A Proof of Goldbach's strong conjecture

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Abstract : The proof includes a series and its aspects, treated in a special way. The concept of Unique Path of primes is explained and its effects are shown. In the midway of the proof, it is postponed for a while and a deviation from the course is taken to introduce a probably new axiom. Afterwards the proof restarts
again and using the axiom and other results the conjecture is proved.

Please note : Every symbol 'p' with or without any suffix denotes some prime number. a|b means a divides b and a®b means a doesn't divide b. n is a natural number. The word 'prime' will hereafter mean prime number and 'even' mean even positive integer. The sign '∃' means 'there exist'.

First stage : There is at least one prime p ($3 \le p < n$) for every $2n > 6$ such that p®2n.
Proof : For any even $2n > 6$, at least one of the evens 2n-2 and 2n+2 is not an integral power of 2.
Now n-1 or n+1 is divisible by at least one prime p ($3 \le p < n$).
So $p|2(n-1) \Rightarrow$ p®$2(n-1)+2 \Rightarrow$ p®2n, or alternatively $p|2(n+1) \Rightarrow$ p®$2(n+1)-2 \Rightarrow$ p®2n
Suitably using any of the above two alternative results we can prove the claim.

Second stage : Concider a prime p_1 ($3 \le p_1 < n$) such that p_1®2n. Now let $2n-p_1$ is divisible by a prime p_2, where $p_2 < n$.
So there can be a series

$\exists p_2$, such that $p_2|2n-p_1$, where $p_2 < n$

$\exists p_3$, such that $p_3|2n-p_2$, where $p_3 < n$

… … …

… … … $\exists p_k$, such that $p_k|2n-p_{k-1}$, where $p_k < n$

The primes p_2, p_3,… are taken in such a manner, as far as possible, that each one is different from all the other primes (including p_1) appearing previous to itself in the series.

It can easily be proved that no such prime divides 2n (since any $p_k \neq p_{k-1}$).

The operation of getting p_2,p_3,… must end at some p_k, otherwise there will be infinite number of different primes<n; k is a finite positive integer. We henceforth shall call p_1 as 'starting prime'. We further call p_2,p_3,…, p_k (all being different, where p_k is the last of them) as different outputs or simply as outputs.

Let the course of the proof be postponed for a while to discuss
a topic. It is a common sense that
we can omit anything from a
written or mentioned expression.
For that purpose we simply need
to wipe out or erase the purported

object from the expression. But
when the question comes to the dealing with its logical aspect, we need to introduce an axiom. Namely •••
Axiom of omission : We can omit or erase anything from an expression or a system of expressions in some context, if the rest of it bears some logical meaning in that same context.

It is a different question what the effect of this axiom should be in the context of other mathematical topics. It is just a logical interpretation of certain human discretion taken in common sense perspectives.

For our purposes in the following discussions we shall quite justifiably interpret the 'omission of something from something' as the 'imagination of no existence of former in anywhere of the later', judging only by whatever would be obvious from mentioned things there.

Return to the proof •••••

Observation (1) : For a particular p_1 we can choose arbitrarily particular $p_2, p_3, \ldots, p_k$ (p_k being the last available different output for the series where p_1 is the starting prime) and in this way they constitute a Unique Path : an ordered list of successive particular selections (within the scopes available) from the prime factors of various $2n-p_t$'s, p_t's starting from p_1, where p_1 is also included in the same list and put in the first place. Such Unique Path is always strictly ordered.

Observation (2) : p_k being the last different output ($<n$) { p_k available from $2n-p_{k-1}$ as a factor of it } in
the series, proceeding similarly beyond it we get p_{k+1} from $2n-p_k$, where $p_{k+1}|2n-p_k$ with the exception that in this case p_{k+1} not necessarily $<n$. Besides, as this step goes past that of the last different output, the stipulation for being different from the previously appeared primes vanishes automatically.

Now $p_{k+1}<n$ implies p_{k+1} is a recycled prime, i.e, p_{k+1} is one of $p_1,p_2,p_3,\ldots, p_k$ (since p_k is the last available different output for the series of k-1 unique steps, p_1 being the starting prime).

We define 'a list' as a successive mentioning of items (ignore the commas) and 'a choice' as a selection of mentioned item/s. Evidently a list is an expression. We claim that,

we choose only one item from a particular list⇒ we omit the rest of the items from the list (provided the list contains more than one items as it's elements)
Proof : If not so.

Since a list is an expression and we have to choose from the list, if we retain at least another item in the list, other than the one intended for this choice,

there will be at least two mentioned items for a choice, where none can be excluded. But we have to choose (i.e, select the mentioned) only one item as per requirement. So there is a contradiction.

Hence our claim is true.

Let p_{k+1} is a recycled prime. Therefore it can be taken from either (1) the set of all elements of the Unique Path, and there is no uniquely particular order for its elements, which implies more than one lists, of different particular orders for their respective elements, can be made with the elements (all and nothing more for every such list) of the set ••••

We will now prove a lemma.

Lemma : We can choose an element from a non empty set ⇒ we can choose it from a list that can be formed with the elements of the same set, where the list includes the element in question.

Proof : If we can choose the element from a non empty set then the element is a member of the set.

So the list described above can be formed from the above set ⇒ the element in question is mentioned somewhere in the list and therefore we can undoubtedly choose it from that list.

This proves the lemma.

Now the above lemma leads to a more specific option for choosing the recycled prime, so to say that {continuing from (1)} ••••, or (2) it can be taken from the Unique Path only, where the existence of every element (other than the first) of this Unique Path⇔the existence of corresponding step in the series from which the Unique Path derives.

Explanation : Take the case (1).

Choosing p_{k+1} from the set of all elements of the Unique Path implies choosing it from at least one list having the elements of the set (all and nothing more).

If the recycled prime only to be taken from a list having an order (a list always has some particular order for its elements) different from that of the Unique Path, i.e, relative positions of other elements of the list w.r.t the recycled prime to be chosen as p_{k+1}, are different from those of the Unique Path,

then either the other list (which is not the Unique Path) cannot be derived by any series such as mentioned before, or (if it at all could be derived so) the list is a different Unique Path made with the same elements (all and nothing more) of that original one (any such Unique Path remains unique only w.r.t one time particular selection of primes placed in a particular order, those which can be attributed to that Unique Path within available scopes).

Both the possibilities contradict the hypothetical situation of existence of the series or related uniqueness of the original Unique Path. [For the same reason a list formed from a proper subset of the set mentioned in case (1) is unacceptable here]

Again, since each step in a series corresponds to an unique element of the associated Unique Path and conversely (as obvious from the series),

therefore the existence of every element (other than the first) of the Unique Path⇔the existence of corresponding step in the series from which the Unique Path derives.

Summing up we can say, the case (2) is more definite and the only acceptable one as compared to other possibilities discussed above for our purpose in this context (i.e, when p_{k+1} is a recycled prime).

Further, we claim : the omission of an output from an Unique Path⇒the omission of corresponding generating step from the series from which the Unique Path derives.•••••(3)

Proof : let the proposition is not true.

Then despite the omission of an output from the Unique Path, there exists corresponding generating step of it in the series mentioned above.

But the existence of the above step in the series⇒the existence of the output in question in the Unique Path {from the case (2) above}, which contradicts the hypothesis that the output is omitted from the Unique Path.

Therefore our claim is true.

Since p_{k+1} is a singularly mentioned prime (as evident from its identity), a recycled prime taken as p_{k+1} is also a singularly mentioned identity which implies we have to choose only one output from the list of different outputs.

Now for an arbitrarily particular p_1 we obtain a Unique Path of successive particular selection of primes {as described in Observation (1)}, which must contain a unique starting prime and a unique last output w.r.t the path itself (otherwise we will deny that it is a Unique Path).

Since a Unique Path itself is an ordered list,

omission of any of these two from this Unique Path' (i.e, the imagination that there is no existence of any one of these two in this Unique Path) doesn't make any logical meaning for the residual expression (i.e, the list) containing any of the rest except p_1 or p_k, in the process regarding the choice of a recycled prime as was stated earlier, where existence of the Unique Path and thereby the series from which it derives, should be taken as presuppositions (i.e, as necessary conditions) for the choice of a recycled prime to appear as p_{k+1}.

[Clarification : The above omissions implies the Unique Path is devoid of a starting prime and/or a last output$\Rightarrow$the series is devoid of corresponding generating step of the last output {from claim (3)} or the very first step which is unique and unavoidable for any such series, and thus the whatever (allowing provisions for the possibilities of the omissions of outputs different from p_1 and p_k) such residual part of the Unique Path in the context of whatever logical set up and conclusion that have been established upto this point, goes undefined.]

Therefore by Axiom of omission, any of the above two omissions, i.e, p_1 or p_k, from the Unique Path in the context described in the clarification, is impossible,

which means after every possible omission from the above mentioned ordered list (i.e, the Unique Path) there remains at least two primes to choose as p_{k+1}, none of which is omittable.

So we can say that we can't omit, all primes other than that intended one for recycling purpose, from the Unique Path and as the very choice from this ordered list, in question, requires at least one of the primes p_1 or p_k be removed from it besides all other possible omissions,

and since the recycled prime in question must be chosen from that Unique Path only {from case (2) discussed before},

it implies we can't choose only one recycled prime (therefore any recycled prime at all, in fact) as p_{k+1} from that Unique Path.

Summing up the above discussions

we conclude that p_{k+1} can't be recycled, that implies p_{k+1} isn't$<n$, and since $p_k<n$,

we are bound to accept the conclusion that $p_{k+1}>n \Rightarrow p_{k+1}=2n-p_k$.

[$2n-p_k$ can't have a factor that is grater than n and smaller than itself, and $p_{k+1}\neq n$ for obvious reasons.]

Therefore, $2n=p_k+p_{k+1}$

Take a look back at the beginning of the Second stage •••

Contrary to what we have assumed at there, if p_2 isn't $<n$, then as $p_1<n$, p_2 becomes$>n$. This implies $2n=p_1+p_2$

[Reasons are similar as above]

Finally, over the question whether the integers 6 & 4 comply to Goldbach's strong conjecture, we write $6=3+3$ and $4=2+2$

Therefore Goldbach's strong conjecture holds for every $2n \geq 4$.