The logical content of inequalities

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Abstract

Objective. From a theoretical point of view, the demarcation between science and (fantastical) pseudoscience is in necessary for both practical and theoretical reasons. One specific nature of pseudoscience in relation to science and other categories of human reasoning is the resistance to the facts.

Methods. Several methods are analyzed which may be of use to prevent that belief is masqueraded genuinely as scientific knowledge.

Results. Modus ponens, modus tollens and modus conversus are reanalyzed. Modus sine, logically equivalent to modus ponens is developed. Modus inversus and modus juris are described in detail.

Conclusions. In our striving for knowledge, there is still much more scientific work to be done on the demarcation between science and pseudoscience.

Keywords: non strict inequality, quantum mechanics, science, pseudoscience

1. Introduction

Acquiring long lasting and possibly generally valid scientific knowledge is concerned with problems which are closely related to central problems of science as such. At first blush, different scientific methods like inductive and deductive reasoning, systematic observation and experimentation and other methods of inquiry does not guarantee automatically the discovery and justification of new truths. Clear and sometimes formal standards or normative criteria for identifying advances and improvements in science with respect to mathematics are necessary too. In contrast to natural sciences, there is a widespread view that investigating fundamental questions concerning mathematics is to some extent problematic since the status of mathematical knowledge appears to be ultimate and therefore less open to revision than natural sciences. Narrowly speaking, even if the methods of investigation of natural sciences (more or less induction) may differ markedly from the methods of investigation in the mathematics (more or less deduction) there is usually a lot of overlap between them. both have at least on point in common, the (to many times possibly fruitless) hunt for the truth. The problem of course is, what is the truth and, in a way, easy to state, is there an absolute truth at all? The origins of the problems closely connected to the truth are traceable to ancient times and this simple statement masks a great deal of controversy. Is there at least one single knowledge, statement or axiom et cetera, which can or which must be accepted as being true by all scientist, since the axiomatic method is one of the crucial tools for mathematics? In an attempt to find a logically consistent answer to problems like this it is necessary to consider a number of distinct ways of answering questions about the nature of truth and the preservation of truth?

Strategically, proceeding axiomatically has as one of many advantages to develop a theorem or a theory in a rigorous way from some fundamental principles. This fundamental insight underpins the possibility of an axiomatic system (1) to serve, for later investigations, as a tool for discovery of errors inside a theory. As a matter of fact, Hilbert's demand to find a complete and consistent set of axioms for all mathematics was counteracted to some extent by Gödel's incompleteness theorems (2). But one way to handle these difficulties is to reject the possibility for the premises to be true but conclusion false.

2. Material and Methods

Inequalities are widely used in many branches of physics and mathematics. In general, an inequality is a relation that holds between two values which are not equal, which are different. In mathematics, analytic number theory often operates with inequalities. Usually, an inequality is denoted by the symbols < or > or by the symbols < or >..

2.1 Definitions

To date, mathematics is more or less a product of human thought and mere human imagination and belongs more to the world of human thought and mere human imagination then to experimentally determined sciences. In the following, it is of principal use to ground mathematics on nature grounded entities to disable the possibility to regard mathematics as a religion whose language are numbers, definitions et cetera than as science.

Definition 2.1.1. (Number +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number (3). The number +1 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1+0 \equiv \quad -i^2 \quad = \quad +1 \tag{1}$$

while "=" denotes the equals sign (4) or equality sign (5,6) used to indicate equality and "-" (7) (8) (5) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus (4) signs used to represent the operations of addition and the notions of positive as well.

Definition 2.1.2. (Number +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary. Let the arithmetic operation subtraction be signified by the minus sign (−). The number +0 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1-1 \equiv \quad -i^2 + i^2 \quad \equiv \quad +0 \tag{2}$$

Definition 2.1.3. (The sample space)

The sample space of an experiment or random variable or random trial is the set of all possible outcomes or results of that experiment or random variable or trial. Let A_t denote the sample space of a (Bernoulli distributed) random variable A which can take the values either +1 or +0 at a Bernoulli trial (period of time) t. Let B_t denote the sample space of another (Bernoulli distributed) random variable B which can take the values either +1 or +0 at the same Bernoulli trial (period of time) t. Let P_t denote the sample space of **the premise**, a (Bernoulli distributed) random variable P which can take the values either +1 or +0 at a Bernoulli trial (period of time) t. Let C_t denote the sample space of **the conclusion**, another (Bernoulli distributed) random variable C which can take the values either +1 or +0 at the same Bernoulli trial (period of time) t. In general, we define the sample space A at a Bernoulli trial/period of time t as

$$A_t \quad \equiv \quad \{+0_t \quad , \quad +1_t\} \tag{3}$$

We define the sample space of B at a Bernoulli trial/period of time t as

$$B_t \equiv \{+0_t \quad , \quad +1_t\} \tag{4}$$

We define the sample space of a **premise P** at a Bernoulli trial/period of time t as

$$P_t \equiv \{+0_t \quad , \quad +1_t\} \tag{5}$$

We define the sample space of a **conclusion C** at a Bernoulli trial/period of time t as

$$C_t \equiv \{+0_t \quad , \quad +1_t\} \tag{6}$$

Definition 2.1.4. (Strict inequalities)

A strict inequality (9) is an inequality where the inequality symbol is either < (strictly less than) or > (strictly greater than). Consequently, a strict inequality has no equality conditions. In terms of algebra, we obtain

$$A_t < B_t \tag{7}$$

The notation A_t is < B_t means that "A_t is strictly less than B_t". The following table (**Table 1**) may illustrate this relationship under the conditions above.

Table 1. The strict inequality $A_t < B_t$.

$A_t < B_t$		Conditioned B_t		
		$B_t = +1$	$B_t = +0$	Total
Condition A_t	$A_t = +1$	0	0	
	$A_t = +0$	**1**	0	
	Total			1

The strict inequality $A_t < B_t$ describes the complementary part of the conditio sine qua non relationship *without* A_t *no* B_t. In other words, it is $p(A_t < B_t) + p(A_t \geq B_t) = 1$. Equally there may exist conditions where

$$A_t > B_t \tag{8}$$

The notation $A_t > B_t$ means that "A_t is strictly greater than B_t". The following table (**Table 2**) may illustrate this relationship under the conditions above.

Table 2. The strict inequality $A_t > B_t$.

$A_t > B_t$		Conditioned B_t		
		$B_t = +1$	$B_t = +0$	Total
Condition A_t	$A_t = +1$	0	**1**	
	$A_t = +0$	0	0	
	Total			1

As can be seen, the strict inequality $A_t > B_t$ describes the complementary part of the conditio per qaum relationship *if* A_t *then* B_t. In other words, it is $p(A_t > B_t) + p(A_t \leq B_t) = 1$.

Definition 2.1.5. (Non strict inequalities)

In contrast to strict inequalities, a non-strict inequality is an inequality where the inequality symbol is $\geq$ (either greater than or equal to) or $\leq$ (either less than or equal to). Consequently, a non-strict inequality is an inequality which has an equality condition too. In terms of algebra, we obtain

$$A_t \leq B_t \tag{9}$$

The notation $a \leq b$ means that "**either** A_t is less than B_t **or** A_t is equal to B_t". The following table (**Table 3**) may illustrate this relationship under the conditions above.

Table 3. The non-strict inequality $A_t \leq B_t$.

$A_t \leq B_t$		Conditioned B_t		
		$B_t = +1$	$B_t = +0$	Total
Condition A_t	$A_t = +1$	1	**0**	
	$A_t = +0$	1	1	
	Total			1

As can be seen, the non-strict inequality $A_t \leq B_t$ describes the **conditio per quam** relationship *if* A_t *then* B_t. In other words, it is $p(A_t \leq B_t) = 1 - p(A_t > B_t)$. The notation

$$A_t \geq B_t \tag{10}$$

means that "**either** A_t is greater than B_t **or** A_t is equal to B_t". The logical content of the non-strict inequality $A_t \geq B_t$ is clearly demonstrated by the following table (**Table 4**).

Table 4. The non-strict inequality $A_t \geq B_t$.

$A_t \geq B_t$		Conditioned B_t		
		$B_t = +1$	$B_t = +0$	Total
Condition A_t	$A_t = +1$	1	1	
	$A_t = +0$	0	1	
	Total			1

As can be seen, the non-strict inequality $A_t \geq B_t$ describes the conditio sine qua non relationship *without* A_t *no* B_t. In other words, it is $p(A_t \geq B_t) = 1 - p(A_t < B_t)$.

Definition 2.1.6. (Russell's paradox)

Considering Cantor's power class theorem, Russell was the first to discuss a contradiction arising in the logic of sets or classes at length in his published works (10,11). Some sets are members of themselves, while other are not. The empty class or set must not be a member of itself. Thus far, according to Russell's paradox (10,11), let R be the set of all sets (with certain properties). *Either* R is a member of itself *or* R is not a member of itself. Furthermore, either R contain itself or R does not contain itself. The following table may provide a preliminary overview (Table 5).

Table 5. Russell's paradox.

R is the set of all sets.		R contains itself		
		Yes	No	Total
R is member of itself	Yes			U
	No	c	d	$\underline{U}$
	Total	W	$\underline{W}$	1

Scholium.

Russell's paradox (also known as Russell's antinomy), discovered by Bertrand Russell in 1901, demands that if R is not a member of itself (case $\underline{U}$), then R's definition dictates that it must contain itself (case c), and if R contains itself (case W), then R contradicts its own definition as the set of all sets that are not members of themselves.

Russell's conclusion is not justified and incorrect.

Reasons.

First of all, Russell is mismatching being member of itself and containing itself, both notions are due to Russell understanding not identical. In particular, if containing itself and being member of itself are two different and not identical or equivalent notions, then it is possible for the set of all set R to contain itself while *being a member* of itself or *not being a member* of itself. In the same respect, if containing itself and being member of itself are two different notions then it is possible for the set of all sets not being member of itself (case $\underline{U}$) while containing itself (case c) or not containing itself (case d). But even in the case if R just contains itself, Russell's conclusion is incorrect. In this case we obtain the following situation (Table 6).

Table 6. Russell's paradox.

R is the set of all sets.		R contains itself		
		Yes		Total
R is member of itself	Yes			U
	No	c		$\underline{U}$
	Total	W		1

Even if R contains itself, according to Russel, R must not be a member of itself otherwise being member of itself and containing itself would mean the same.

Definition 2.1.7. (The ten commandments of a fair scientific engagement)

I) The axiom principium identitatis (i. e. +1=+1) is the only principle you must respect. You shall not respect any other axioms before principium identitatis.

II) You shall not tolerate any individual unscientific behavior or any other errors in science.

III) You shall not misuse principium identitatis.

IV) You shall make sure that at least every seventh of your publications must start with or must be devoted to principium identitatis.

V) You shall honor your former scientific predecessors since the beginning of time and your present scientific competitor too. Without those, you would not be there, where you are today.

VI) You shall not put into question the reputation or the integrity of your scientific colleagues.

VII) You shall not work on two different scientific projects, articles et cetera at the same time.

VIII) You shall not forget to give credit or reference to another scientist.

IX) You shall not bear false witness against your former scientific predecessors, your scientific competitor or yourself.

X) You shall respect the work of the colleagues you are working night or day together.

2.1. Methods

Detecting inconsistencies and inadequacies in scientific theories and resolving contradictions is of particular importance within science itself. Experiments or experience can help us many times to decide upon the truth or falsity of natural laws but do not provide any help to trace these inconsistencies and inadequacies back to the fundamental axioms from which they spring. Unfortunately, even peer-reviewed published or proposed theorems or statements of science and mathematics are not automatically correct. Whenever we find that a system has been questioned somehow, we shall test the same again and reject it if possible, as circumstances may require. It is necessary to prove these theorems while using rigorous proof methods of science and mathematics which are acceptable beyond any shadow of doubt. In order to formulate methodological rules which, prevent us to adopt inconsistencies and inadequacies in scientific theories it necessary to consider that the results of (thought) experiments are either to be rejected, or to be accepted.

2.2.1. Direct Proof

A direct (mathematical) proof demonstrates the truth or falsehood of a given equation, statement by a straightforward combination of established facts.

2.2.2. Proof by modus ponens

Explicitly, the modus ponens statement is $P_t \rightarrow C_t$ or "**If *P_t is true*, then C_t *is true***" (Table 7). In other words, modus ponens demands that $(P_t \rightarrow C_t) = +1$ and that $P_t = +1$ is true.

Table 7. **Modus ponens**: *if* the premisse P_t is true *then* the conclusion C_t is true ($P_t \rightarrow C_t$).

$P_t \rightarrow C_t$		C_t		
		$C_t = +1$	$C_t = +0$	Total
P_t	$P_t = +1$	1	0	
	$P_t = +0$	1	1	
	Total			1

The modus ponens demands that $P_t = +1$ (Table 7). In this case, the conclusion is that $C_t = +1$ too.

Table 8. **Modus ponens**: *if* the premisse P_t is true *then* the conclusion C_t is true ($P_t \rightarrow C_t$). $P_t = +1$. Ergo: $C_t = +1$.

$P_t \rightarrow C_t$		C_t		
		$C_t = +1$	$C_t = +0$	Total
P_t	**$P_t = +1$**	**1**	**0**	
	$P_t = +0$	1	1	
	Total			1

The proof by modus ponens in classical two-valued logic can be clearly demonstrated by use of the following table 9.

Table 9.

Proof by modus ponens.		
Claim.		
	(Premise 1)	$P_t \rightarrow C_t$
Proof.		
	(Premise 2)	P_t
Decision.		
	(Conclusion)	C_t
Quod erat demonstrandum.		

Scholium.

Modus ponens is grounded on the preservation of truth but the same has been criticized (12) too. Many times, modus ponens is used for time depended processes too where an antecedent is prior in time to a consequent. An inappropriate use of modus pones under these conditions can lead to contradictions. It is necessary to apply modus ponens especially on events which occur together, at the same (period of time) t. The following example may formalize modus ponens in more detail. For the sake of simplicity, we define $P_1 = (+1=+1)$, we define $P_2 = (+2=+2)$ and we define $C_1 = (+3=+3)$. The proof by modus ponens is as follows.

Proof by modus ponens I.

Claim.

Premise 1: $P_1 \rightarrow C_2$: *if* the premisse $P_1 = (+1=+1)$ is true *then* the conclusion $C_1 = (+3=+3)$ is true.

Proof.

Premise 2: P_1 : +1 = +1 is true.

Adding +2 we obtain +2 +1 = +2 + 1 or +3=+3.

Decision. (Conclusion)

$C_1 = (+3=+3)$ is true.

Quod erat demonstrandum.

Proof by modus ponens II.

Claim.

Premise 1: $P_2 \rightarrow C_1$: *if* the premisse P_2 = (+2=+2) is true *then* the conclusion C_1 = (+3=+3) is true.

Proof.

Premise 2: P_2 : +2 = +2 is true.

Adding +1 we obtain +2 +1 = +2 + 1 or +3=+3.

Decision. (Conclusion)

C_1 = (+3=+3) is true.

Quod erat demonstrandum.

Modus ponens allows that one and the same conclusion C_1 = (+3=+3) is true and can be deduced from different points of view, from different premisses. In the case of the premise P_1 it is true that +1=+1. The premise P_2 is because of premise P_1 not incorrect, because P_2 : +2 = +2 is also true, the premise P_2 is just not used for the proof performed. However, as can be seen in the second proof, the premise P_2 can used for the proof too without any restriction. Thus far, modus ponens cannot be misused for claims that from something incorrect or non-existent (P_1 = +0) something correct (C_1 = +0) follows. This would provide evidence of *creatio ex nihilio*.

2.2.3. Proof by modus securus - anti modus ponens

In point of fact, modus ponens demands that "**if P_t is true then C_t is true**" or $P_t \rightarrow C_t$. Thus far, if the negation of modus ponens is true i. e. $\neg(P_t \rightarrow C_t)$ = true, then the original modus ponens proposition (and by extension the contrapositive) is false. The following table (Table 10) shows the case, when modus ponens is false.

Table 10. **Modus securus - Negation of modus ponens (Anti modus ponens)**:

Premisse P_t is true and conclusion C_t is false $\neg(P_t \rightarrow C_t)$.

$\neg(P_t \rightarrow C_t)$		C_t		
		$C_t = +1$	$C_t = +0$	Total
P_t	$P_t = +1$	0	**1**	
	$P_t = +0$	0	0	
	Total			1

Either modus ponens ($P_t \rightarrow C_t$) is true *or* modus securus ($\neg(P_t \rightarrow C_t)$) is true but not both.

Table 11.

Proof by modus securus.

Claim.		
	(Premise 1)	$\neg(P_t \rightarrow C_t)$
Proof.		
	(Premise 2)	P_t
Decision.		
	(Conclusion)	$\neg C_t$

Quod erat demonstrandum.

2.2.4. Proof by modus sine

Modus ponens and modus sine are more than only closely related. Modus ponens and modus sine are logically equivalent: if a statement according to modus ponens is true, then the same statement according to modus sine (Table 12) is true, and vice versa.

Table 12. **Modus sine**: *Without* premisse P_t is false *no* conclusion C_t is false ($\neg P_t \leftarrow \neg C_t$). $P_t = +0$. Ergo: $C_t = +0$.

$\neg P_t \leftarrow \neg C_t$		C_t		
		$C_t = +0$	$C_t = +1$	Total
P_t	$P_t = +0$	1	1	
	$P_t = +1$	0	1	
	Total			1

Modus sine doesn't allow us to draw a false conclusion form a true premise. The proof by modus sine in classical two-valued logic can be demonstrated by use of the following table

Table 13.

Proof by modus sine.

Claim.		
	(Premise 1)	$\neg P_t \leftarrow \neg C_t$
Proof.		
	(Premise 2)	$\neg P_t$
Decision.		
	(Conclusion)	$\neg C_t$

Quod erat demonstrandum.

Table 14.

Trial t	P_t	C_t	$\neg P_t$	$\neg C_t$	Implication $(P_t \rightarrow C_t)$	Modus sine $(\neg P_t \leftarrow \neg C_t)$	Contrapositive $(\neg C_t \rightarrow \neg P_t)$
1	1	1	0	0	**1**	**1**	**1**
2	1	0	0	1	0	0	0
3	0	1	1	0	1	1	1
4	0	0	1	1	**1**	**1**	**1**

In general, we obtain *the logical equivalence* of

$$(P_t \rightarrow C_t) \quad = \quad (\overline{P_t} \cup C_t) \quad = \quad (\neg P_t \leftarrow \neg C_t) \tag{11}$$

2.2.5. Proof by modus tollens

Modus tollens and modus ponens are closely related. Theophrastus was the first to explicitly describe the argument form modus tollens (13). A proof by modus tollens is determined by the secured relationship $P_t \rightarrow C_t$. The logical consequence is that the negation of C_t implies the negation of P_t is valid. Following Popper, "… it is possible by means of purely deductive inferences (with the help of the modus tollens of classical logic) to argue from the truth of singular statements to the falsity of universal statements." ((14), p. 19). In other words, "By means of this mode of inference we falsify the whole system (the theory as well as the initial conditions) which was required for the deduction of the statement p, i.e. of the falsified statement." ((14), p. 56). In particular and in contrast to a proof by contrapositive, the modus tollens statement demands that $P_t \rightarrow C_t$ or that the premise "**If *P_t* *is true*, then C_t *is true***" (Table 15) is given.

Table 15.

Proof by modus tollens.

Claim.

	(Premise 1)	$P_t \rightarrow C_t$
Proof.		
	(Premise 2)	$\neg C_t$
Decision.		
	(Conclusion)	$\neg P_t$

Quod erat demonstrandum.

In other words, modus tollens demands that $(P_t \rightarrow C_t) = +1$ and that $\neg C_t = +1$ is true. In this case, the conclusion is justified that $\neg P_t = +1$ and is clearly demonstrated by use of the following table 16.

Table 16. **Modus tollens**: *if* the premisse P_t is true *then* the conclusion C_t is true (P_t -> C_t). C_t =+0. Ergo: P_t =+0.

$P_t \rightarrow C_t$		C_t		
		$C_t = +1$	$C_t = +0$	Total
P_t	$P_t = +1$	1	**0**	
	$P_t = +0$	1	**1**	
	Total			1

The modus tollens rule may be written as a theorem of propositional logic as

$$\left(\left(P_t \rightarrow C_t \right) \cap \neg C_t \right) \rightarrow \left(\neg P_t \right) \tag{12}$$

Table 17.

Trial t	P_t	$\neg P_t$	C_t	$\neg C_t$	$P_t \rightarrow C_t$	$((P_t \rightarrow C_t) \cap \neg C_t)$	$\neg P_t$	$((P_t \rightarrow C_t) \cap \neg C_t) \rightarrow (\neg P_t)$
1	1	0	1	0	1	0	0	1
2	1	0	0	1	0	0	0	1
3	0	1	1	0	1	0	1	1
4	0	1	0	1	1	1	1	1
…	…	…	…	…	…	…	…	…

2.2.6. Proof by contraposition

A proof by contraposition is based on the fact that the statement "*if* the premisse P_t is true *then* the conclusion C_t is true" is **logically equivalent** to the statement "*if* the premisse P_t is not true *then* the conclusion C_t is not true". (Table 18). The proof by contraposition is not identical with the proof by modus tollens. In other words, in a proof by contraposition we show that C_t is false and then conclude that P_t is false too.

Table 18. **Proof by contraposition**: *if* the conclusion C_t is not true *then* the premisse P_t is not true ($\neg C_t \rightarrow \neg P_t$).

$\neg C_t \rightarrow \neg P_t$		P_t		
		$P_t = +0$	$P_t = +1$	Total
C_t	$C_t = +0$	1	**0**	
	$C_t = +1$	1	1	
	Total			1

Modus ponens and the proof by contrapositive are *logically equivalent* and are determined by the minimum demand that

$$(P_t \cap C_t) = (\neg P_t \cap \neg C_t) = +1 = TRUE \tag{13}$$

The case $P_t = +1$ and $C_t = +0$ is neither compatible with the equation

$$(P_t \cap C_t) = (1 \cap 0) = +0 \tag{14}$$

nor with the equation

$$(\neg P_t \cap \neg C_t) = (0 \cap 1) = +0 \tag{15}$$

In general, logical equivalence doesn't care whether a glass is half-full or half-empty. This is a matter of personal taste. A statement and its contrapositive are logically equivalent or it is

$$(P_t) \rightarrow (C_t) \equiv (\neg C_t) \rightarrow (\neg P_t) = +1 = TRUE \tag{16}$$

The contrapositive of a certain statement has the same truth value (truth or falsity) as the statement itself. If a contrapositive is true, then its statement is true (and vice versa). If a contrapositive is false, then its statement is false (and vice versa).

Table 19.

Proof by contraposition.		
Claim.		
	(Premise 1)	$\neg C_t \rightarrow \neg P_t$
Proof.		
	(Premise 2)	$\neg C_t$
Decision.		
	(Conclusion)	$\neg P_t$
Quod erat demonstrandum.		

2.2.7. Modus inversus

The proof by inversion (modus inversus) is a valid rule of inference or a proof method "by which from a given proposition another is derived having for its *subject* the contradictory of the original subject and for its *predicate* the contradictory of the original predicate." (Toohey 1948, p. 51). In general, the inverse of the statement $P_t \rightarrow C_t$ ("**If *P_t* *is true*, then *C_t* *is true***") is known to be the statement or the equation $\neg P_t \rightarrow \neg C_t$ or in spoken language: "**If *P_t* *is false*, then *C_t* *is false***" while the logical equivalent is viewed by the table (Table 20). In this context, it is worth to point out, that the basic relationship $(P_t \rightarrow C_t) = (\neg\neg P_t \rightarrow \neg\neg C_t)$ is valid. In other words, a direct proof provided without any technical errors which is grounded on something false must end up in something false.

Table 20. **Modus inversus**: *if* the premisse P_t is false *then* the conclusion C_t is flase $(\neg P_t \rightarrow \neg C_t)$. P_t =+0. Ergo: C_t =+0.

$\neg P_t \rightarrow \neg C_t$		C_t		
		$C_t = +0$	$C_t = +1$	Total
P_t	$P_t = +0$	1	0	
	$P_t = +1$	1	1	
	Total			1

The proof by modus inversus is viewed by the following table (Table 21). The logical equivalent of modus inversus is $(P_t \leftarrow C_t)$.

Table 21.

Proof by modus inversus.		
Claim.		
	(Premise 1)	$\neg P_t \rightarrow \neg C_t$
Proof.		
	(Premise 2)	$\neg P_t$
Decision.		
	(Conclusion)	$\neg C_t$
Quod erat demonstrandum.		

2.2.8. Proof by modus juris (anti modus inversus)

In point of fact, modus inversus demands that "*if* the premisse P_t is false *then* the conclusion C_t is flase ($\neg P_t \rightarrow \neg C_t$). P_t =+0. Ergo: C_t =+0. Thus far, if the negation of modus inversus is true i. e. $\neg(\neg P_t \rightarrow \neg C_t)$ = true, then the original modus inversus proposition (and by extension the contrapositive) is false. The following table (Table 22) shows the case, when modus inversus is false.

Table 22. **Modus juris (Negation of modus inversus)**: Premisse P_t is false and conclusion C_t is true $\neg(\neg P_t \rightarrow \neg C_t)$.

$\neg(\neg P_t \rightarrow \neg C_t)$		C_t		
		$C_t = +0$	$C_t = +1$	Total
P_t	$P_t = +0$	0	1	
	$P_t = +1$	0	0	
	Total			1

Either modus inversus ($\neg P_t \rightarrow \neg C_t$) is true *or* modus juris ($\neg(\neg P_t \rightarrow \neg C_t)$) is true but not both.

Table 23.

Proof by modus juris.

Claim.

(Premise 1) $\neg(\neg P_t \rightarrow \neg C_t)$

Proof.

(Premise 2) P_t

Decision.

(Conclusion) $\neg C_t$

Quod erat demonstrandum.

2.2.9. Modus conversus

The inverse of $P_t \rightarrow C_t$ is $\neg P_t \rightarrow \neg C_t$ and logically equivalent to the contrapositive ($\neg C_t \rightarrow \neg P_t$) of the converse ($C_t \rightarrow P_t$). The contrapositive of the statement "*if* the premisse P_t is false *then* the conclusion C_t is false" is logically equivalent to the statement "*without* premisse P_t is true *no* conclusion C_t is true". In logic, the converse of an implicational statement is the result of reversing its two parts of $P_t \rightarrow C_t$ to $C_t \rightarrow P_t$ (Table 24).

Table 24. **Modus conversus**

$C_t \rightarrow P_t$		P_t		
		$P_t = +1$	$P_t = +0$	Total
C_t	$C_t = +1$	1	0	
	$C_t = +0$	1	1	
	Total			1

The dominance of modus ponens over the other modi is not justified. One disadvantage of modus ponens is that conclusions with false antecedents are considered true. In opposite to modus ponens, modus conversus does not allow to conclude a true conclusion ($C_t = +1$) from a false premise ($P_t = +0$). Table 25 provides us with an overview.

Table 25.

			Implication	Converse			Contrapositive	Inverse
Trial t	P_t	C_t	$(P_t \rightarrow C_t)$	$(C_t \rightarrow P_t)$	$\neg P_t$	$\neg C_t$	$(\neg C_t \rightarrow \neg P_t)$	$(\neg P_t \rightarrow \neg C_t)$
1	1	1	**1**	**1**	0	0	**1**	**1**
2	1	0	0	1	0	1	0	1
3	0	1	1	0	1	0	1	0
4	0	0	**1**	**1**	1	1	**1**	**1**

Note that the converse of $P_t \rightarrow C_t$ is $C_t \rightarrow P_t$. The contrapositive of $P_t \rightarrow C_t$ is $\neg C_t \rightarrow \neg P_t$ and has the same truth values as $P_t \rightarrow C_t$ or it is $P_t \rightarrow C_t = \neg C_t \rightarrow \neg P_t$. The inverse of $P_t \rightarrow C_t$ is $\neg P_t \rightarrow \neg C_t$. The converse of $P_t \rightarrow C_t$ and the inverse of $P_t \rightarrow C_t$ have the same truth values or it is $C_t \rightarrow P_t = \neg P_t \rightarrow \neg C_t$. In general, the inverse of premise $P_t \rightarrow C_t$ is the same as the contrapositive of the converse. **Besides of all differences, the implication, the converse, the contrapositive and the inverse agree all completely at the trial t=1 and the trial t=4**.

2.2.10. Proof by contradiction (Reductio ad absurdum)

In point of fact, it is difficult for scientists prove a theorem, a theory et cetera to be true for ever. Regardless of how many positive examples appear to support a theorem or a theory, one single counter-example or one single contradictory instance to a theory is sufficient enough to falsify the general validity of a theorem or of a theory et cetera. A proof by contradiction (15,16) is such a scientific proof method which is able to proof the general the falsity or the truth of a statement, an equality, a principle (P) et cetera. Reduction to the impossible is a style of reasoning found repeatedly in Aristotle's Prior Analytics (17). Throughout the history of philosophy and mathematics from classical antiquity onwards there have been circumstances where a thesis had to be accepted because its rejection would be untenable. “The proof ... *reductio ad absurdum*, which Euclid loved so much, is one of the mathematician’s finest weapons” ((18), p. 94). A contradiction in a formal axiomatic system can prove any theorem true. In other words, according to the to *the principle of explosion* from a contradiction, anything follows (*ex contradictione sequitur quodlibet*). Even if there are trials advocated especially by the Peruvian philosopher Francisco Miró Quesada to establish something like a system of *paraconsistent logic* (19) which attempts to *deal with contradictions* (20), the progress has been very slow. In short, a proof by contradiction demands to assume that P_t is false. In the following assume that $\neg P_t$ is true and derive a contradiction. Since P_t cannot be both true and false, P_t is false.

Table 26.

Proof by contradiction.		
Claim.		
	(Premise 1)	P_t is false.
Proof.		
	(Premise 2)	$\neg P_t$ is true.
Decision.		
	(Conclusion)	Derive a contradiction from $\neg P_t$ is true.
Quod erat demonstrandum.		

Something impossible or incorrect cannot be derived from something correct as long as there are nor technical or other errors inside a proof.

2.2.11. Proof by counterexample

Can we learn anything from scientific theories or from experiments? Theoretically, one single experiment (21) has the potential to refute a whole theory. Historically, no theory has been refuted by one single experiment. In philosophy, mathematics, physics or in science as such, it is not all the time possible to prove all scientific claims in time beyond any doubt. The proof by a counterexample (22–25) is a valid proof methodology to infer consequences of scientific claims or theories and to demonstrate clearly that a certain scientific position is wrong by showing that it does not apply in certain cases. A counterexample which is able to derive a logical contradiction in the absences of technical and other errors out of a theorem or a theory refutes the same.

2.3. Axioms

2.3.1. Principium identitatis

$$+1 \qquad = \qquad +1 \tag{17}$$

2.3.2 Principium contradictionis

Contradictions are an objective and important feature (Barukčić, 2019c) of objective reality. Still, contradictions in theorems, arguments and theories would allow us to conclude everything desired. In contrast to religion and other domains of human culture, one very important and at the end to some extent normative criteria to achieve some advances and progress in science is depended on detecting contradictions in science and eliminating the same too. The most important point is that even if we are surrounded by contradictions a co-moving observer (Barukčić, 2019c) will always find that something is *either* +1=+1 *or* +0=+0 but not both, i. e. it is not +1 = +0. The simplest form of Aristotle's law of contradiction (Barukčić, 2019a; Barukčić, 2019b; Barukčić, 2019c;) is defined as

$$+0 \qquad = \qquad +1 \tag{18}$$

However, according to Popper, a philosopher of science of the 20th century, contradiction is the demarcation line between science and 'non-science'. "We see from this that if a theory contains a contradiction, then it entails everything, and therefore, indeed, nothing[...]. A theory which involves a contradiction is therefore entirely useless as a theory". ((14), p. 429). In particular, to face the threat of a logical or scientific *Armageddon* and the breakdown of any logical coherence posed by accepting the contradiction +0 = +1, it is necessary to formulate the same clearly. Far from reduced to the silence of deep space given due to the explosive effect of *ex contradictione quodlibet*, there are circumstances of special theory of relativity where it is possible to allow a kind of inconsistency without logical incoherence (20). A proof can be based on *principium contradictionis*, the premise like +0 = +1 can justify further conclusions. A sound argument would follow if the conclusions were logically derived from the premises (26) without any technical errors. The result would have to be a mistake, since we started with a mistake. In contrast to a sound argument a valid argument is a sound argument while all the premises are true. If anything after the false premise is true and logically consistent then in the absence of any technical errors the conclusion itself must be false too.

3. Results

Theorem 3.1. ($A_t < B_t$ and disjunction)

The strict inequality $A_t < B_t$ is not identical with an inclusive disjunction, also known as alternation.

Proof.

The truth table of logical disjunction is ($A_t \cup B_t$) is defined as follows (Table 27).

Table 27. **Logical disjunction**

$A_t \cup B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ yes	$b_t =$ yes	
	$A_t = +0$	$c_t =$ yes	$d_t =$ no	
	Total			1

The strict inequality $A_t < B_t$ is defined as follows (Table 28).

Table 28. **Strict inequality** $A_t < B_t$

$A_t < B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ no	$b_t =$ no	
	$A_t = +0$	$c_t =$ yes	$d_t =$ no	
	Total			1

Logical disjunction and the strict inequality agree on in case $c_t = ((A_t = +0) \cap (B_t = +1))$ but not in general. The strict inequality $A_t < B_t$ is not identical with of inclusive disjunction.

Quod erat demonstrandum.

Remark 1.

This proof is of importance for quantum theory too.

Theorem 3.2. ($A_t > B_t$ and disjunction)

The strict inequality $A_t > B_t$ is not identical with an inclusive disjunction, also known as alternation.

Proof.

The truth table of logical disjunction is ($A_t \cup B_t$) is defined as follows (Table 29).

Table 29. **Logical disjunction**

$A_t \cup B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ yes	$b_t =$ yes	
	$A_t = +0$	$c_t =$ yes	$d_t =$ no	
	Total			1

The strict inequality $A_t > B_t$ is defined as follows (Table 30).

Table 30. **Strict inequality** $A_t > B_t$

$A_t > B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ no	$b_t =$ yes	
	$A_t = +0$	$c_t =$ no	$d_t =$ no	
	Total			1

Logical disjunction and the strict inequality $A_t > B_t$ agree on in case $b_t = ((A_t = +1) \cap (B_t = +0))$ but not in general. The strict inequality $A_t > B_t$ is not identical with of inclusive disjunction.

Quod erat demonstrandum.

Theorem 3.3. ($A_t \leq B_t$ and material implication)

The strict inequality $A_t \leq B_t$ is identical with material implication.

Proof.

The truth table of material implication is ($A_t \rightarrow B_t$) is defined as follows (Table 27).

Table 31. **Logical disjunction**

$A_t \rightarrow B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ yes	$b_t =$ no	
	$A_t = +0$	$c_t =$ yes	$d_t =$ yes	
	Total			1

The non-strict inequality $A_t \leq B_t$ is defined as follows (Table 28).

Table 32. **Non-strict inequality** $A_t < B_t$

$A_t \leq B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	$a_t =$ yes	$b_t =$ no	
	$A_t = +0$	$c_t =$ yes	$d_t =$ yes	
	Total			1

Material implication and non-strict inequality $A_t \leq B_t$ agree in all cases. The non-strict inequality $A_t \leq B_t$ is identical with material disjunction.

Quod erat demonstrandum.

Theorem 3.4. ($A_t \geq B_t$ and disjunction)

The non-strict inequality $A_t \geq B_t$ is identical with conditio sine qua non.

Proof.

The truth table of conditio sine qua non is ($A_t \leftarrow B_t$) is defined as follows (Table 29).

Table 33. **Conditio sine qua non**

$A_t \leftarrow B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	a_t= yes	b_t= yes	
	$A_t = +0$	c_t= no	d_t= yes	
	Total			1

The non-strict inequality $A_t \geq B_t$ is defined as follows (Table 30).

Table 34. **Non-strict inequality** $A_t \geq B_t$

$A_t \geq B_t$		B_t		
		$B_t = +1$	$B_t = +0$	Total
A_t	$A_t = +1$	a_t= yes	b_t= yes	
	$A_t = +0$	c_t= no	d_t= yes	
	Total			1

Conditio sine qua non and the non-strict inequality $A_t \geq B_t$ agree on all cases. The non-strict inequality $A_t \geq B_t$ is identical with conditio sine qua non.

Quod erat demonstrandum.

Remark 2.

This proof demonstrates clearly the equivalence of the non-strict inequality $A_t \geq B_t$ and conditio sine qua non. Especially, it is not true, that the non-strict inequality $A_t \geq B_t$ is identical with (an inclusive/exclusive) disjunction. The non-strict inequality $A_t \geq B_t$ can be simplified as *either* ($A_t = B_t$) *or* ($A_t > B_t$) but not both at the same trial. Furthermore, both cases *either* ($A_t = B_t$) *or* ($A_t > B_t$) are a determining part of the non-strict inequality $A_t \geq B_t$. If the case ($A_t > B_t$) is not allowed, then the use of the non-strict inequality $A_t \geq B_t$ is not allowed too, since the same demands that it must be possible that ($A_t > B_t$) too. Using the non-strict inequality $A_t \geq B_t$ without allowing the case ($A_t > B_t$) implies a mis-use of the non-strict inequality $A_t \geq B_t$ and can be the source of many contradictions. The variance $\sigma(X_t)^2$ is defined as $\sigma(X_t)^2 = E((X_t)^{2)} - E(X_t)^2$ and can take the values $\sigma(X_t)^2 \geq 0$, where E denotes the expectation values. In principle, it is allowed that *either* $\sigma(X_t)^2 > 0$ *or* $\sigma(X_t)^2 = 0$, but not both at the same trial t / (period of) time t.

Theorem 3.5. (The preservation of contradiction.)

We inevitably make mistakes and have false beliefs. To prevent lapsing into absurdity, hypothesis can be tested only on clear foundations. In general, it is claimed that from a contradictory premise, anything follows, which contradicts the principle of the preservation of contradictions. However, in the absence of technical errors, if something contains a contradiction then everything else derived from such a contradiction should obtain a contradiction too (preservation of contradiction) otherwise it must be possible without one exemption the derive a true conclusion form a false premise.

Claim.

In the absence of technical errors, it is generally valid, that the contradiction is preserved.

Proof by modus juris.

In opposite to our claim, we assume the negation of the same or in other words: it is generally valid, that the contradiction is not preserved. Consequently, starting with a false premise (P_t : +1=+0 is false), we are not able to find one single case where it is not possible to derive a true conclusion (C_t : +3=+3 is true) from a false premise. The proof of this theorem is performed by *modus juris* and logically structured as follows.

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (+3 & = & +3) \\ \multicolumn{5}{c}{Premise\ 1: \quad P_t = (+1 = +0)\ is\ false \quad and \quad C_t = (+3 = +3)\ is\ true.} \end{array} \tag{19}$$

The premise 2 of our proof by *modus juris* is

$$+1 = +0 \tag{20}$$

false. Adding +2 on both sides of the equation, it is

$$(+1) + (+2) = (+0) + (+2) \tag{21}$$

or

$$(+3) = (+2) \tag{22}$$

Quod erat demonstrandum.

Remark 3.

We started with the proof above with a false premise (P_t: +1=+0). Our expectation was, that this contradiction will not be preserved with the consequence that we should be able to derive a true conclusion but we failed. In the absence of technical errors, it was not possible to derive a true conclusion (C_t: +3=+3 is true) from a false premise, which completes our proof. From a contradiction, a contradiction follows. It was mentioned before that modus ponens allows to derive a true conclusion from different premises. If one concrete and single premise (i. e. P_t is true) is used to derive a true conclusion, then the rest (i. e. P_t is false) of many other possible but true premises is equally not used for these purposes. Modus pones just don't care about the rest of all other possible premises to derive a true conclusion, modus ponens considers only one single premise and insists that from such a single and true premise a true conclusion must be drawn. To be precise, the conclusion that modus ponens allows to derive a true conclusion from a false premise is incorrect. As a result, obtaining of true and long-lasting scientific knowledge conducted through most simple step-by-step proofs has the potential to overcome obscurity and confusion in science. Searching for true scientific knowledge is a risky gesture. Still, *either* modus inversus ($\neg P_t \rightarrow \neg C_t$) is generally true *or* modus juris ($\neg(\neg P_t \rightarrow \neg C_t)$) is generally true but not both. The proof demonstrated that modus juris, the negation of modus inversus, is not generally true. Consequently, we must accept that modus inversus is generally true and of use for further purposes. Nonetheless, contemporary approaches taken to develop *a system of paraconsistent logic* (20,27–32) which we have so far seen need to ensure the preservation of truth. Despite the fact that paraconsistent logic is to some extent the rejection of classical logic, even a system of paraconsistent logic cannot avoid the explosion principle (*ex contradictione quodlibet*) when faced with a contradiction, where a contradiction is present. The contradiction is preserved especially according *modus inversus* ($\neg P_t \leftarrow \neg C_t$) too which is the logical equivalent of *modus ponens* ($P_t \rightarrow C_t$). *Modus inversus* as the other side of modus ponens demands that *without* a false premisse P_t *no* false conclusion.

Theorem 3.6. (The rule of the multiplication by zero is incorrect)

Claim.

In the absence of technical errors, today's rule of the multiplication by zero leads to a contradiction and is incorrect.

Proof by modus inversus.

The proof by modus inversus is logically structured as follows.

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (+3 & = & +2) \\ \multicolumn{5}{c}{Premise\ 1:\quad if \quad P_t = (+1 = +0)\ is\ false, \quad then \quad C_t = (+3 = +2)\ is\ false.} \end{array} \tag{23}$$

The premise 2 of our proof by modus sine is

$$+1 \quad = \quad +0 \tag{24}$$

false. Adding +2 on both sides of the equation, it is

$$(+1) + (+2) \quad = \quad (+0) + (+2) \tag{25}$$

or

$$(+3) \quad = \quad (+2) \tag{26}$$

which is correct according to *the proof by modus inversus* (*if* P_t (+1=+0) is false *then* C_t (+3=+2) is false). The following changes of this equation are mathematically without any technical error and correct. These changes must preserve this contradiction. Multiplying by zero, it is

$$(+3) \times (+0) \quad = \quad (+2) \times (+0) \tag{27}$$

According to our today's rules of the multiplication by zero, this is equivalent with

$$(+0) \quad = \quad (+0) \tag{28}$$

In other words, our today's rules of the multiplication by zero equalizes differences and are able to change something false to something true without any logical necessity. Thus far, it is

$$(+1 - 1) \quad = \quad (+1 - 1) \tag{29}$$

or

$$(+1) \quad = \quad (+1) \tag{30}$$

Quod erat demonstrandum.

Remark 4.

We started with something incorrect and derived something correct. Thus far, one conclusion could be that from contradictory premises, anything follows, even something which is true. This is not convincing. The problem is today's rule of the multiplication by zero which changes +3 = +2 to +0=+0 and at the end to +1=+1. Still, *either* modus inversus ($\neg P_t \rightarrow \neg C_t$) is true *or* modus juris ($\neg(\neg P_t \rightarrow \neg C_t)$) is true but not both. The consequence is, that we must reject today's rule of the multiplication by zero as incorrect. In general, *without* a false premisse P_t *no* false conclusion C_t (*modus inversus*) with the consequence that *a true premise excludes a false conclusion*. But not as demonstrated above if we multiply by zero. Consequently, we must reject either modus ponens or today's rule of the multiplication by zero since modus inversus is the logical equivalent of modus ponens.

Theorem 3.7. (The factorial operation is not consistent)

Saitho's equation published as

$$+\frac{1}{0} = +\frac{0}{0} \tag{31}$$

is grounded on a logical contradiction and refuted.

Proof by modus inversus.

The proof by *modus inversus* is logically sound and demands that *if* P_t (+1=+0) is false *then* C_t (+1!=+0!) is false too. We define in this context

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (!\,(+1) & = & !\,(0)) \\ \multicolumn{5}{c}{Premise\ 1:\quad If\quad P_t = (+1 = +0)\ is\ false,\quad then\quad C_t = (1! = 0!)\ is\ false.} \end{array} \tag{32}$$

The premise 2 of our proof by modus sine is

$$+1 = +0 \tag{33}$$

false. Following Christian Kramp (1760 – 1826), the factorial (33) of a positive integer n is denoted by *n!*. Today, the value of 0! is 1, and the value of 1! is 1 too. Thus far, taking the factorial of the equation before, we obtain

$$(+1)! = (+0)! \tag{34}$$

or

$$+1 = +1 \tag{35}$$

which is false too.

Quod erat demonstrandum.

Remark 5.

In general, since *either* modus inversus ($\neg P_t \rightarrow \neg C_t$) is true *or* modus juris ($\neg(\neg P_t \rightarrow \neg C_t)$) is true but not both, we proof the correctness of modus inversus. In the proof above and logic of modus inversus, it is P_t = (+1=+0) = false, ergo C_t (+1! = +0!) must be false too but it is not. A technical error is not apparent. This contradiction is due to the fact that the factorial has the potential to equalize differences.

Theorem 3.8. (Refutation of Saitho's equation 1/0 = 0/0)
Saitho's equation published as

$$+\frac{1}{0} = +\frac{0}{0} \tag{36}$$

is grounded on a logical contradiction and refuted.

Proof by modus inversus.
The proof by *modus inversus* is logically sound. We define in this context

$$\begin{array}{ccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (!\,(+1) & = & !\,(0)) \end{array}$$
$$Premise\ 1:\quad Without\quad P_t = (+1 = +0)\ is\ false\quad no\quad C_t = \left(+\frac{1}{0} = +\frac{0}{0}\right)\ is\ false. \tag{37}$$

The premise 2 of our proof by modus sine is

$$+1 = +0 \tag{38}$$

false. Dividing by zero, we obtain

$$+\frac{1}{0} = +\frac{0}{0} \tag{39}$$

Quod erat demonstrandum.

Remark 6.

Modus inversus ($\neg P_t \leftarrow \neg C_t$) is the logical equivalent of *modus ponens* ($P_t \rightarrow C_t$) and demands that *without* a false premisse P_t (in our case +1=+0) *no* false conclusion C_t (+(1/0) = +(0/0)). In the proof above, it is P_t = (+1=+0) = false. Ergo: C_t = (+(1/0) = +(0/0))= false too, according to the proof by modus ponens.

Theorem 3.9. (Refutation of Anderson's Nullity)

Modus ponens demands that the premise "**If *P_t is true*, then C_t *is true***" or ($P_t \rightarrow C_t$) = true (Table 5) is given. The inverse of the statement $P_t \rightarrow C_t$ ("**If *P_t is true*, then *C_t is true***") is the statement or the equation $\neg P_t \rightarrow \neg C_t$ or in spoken language: "**If *P_t is false*, then *C_t is false***" (Table 16). Thus far, under circumstances where P_t = +0 = false (i.e. $\neg$ P_t = +1= true), the conclusion is that $\neg$ C_t = +1 as demonstrated by the table (Table 16).

Claim.

Andersons's Nullity is self-contradictory and refuted.

Proof by modus inversus.
The proof by modus inversus is logically structured as follows.

$$\begin{array}{lccccc} P_t & \equiv & (+1 & = & +0) \\ C_t & \equiv & (Nullity & = & Nullity) \\ Premise\ 1: & if & P_t = (+1 = +0)\ is\ false, & then & C_t = (Nullity = Nullity)\ is\ false. \end{array} \tag{40}$$

The premise 2 of our proof by modus sine is

$$+1 \quad = \quad +0 \tag{41}$$

false. Multiplying this equation by Andersons's Nullity we obtain

$$(+1) \times (Nullity) \quad = \quad (+0) \times (Nullity) \tag{42}$$

According to Anderson's Axiom 15 (34), it is Nullity × 1 = Nullity. We obtain

$$(Nullity) \quad = \quad (+0) \times (Nullity) \tag{43}$$

According to Anderson's Axiom 15, it is Nullity × 0 = Nullity (34). We obtain a conclusion C_t which is correct as

$$(Nullity) \quad = \quad (Nullity) \tag{44}$$

Quod erat demonstrandum.

Remark 7.
A consistent logical or mathematical operation is one that does not entail any contradiction. Consistently with the theorem above is that from contradictory premise or statement (+1=+0), anything follows (*ex contradictione sequitur quodlibet (ECSQ)*). Historically, ex contradictione sequitur quodlibet (or the Principle of Explosion) is ascribed to William of Soissons, a 12th century French logician who lived in Paris. The premise P_t is not only not given, the premise P_t = (+1=+0) is false. The consequence of such a false premise is that the conclusion C_t must be false to, but it is not. This is a contradiction. Anderson's nullity is self-contradictory and refuted.

4. Discussion

The change of objective reality appears to be a consistent process, to date, an end is not in sight. The nature of scientific inquiry of objective reality by which scientific knowledge is generated varies much across disciplines and often also invoke the incompatibility of opposed properties. The scientific success achieved depend to a very great extent. on the scientific methods used. Therefore, considerations accounting for the very nature of truth and falsity must be able to rely on logically sound scientific methods too.

Modus ponens is one of the basic rules of inference and demands something like "If P_t, then C_t". In other words, from P_t, we can infer C_t. If it were possible to have P_t true and C_t false then modus ponens inference would be invalid. What we think, what we write, what we talk, our everyday reasoning is supported by modus ponens too. Loosing modus ponens would indicate a sever loss. Philosopher's aimed to show that modus ponens is not a generally valid (12) form of inference. Many times, similar to other paradoxes, in one or other way, such trials rest on confusions and are easily circumvented.

Example.

Premise 1: If it is raining today on the street X (P_t is true), **then** the street X is wet (1000000 light years later).

Premise 2: It is raining on the street X today (P_t is true),

Conclusion: The street X is wet (1000000 light years later).

Such a conclusion is of course not justified. But this does not disprove modus ponens. We just don't know today, whether the street X is still existing 1000000 light years later. But even if the street X is still existing 1000000 light years later, what has today's rain to do with the street which is wet 1000000 light years later. Modus ponens can lead to inconsistencies if some basic assumptions are not considered by the user. It is necessary to make sure that events analyzed occur at the same (period of) time t.

To date, the misuse of non-strict inequalities finds its own melting point in the mathematical formulations of Heisenberg's uncertainty principle, of Bell's theorem/inequality, in CHSH inequality et cetera of quantum mechanics. It is more than strange to ground any scientific knowledge on such an inconsistency.

5. Conclusion

Non-strict inequalities have their own interior logic which must be respected in detail otherwise a theory or theorem will end up at a contradiction.

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