ , are irrational numbers.

The figures in this paper have been fully verified by WolframAlpha.

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