A graph for describing events in TSR (v1, 2019-08-26)

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Abstract. We present a graphical approach which illustrates a chain of events within the theory of special relativity (TSR). The graph represents an alternative to the 'world line', and presents the standard time & position–parameters. Further, the graph provides the so-called 'proper time', which we refer to as 'internal time'. The object we follow has a velocity being piecewise constant; a condition that can be relaxed. The travelling twin provides an example.

Key words: Event graph, internal time, embedded clock, dimensions of time, travelling twin.

1 Introduction

In previous works, Hokstad (2018a, 2018b, 2019) we have studied a time vector in TSR to describe an event in the perspective of various moving reference frames (RF), which are treated in a symmetric way. Now we build on this work, but focus on describing the *chain of events* of an object in the perspective of one specific RF.

The velocity of the moving object is a piecewise constant, and we imagine this object to have a built in ('embedded') clock showing the 'internal time' (= 'proper time'). This time is known to be independent of the 'perspective', *i.e.* the chosen RF.

A graph will illustrate the development of both this embedded clock reading, and the standard space and time parameters of the event chain. We use the 'travelling twin' to illustrate the approach, applying two different perspectives.

We have just one space coordinate, but a generalization to three space coordinates is simple. In general, the object could also have an arbitrary (changing) velocity. – However, the approach just describes the movement of a mass-less object, (not considering forces).

The approach points to the essential distinction between [1)] the embedded clock reading, which is independent of the chosen RF, and [2)] the two other parameters, (time and position) depending on the chosen perspective (*i.e.* RF). This distinction might affect how we interpret time in TSR.

2 A graphical approach

We start out with a RF, *K* having synchronized clocks at virtually any position. The clock readings ('time') on *K* is denoted t, and there is one space coordinate (x). Further, there is a moving object at speed, v along the x-axis. For simplicity, we assume that at $t = 0$ this object is positioned at the origin. It follows that at time t, its position equals $x_t = vt$.

Now (t, x_t) for $t \geq 0$ defines a chain of events, and we will provide a graphical representation of this. In previous works we have argued for describing an event by the time vector

$$\vec{t} = \begin{pmatrix} \tau_t \\ x_t/c \end{pmatrix} \tag{1}$$

Here c = speed of light, and [1]

$$\tau_t = \sqrt{t^2 - (x_t/c)^2} = t\sqrt{1 - (v/c)^2} \tag{2}$$

For a constant v the absolute value of the time vector (1) equals

$$|\vec{t}| = t.$$

[1] Note that τ_t equals the clock reading, which we in previous work have denoted *basic clock (BC)* time, t^{BC}. But t^{BC} represented the clock readings of auxiliary clocks, which we introduced in order to define the time vector, (1). In contrast, τ_t represents the 'internal time' of the moving object, which is in the focus of our experiment.

Eq. (2) relates the three main parameters of an event: t, x_t/c and τ_t. Note that all three parameters represent time, as x_t/c equals the time required for light to go the distance x_t (measured in light years)

It is well known that the time τ_t is invariant under the Lorentz transformation. Any event has different (t, x_t) values in the various moving RFs performing observations. The parameter, τ_t, however, will not depend on the chosen RF. We will therefore refer to τ_t as the *internal* time of the event, (t, x_t).

Observe the strong link to Minkowski's space-time, (Minkowski, 1909). Minkowski refers to our τ_t as 'proper time, and our t as 'coordinate time', (Petkov, 2012).

Recalling that $x_t = vt$, we further introduce $\varphi_v \in (-\pi/2,\ \pi/2)$ by

$$\sin\varphi_v = \frac{v}{c} = \frac{x_t/c}{t} \tag{3}$$

It follows that

$$x_t/c = \sin\varphi_v\, t \tag{4}$$

$$\tau_t = \cos\varphi_v\, t \tag{5}$$

Fig. 1 presents main features of our graphical approach. We draw a straight line at angle φ_v from the origin. The length of this line equals the time t on our RF, and we then read the objects position (divided by c) along the vertical axis, and its internal time, τ_t along the horizontal axis. So this graph provides simultaneous values of the three parameters, t, x_t/c and τ_t. We may draw the line using

$$\frac{x_t}{c} = \frac{v/c}{\sqrt{1-(v/c)^2}}\tau_t = \tan\varphi_v\, \tau_t \tag{6}$$

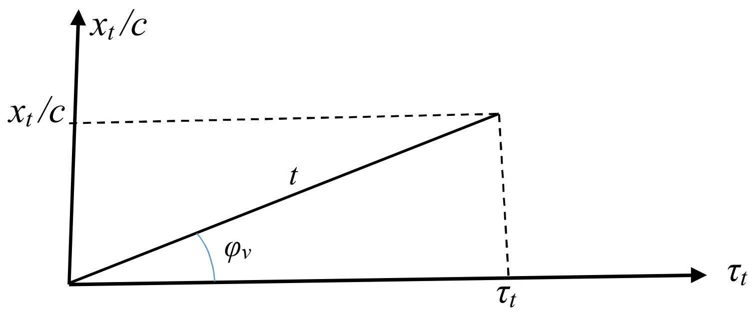

Figure 1. A chain of events, (t, x_t) on a specific RF; describing an object at a constant speed. The abscissa gives the reading of the object's 'embedded clock' (=internal time).

Similarly, we can illustrate the chain of events for an object having a *piecewise constant* v, see the example in Fig. 2. In this figure the object is after a certain time brought to rest (v=0), giving a line parallel with the abscissa, τ_t, *i.e.* a period where $\Delta x_t = 0$ and $\Delta t = \Delta\tau_t$. Next, the object again approaches the origin; thus, having a negative v. Thereafter the 'object', (or now rather 'event') has velocity, $v=c$, and the line is orthogonal to the coordinate, τ_t; (so here $\Delta\tau_t = 0$). *etc.*

The length of this 'time line' of events (t, x_t) in Fig. 2 still corresponds to time on the RF. We now first comment that the time, t no longer has to be equal to $|\vec{t}\,|$. In the general case, (nonconstant, v), the time, t will not be entirely given by $\vec{t}$, but also depend on the past history. But for a given $\vec{t}$, its absolute value, $|\vec{t}\,|$, represents the *minimum* attainable value of t, which corresponds to have the same velocity all the way. (This claim is well illustrated in Fig. 2.)

Secondly, we stress that in this approach, the time, t is decomposed into its two orthogonal components:

- τ_t: the reading of the 'embedded' clock; *i.e.* the aspect of time that is completely independent of which RF we apply for describing the movement of the object.
- x_t/c: the distance in space that the object has moved; measured as the time required for light to traverse the distance.

These two dimensions of time are illustrated in Figs. 1 and 2. When the object is at rest with respect to the relevant RF (*i.e.* $v = \varphi_v = 0$), x_t/c remains unchanged and $\Delta t = \Delta\tau_t$. When the 'object' has speed *c,* ($\varphi_v = \pi/2$), its 'internal clock' is unchanged and the increase in *t* equals the increase in x_t/c. And, in general, we can in any period of constant *v,* decompose *t* into its two components, x_t/c and τ_t.

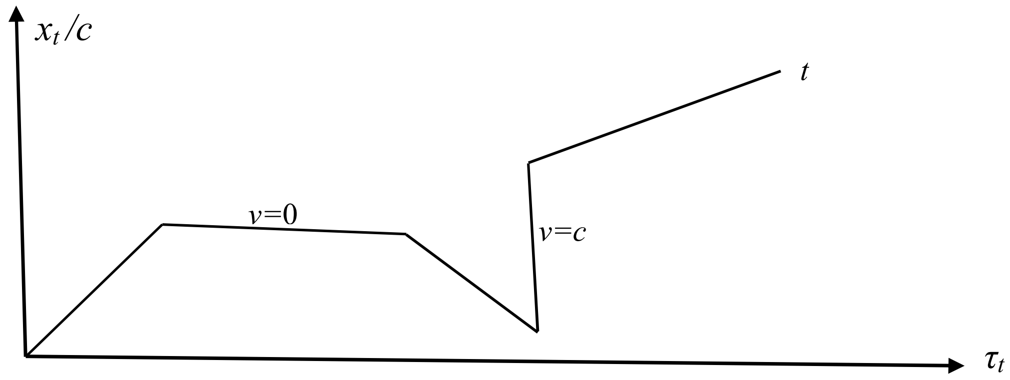

Figure 2. A graph illustrating an object's movement, when the velocity, *v* is piecewise constant. The length of the graph (the 'time line') equals the elapsed time, *t* on the RF. The two coordinate axes give the 'components' of the 'tme', *t*.

Thirdly: Often (t, x_t) are seen as the main parameters of the experiment; thus taking the perspective of the RF. However, as pointed out, the internal time, τ_t is independent of which RF observes the object/clock. So if it is the phenomenon as such that is of main interest, (independent of the RF making the observations), we might choose τ_t as the primary parameter. In a RF at velocity, -*v,* relative to the object, we will then observe parameters (t, x_t), where x_t is given by (6), and *t* is given by

$$t = \tau_t/\sqrt{1-(v/c)^2} = \tau_t/\cos\varphi_v \tag{7}$$

Thus, the event parameters (t, x_t) on the chosen RF are found from the internal time, τ_t and the velocity, *v* between the object and the observing RF; and so we see τ_t as the main time parameter. These considerations seem relevant *e.g.* when discussing the case of the μ-mesons, (*cf.* Hokstad, 2019).

3 Example: the travelling twin

The travelling twin paradox is frequently discussed, *e.g.* see Schuler and Robert (2014). As in Hokstad (2018a, 2018b, 2019) we apply the numerical example of Mermin (2005). The velocity of the travelling twin's rocket is $v = 0.6c$, giving $\sqrt{1-(v/c)^2} = 0.8$. The distance from the earth to the star equals 3 light years, and so by the arrival to the star the clocks of the RF to the earthbound twin, will read $t = 3c/0.6c = 5$ years. At this instant the clock of the travelling twin reads $\tau_t = t\sqrt{1-(v/c)^2} = 5\cdot 0.8 =$ 4 years. We first illustrate our approach from the perspective of the earthbound twin, *i.e.* choosing his RF for describing the event, (RF_1); next we consider the perspective of the travelling twin on his travel *from* the earth; this RF is denoted RF_2.

3.1 The perspective of the earthbound twin

Fig. 3 presents the graphs describing the travels of the two twins from the perspective of the earthbound twin (RF_1). The travelling twin's movement to the star is described by the blue line at an angle φ_v:

$$\sin\varphi_v = v/c = 0.6.$$

For the return travel, back to $x_t/c = 0$, we replace *v* by -*v*. Thus, the figure illustrates the well-known results that by the return it has elapsed a total time $t = 5 + 5 = 10$ years on RF_1, and the embedded clock of the travelling twin at this instant shows, $\tau_{10} = 8$ years. Further, the red line illustrates how the time and position develop for the earthbound twin on RF_1: here $x_t/c \equiv 0$ and $t \equiv \tau_t$.

We note that the final value of *t* (of course) is identical for the two chains of events (=10 years); the internal time, τ_{10}, however, differs.

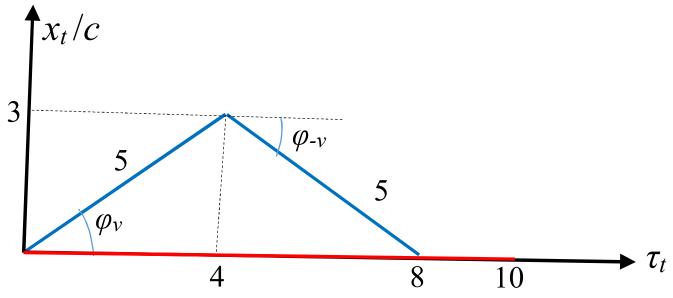

Figure 3. The graph of the travelling twin's journey, (blue line); seen from the perspective of the earthbound twin (RF_1). The maximum distance from earth equals 3 light years (x_t/c =3 years), and at the arrival here the clocks of RF_1 show t = 5 years, while the embedded clock of the travelling twin shows $\tau_5 = 4$ years. By the return t = 10 years and $\tau_{10} = 8$ years. - The red line follows the earthbound twin in RF_1, having $x_t/c \equiv 0$.

3.2 The perspective of the travelling twin

As pointed out *e.g.* in Hokstad (2019), we should - within the framework of the TSR - rather introduce three RFs to describe the travelling twin example: the RF of the earth, the RF of the travelling twin on his travel *from* the earth, and, finally, the RF of the return from the star and *back* to the earth. Further, we must calibrate the clocks on the last two RFs at the instant of the turning at the star. (So strictly speaking it is not the travelling twin that is returning, but rather a clock that is calibrated with his at the star). In this way we get an experiment compatible with the TSR.

Our graphical approach applies the perspective of the same RF during the entire chain of events. So in the following example we choose the perspective of the travelling twin's RF on his travel *away from the earth*. We denote this RF_2, and thus apply this for the entire journey.

In RF_2 (see Fig. 4) the earthbound twin is moving along the negative x_t/c-axis at an angle φ_{-v}, where

$$\sin\varphi_{-v} = -v/c = -0.6,$$

His movement is described by the red line. Now when his internal clock reads $\tau_t = 10$ years, it follows that the parameters of this event in RF_2 equals x_t/c = -7.5 years and t = 12.5 years. Thus, the red line gives a full description for the 'journey' of the earthbound twin within RF_2.

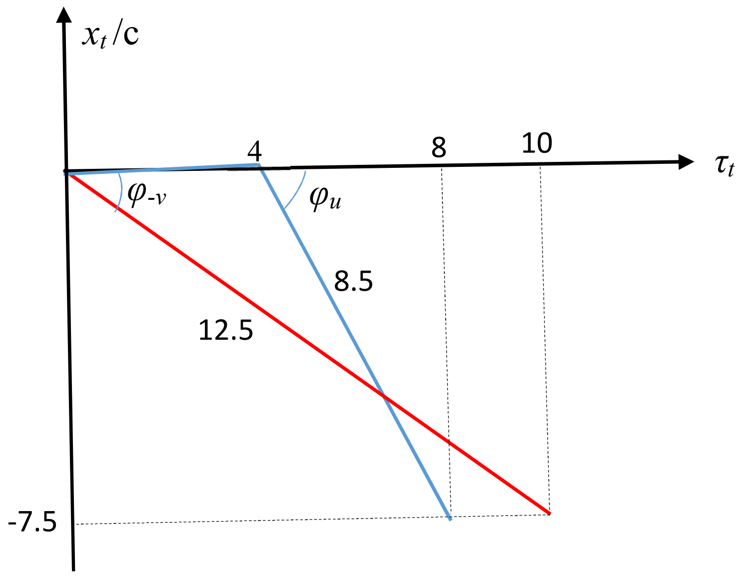

Figure 4. The graphs of the two event chains on RF_2, (the RF of the travelling twin on his travel *away* from earth). The red line describes the earthbound twin's movement in RF_2. The blue line describes the movement of the travelling twin in the same RF. At $\tau_t = 4$ years (and still x_t/c = 0) he starts his return travel.

Following the travelling twin, he will for the first four years remain at the origin of RF_2, (since $v = 0$), see blue line in Fig. 4. Next, for $\tau_t \geq 4$, the blue line illustrates the *returning* clock. This will have a speed v relative to RF_1. Further, the speed between RF_1 and RF_2 is also equal to v. Thus, applying the rule for 'adding' velocities in TSR, the velocity we observe in RF_2 of the returning clock becomes:

$$u = -\frac{2v}{1+(v/c)^2}$$

Inserting $v/c = 0.6$ here, we get $u/c = -15/17$. So now we have an angle, φ_u given by

$$\sin\varphi_u = u/c = -15/17,$$

Thus, also

$$\cos\varphi_u = \sqrt{1-(v/c)^2} = 8/17$$

$$\tan\varphi_u = -15/8$$

This determines the blue line in Fig. 4 for $\tau_t \geq 4$. Above we found that the chain of events on RF_2 ends at x_t/c = -7.5 years. This information explains the remaining part of the blue graph in Fig. 4. Now we see that when x_t/c = -7.5 years for this event chain, the time on RF_2 equals $t = 4 + 8.5 = 12.5$ years. Further, the internal clock of the returning RF reads $\tau_t = 4 + 4 = 8$ years. All this can of course also be formally verified, using the Lorentz transformation, and is in full agreement with standard results.

Finally note that in this travelling twin example we actually follow two chains of events, (the red and blue trajectory) which differ even if they start and end up at the same location. We now summarize main observations provided by Figs. 3 and 4:

- The red chain of events have the same total duration, t, and also end up at the same 'location', x_t/c for both perspectives, (RF_1 and RF_2). The same statement is valid for the blue chain of events.
- However, the values of both these parameters differ according to the perspective: For the total duration, t, we get 10 years and 12.5 years, respectively. For x_t/c we get 0 and -7.5 years, respectively. Thus, both the red and blue event chain are described differently by the two RFs.
- For the 'internal time', τ_t, however, we get the 'opposite' result. The red and blue chain of events have different τ_t - value in both the perspective of RF_1 (Fig. 3) and RF_2 (Fig. 4). But the blue graph gives the same τ_t (= 8 years) in *both* perspectives, and similarly the red graph gives the same τ_t(= 10 years) in *both* perspectives. This speaks in favor of considering τ_t as the primary time parameter.

4. Generalizations

We consider a couple of rather obvious generalizations of the results in Ch. 2.

4.1 Three space coordinates

The suggested approach assumes that the movement (at constant velocity, v) of the object we follow, occurs along the x_t-coordinate, and thus we introduced only this space coordinate. However, we may introduce three space coordinates (x_t, y_t, z_t) with corresponding velocity coordinates, (v_x, v_y, v_z). Then at time t on the RF we have $(x_t, y_t, z_t) = t\cdot(v_x, v_y, v_z)$. The spatial distance from the origin at time t now equals

$$l_t = \sqrt{x_t^2 + y_t^2 + z_t^2} = t\sqrt{v_x^2 + v_y^2 + v_z^2}$$

where the absolute value of the velocity vector, $\vec{v} = (v_x, v_y, v_z)'$, equals $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$. Now

$$\frac{l_t}{c} = \frac{v}{c}t$$

and we have an analogy to *eqs.* (3), (4) of Ch. 2. The trajectory of the event chain will be as in Ch. 2, just replacing x_t by l_t: the time line still being defined by φ_v with $\sin\varphi_v = v/c$. At any time, t on the RF, we may decompose l_t into its components (x_t, y_t, z_t), according to the direction of the velocity-vector.

4.2 General velocity

The approach so far is based on having a piecewise constant velocity, v. So we may at any instant change the velocity, *i.e.* the direction of the graph. Now let the periods of constant velocity become infinitesimal small. For simplicity we again consider just one space coordinate. Then we can generalize the results of Ch. 2 as follows. Rather than using v, we let

v_t = Velocity at time t.

For an interval (t, $t+dt$) we have $dx_t = v_t \cdot dt$, (that is, $v_t = dx_t / dt$). Thus

$$\frac{x_{t_0}}{c} = \int_0^{t_0} \frac{v_t}{c} dt \tag{8}$$

Similarly, we get

$$\tau_{t_0} = \int_0^{t_0} \sqrt{1-(v_t/c)^2}\ dt \tag{9}$$

Given $\{v_t\}_{t\geq 0}$, we can now for any $t = t_0$ in principle find the coordinates x_{t_0}/c and τ_{t_0} of the graph. Thus, *eqs.* (8), (9) replace the equations (4), (5), which are valid for a constant velocity, only. By using (8), (9) we easily prove the results when v_t is piecewise constant, which we illustrated in Fig. 2, (and later applied in Ch. 3).

5 Summary and Conclusions

We present a graphical approach to describe and treat a chain of events (of a moving object) in the perspective of a specific reference frame (RF). The approach is most easily applied when the velocity is piecewise constant. The graph ('time line') illustrates the three essential time parameters for describing events: the time, t of the RF, further, the position, x_t (divided by the speed of light, c), and, finally, the internal time, τ_t of the moving object, *i.e.* the reading of the 'embedded' clock. We note that the approach is somewhat related to work of Minkowski, *cf.* his 'proper time' and 'world line'.

The graph illustrates well how time, t, for a constant v, is decomposed into its two orthogonal components, τ_t and x_t/c. In general, however, time t is not entirely given by these components, but will also depend on the history.

Further, the approach illustrates the important distinction between, on one side, the two parameters, t and x_t, which depend on the chosen RF, and, on the other side, the internal time, τ_t, which has the same value, irrespective of the chosen RF. The travelling twin case provides a good illustration of this.

This specific feature of the internal time, τ_t suggests that in some cases it could be appropriate to consider this as the 'primary' time parameter. In that case, the t and x_t of the RF where we perform observations will be derived from the τ_t, together with the velocity between the moving object and the RF. We suggest this might affect the way we talk about/interpret time within the STR. The well-known observations of the µ-mesons could be a relevant case in such a discussion.

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