# The Connections for Forms

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**Abstract**

We generalize the notion of connections with help of exterior forms

## 1 The connections of Koszul

The connections of Koszul are defined as applications for à vector fiber bundle $E$:

$$\nabla : \Gamma(M,E) \to \Lambda^1 \otimes \Gamma(M,E)$$

with $\Gamma(M,E)$ the sections of $E$ and $\Lambda^1$ the 1-forms.

$$\begin{array}{rcl} \nabla_{fX}(s) & = & f\nabla_X(s) \\ \nabla_X(fs) & = & Xf.s + f\nabla_X(s) \end{array}$$

## 2 The connections for forms

The connections for forms are defined as applications:

$$\nabla : \Gamma(M,\Lambda^*(TM) \bigotimes E) \to \Gamma(M,\Lambda^*(T^*M) \bigotimes E)$$

$$\nabla_X(\alpha \wedge s) = d\alpha(X) \wedge s + (-1)^{deg(\alpha)}\alpha \wedge \nabla_X(s)$$

$d$ is the differential operator of the forms.

$$\nabla_{X\wedge Y}(s) = X^* \wedge \nabla_Y(s)$$

## 3 Properties

The space of connections for forms is an affine space with vector space, the linear maps of the bundle of forms with values in $E$.

## 4 Bibliography

J.Jost, "Riemannian Geometry and Geometric Analysis", Springer, Berlin, 2008.