# On the equation $8x^2\cos^2 x-\left(4-2\sqrt{2}\right)x^2-1=0$

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Abstract. We give some roots of the equation:

$$8x^2\cos^2 x-\left(4-2\sqrt{2}\right)x^2-1=0 .$$

## 1. Introduction

1.1. If $f(x)=8x^2\cos^2 x-\left(4-2\sqrt{2}\right)x^2-1$ , then $f(-x)=f(x)\ \forall x\in\mathbb{R}$ .

1.2. If $n\in\mathbb{N}$ , then $f\left(\left(n+\frac{1}{2}\right)\pi\right)<0<f(n\pi)$ .

1.3. If $n\in\mathbb{N}$ , then $\exists!\, x>0\wedge n\pi<x<\left(n+\frac{1}{2}\right)\pi$ such that $f(x)=0$ .

1.4. If $x^*>0\wedge f\left(x^*\right)=0$ , then $f\left(-x^*\right)=0$ .

1.5. Graphics of $f(x)$ : Fig.1 : $-1.5<x<1.5$ ; Fig.2 : $-15<x<15$ .

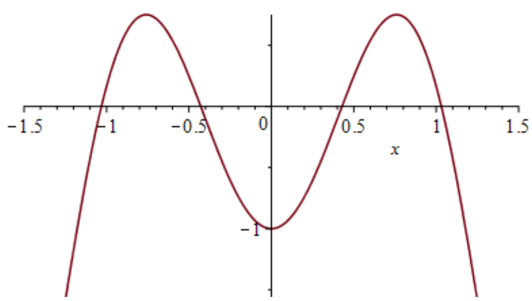


Figure 1.

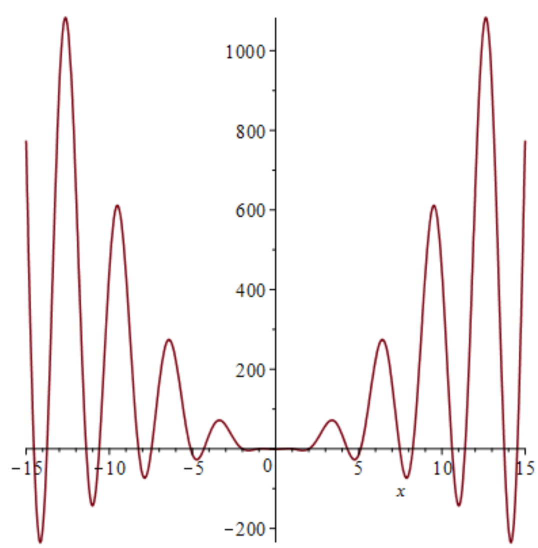


Figure 2.

## 2. Some roots of $f(x)=8x^2\cos^2 x-\left(4-2\sqrt{2}\right)x^2-1=0$ .

2.1. If $x>0\wedge f(x)=0$ , then

$$x=\{0.4284...,1.0311...,2.0057...,4.3102...,5.1118...,7.4581...,8.2492...,\\ 10.6013...,11.3896...,13.7435...,14.5307...,16.8854...,17.6720...,20.0272...,\\ 20.8134...,23.1689...,23.9549...,...\} \tag{1}$$

2.2. The first root:

$$\alpha=0.428448413581805381564431170 8... \tag{2}$$

2.3. The second root:

$$\beta=1.0311737412009999471385735659... \tag{3}$$

## 3. Pi formulas

3.1. If $\alpha=0.4284...$ , then

$$\pi=8\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-2n}\alpha^{2n+1}\cos\left((2n+1)\alpha\right)}{2n+1} \tag{4}$$

3.2. If $\beta=1.0311...,w=1-\beta e^{i\beta}$ , $i=\sqrt{-1}$ , then

$$\pi=\frac{8}{3}\beta+\frac{4}{3}\sum_{n=1}^{\infty}\binom{2n}{n}\frac{(2\beta)^{-2n}\sin(2n\beta)}{n} \tag{5}$$

$$\pi=\frac{8\sqrt{2}}{3}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-3n}}{2n+1}\operatorname{Re}\left(w^n\sqrt{w}\right) \tag{6}$$

3.3. We have

$$\pi=4\alpha+2\sin^{-1}\left(\frac{1}{4\alpha^2}-\frac{1}{\sqrt{2}}\right) \tag{7}$$

$$\pi=\frac{8}{3}\alpha+\frac{8}{3}\sin^{-1}\left(\frac{1}{4\alpha^2\left(\sqrt{2+\sqrt{2}}\cos\alpha+\sqrt{2-\sqrt{2}}\sin\alpha\right)}\right) \tag{8}$$

$$\pi = \frac{8}{5}\alpha + \frac{8}{5}\sin^{-1}\left(\frac{1}{4\alpha^2\left(\sqrt{2+\sqrt{2}}\cos\alpha - \sqrt{2-\sqrt{2}}\sin\alpha\right)}\right) \tag{9}$$

## 4. The modified equation and $\alpha$ root

### 4.1. The modified equation

$$8x^2\cos^2 x - \left(4-2\sqrt{2}\right)x^2 - 1 = 0 \Rightarrow (2x)^2\left(\frac{1}{\sqrt{2}} + \cos(2x)\right) - 1 = 0 \tag{10}$$

### 4.2. Iteration

$$x_{n+1} = \sqrt{\frac{1}{\left(1/\sqrt{2}\right) + \cos x_n}} \quad , x_1 = 0 \Rightarrow x_n \to 2\alpha \tag{11}$$

### 4.3. Representation for $\alpha$ : if $u = 1/\sqrt{2}$ , then

$$2\alpha = \frac{1}{\sqrt{u + \cos\frac{1}{\sqrt{u + \cos\frac{1}{\sqrt{u + \ldots}}}}}} = \sqrt{\frac{1}{u + \cos\sqrt{\frac{1}{u + \cos\sqrt{\frac{1}{u + \ldots}}}}}} \tag{12}$$

Remark: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592\ldots$

# References


1. Frank W.J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark : NIST Handbook of Mathematical Functions. Cambridge University Press, 2010.