# Logical Composition up to Special Relativity Theory <br> Tsuneaki Takahashi 


#### Abstract

Science theory is composed of definitions, logic and facts. Facts are real and not deniable. Logic could have multiple expressions. So we need to choose most appropriate expression. A composition from definitions, facts and logic to Special Relativity Theory is tried here.


## 1. Introduction

Report [1] has developed already actual logic from definitions. This report is another version of it adding definitions, changing and sorting out logic expressions.
Here definitions of systems, words, marks and others are inherited from [1] as long as these are not redefined.

## 2. Logical steps

## Objective

Initially origin of $S^{\prime}$ system ( 0,0 ) is at origin of $S$ system ( 0,0 ) , and $S^{\prime}$ system is moving with velocity $v$ relatively to S system.

On this situation objective is,
To describe $S^{\prime}$ system frame of reference on $S$ system frame of reference.
To get formula of relation between $S$ system indication and $S^{\prime}$ system indication for a time-space point.

## Definitions

When time t is passed, time moves toward time direction also toward space direction with speed c.
Origin of moving time in space recognizes passed time distance at the moving time reached point by moved distance.

Premise
Frame of reference transformation can be done only when
About all related dimension, unit for value is same.

## Time axis of $S^{\prime}$ system

According to time $t$ passing, time moves ct along time axis of S system. While this timing, space zero point of $S^{\prime}$ system moves $x$ along space axis. Then its track could be

$$
x=\mathrm{act}
$$

Possible point it have is $(c t, v t)$. Then

$$
\begin{aligned}
& v t=\mathrm{act} \\
& a=\frac{v}{c}
\end{aligned}
$$

So its track is

$$
\begin{equation*}
x=\frac{v}{c} c t \tag{4}
\end{equation*}
$$

This is space zero line of $S^{\prime}$ system. Then this is time axis of $S^{\prime}$ system.
If a space point $x$ of $S$ system is space point $x^{\prime}$ of $S^{\prime}$ system, its relation is

$$
\begin{align*}
& x^{\prime}=\mathrm{x}-\frac{v}{c} c t \\
& x^{\prime}=\mathrm{x}-v t \tag{5}
\end{align*}
$$

(Fig. 1)


Fig. 1

## Space axis of $S^{\prime}$ system

When moving time reached at point P whose distance is $x$ from origin, moved time distance is $x$ in space for $S$ system origin. Then $S$ system origin recognizes passed time distance $x$ at point P based on (2),
While this timing, $S^{\prime}$ system origin moves $\frac{v}{c} x$ in space. Then moved distance of moving time for $S^{\prime}$ system is

$$
x-\frac{v}{c} x \quad \text { (Fig.2) }
$$



Fig. 2

Then $S^{\prime}$ system origin recognizes passed time distance $x-\frac{v}{c} x$ at point P based on (2), So $S^{\prime}$ system frame of reference is advanced $\frac{v}{c} x$ about time dimension than S system frame of reference
So $S^{\prime}$ system time zero is on the following line

$$
\begin{align*}
& \mathrm{ct}=\frac{v}{c} x \\
& x=\frac{c}{v} c t \tag{6}
\end{align*}
$$

This is time zero line and space axis of $S^{\prime}$ system.
Then if a time point $c t$ of $S$ system is time point $c t^{\prime}$ of $S^{\prime}$ system, its relation is

$$
\begin{equation*}
c t^{\prime}=c t-\frac{v}{c} x \tag{7}
\end{equation*}
$$

(Fig. 3)


Fig. 3

## Scaling

To satisfy a fact light speed constancy for every inertia systems, (5)(7) are used. As elements to be determined, $\alpha, \beta . \gamma$ are applied to (5)(7) [2]

$$
\begin{align*}
& x^{\prime}=\alpha(\mathrm{x}-v t)  \tag{8}\\
& t^{\prime}=\beta \mathrm{t}+\gamma x \tag{9}
\end{align*}
$$

Using these, equations of light speed constancy can be solved.
As result, we get following $\alpha, \beta . \gamma$ determined.

$$
\begin{align*}
& \alpha=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}  \tag{10}\\
& \beta=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}  \tag{11}\\
& \gamma=\frac{-v}{c^{2} \sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{12}
\end{align*}
$$

Comparing (7) (12), $\frac{-v}{c^{2}}$ is multiplier of $x$ included in (7).
Remained

$$
\begin{equation*}
\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{13}
\end{equation*}
$$

is same as $\alpha, \beta$. This is same value as scaling factor derived in [1] considering $S^{\prime}$ system is oblique system. Applying $\alpha, \beta, \gamma$, (8)(9) become Lorentz transformation formula.

## 3. Background of definitions

We can accept definitions unconditionally if these have no contradiction with any facts, But these definition should have meaning why these can be definition. Here we approach to such meanings.

## About definition (1)

Space is expanding and newly expanded space part has same time as existed space have. This means newly expanded space have no own time counted from zero.
On this fact, we can recognize time moved in space from neighbor space.

About definition (2)
When moving time reached to $x$ toward space direction, time passed $\frac{x}{c}$ or time distance $x$ at $x$. This is because why moving time velocity toward to time and toward to space is same $c$.

## 4. Conclusion

Based on (1)(2), basic conceptual formula(5)(7) can be given. Using the formula, solving equation of light speed constancy introduce scaling factor of oblique system.

Solved result including the scaling factor makes Lorentz transformation formula.
These are result on light speed constancy. Then there is no contradiction with
Michelson-Morley experiment which leaded to light speed constancy.

## Reference

[1] viXra:1611.0077
[2] Peter Gabriel Bergmann, Introduction to the Theory of Relativity, (Dover Publication, INC 1976), p19

