# Quantum Model of Mass Function

Xiaozhuan Gao[a], Yong Deng[a,*]

[a] *Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China*

**Abstract**

Dempster-Shafer (D-S) evidence theory has been used in many fields due to the flexibility and effectiveness in modeling uncertainties, which is the extension of classical probability. Recently, quantum probability which can express uncertainty has been used in many fields due to the existing of interference. Especially for human decision and cognition, interference can better model the process of decision. In order to better expand the applications of D-S evidence theory, the paper proposed quantum model of mass function which can consider the interference. In proposed quantum method, quantum mass function uses euler formula to represent. The paper also discusses some operations in quantum model of mass function. Moreover, the paper also discusses the relationship between quantum mass function and classical mass function by using some numerical examples. Classical mass function is the special case when there is no interference in quantum mass function.

*Keywords:* Dempster-Shafer evidence theory, Quantum Theory, Mass Function, Euler formula, Interference

*Corresponding author: Yong Deng, Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China Email address: dengentropy@uestc.edu.cn; prof.deng@hotmail.com

## 1. Introduction

Uncertainty is very important in many fields, which has attracted many researchers attention [1]. There are virous models to handle uncertainty, including fuzzy sets [2], rough sets [3] and Dempster-Shafer (D-S) evidence theory [4, 5]. D-S evidence theory attract more and more attention due to it needs more weaker condition than the Bayesian theory of probability [6]. Besides, D-S evidence theory can handle imprecise and unknown information by assigning probabilities to the power set of events [7]. For this reason, D-S evidence theory has been used in many fields thanks to the flexibility and effectiveness in modeling uncertainties, such as target recognition, decision-making, classifier and so on.

In D-S evidence theory, there are many operations involving superposition of different events or evidences. Hence, considering the superposition of the different events or evidences is also an open issue. Quantum mechanics provides a new view to consider the superposition. Quantum mechanics are different from classical physical mechanics [8, 9, 10]. The classical systems can be obtained accurately, but there are non-classical correlations among the physical attributes of quantum systems, which make these quantities not accurately obtained, but exist as linear superposition state vectors [11, 12]. There are various experiment used to illustrate the properties of quantum mechanics which includes uncertainty, interference and entanglement and so on [12], such as Schrodinger's Cat [13], double-slit experiment [14] and so on [15]. Based on the properties of quantum mechanics, the quantum information is gradually developed, which has been used many fields, such as communication complexity [16] and game theory [17]. Besides, the quantum theory can better describe the way humans make judgments towards uncertainty and decisions under conflict environment [18, 19, 20]. At a more fundamental level, it has become clear that an information theory based on quantum principles extends and completes classical information theory [12].

By the above discussion, it can be seen that quantum theory is a hot and interesting topic. Hence, using D-S evidence theory into quantum mechanics provide a new thought to better analyse the process of superposition and expand the application of evidence theory. Vourdas study the connection between quantum mechanics and D-S evidence theory [21, 22, 23]. Deng [24] proposed a meta mass function expressed by complex numbers in DCS evidence theory. Xiao [25] proposed extension of belief function by using complex number. The paper proposed quantum model of mass function by using euler formula which consider the amplitude and phase angle of variables. Quantum model of mass function can better explain the interference of different variables. Quantum mass function can degenerate classic mass function when all variables are orthogonal, namely, the interference of different variables is 0. The paper also proposed the quantum combination rule which is more application and general than Dempster' combination rule. In this context, the quantum model of mass function provides a promising way to model and handle the process of uncertain information. Consequently, several numerical examples are provided to illustrate the efficiency of the quantum evidence theory.

The paper is organized as follows. In Section 2. the preliminaries D-S evidence theory and euler formula are briefly introduced. Section 3 proposed the quantum model of mass function. In Section 4, there are some numerical example be used to explain the proposed method. Finally, this paper is concluded in Section 5.

## 2. Preliminaries

In this section, the preliminaries of D-S theory [4, 5] will be briefly introduced.

### 2.1. Dempster-Shafer theory

**Definition 2.1.** *( Frame of discernment)*
*Let $\Theta$ be the set of mutually exclusive and collectively exhaustive events $A_i$, namely*

$$\Theta = \{A_1, A_2, \cdots, A_n\} \tag{1}$$

*The power set of $\Theta$ composed of $2^N$ elements of is indicated by $2^\Theta$, namely:*

$$2^\Theta = \{\phi, \{A_1\}, \{A_2\}, \cdots, \{A_1, A_2\}, \cdots, \Theta\} \tag{2}$$

**Definition 2.2.** *( Mass Function)*
*For a frame of discernment $\Theta = \{A_1, A_2, \cdots, A_n\}$, the mass function* m *is defined as a mapping of m from 0 to 1, namely:*

$$m : 2^\Theta \to [0, 1] \tag{3}$$

which satifies

$$m(\phi) = 0 \tag{4}$$

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{5}$$

In D-S theory, mass function is often called a piece of evidence or belief structure or basic probability assignment (BPA). The $m(A)$ represents the belief degree to A, namely, the degree of evidence supports A. In $m(A)$, there is a focal element, namely A. BPA is an essential tool for uncertainty measure, there are some researches for BPA.

**Definition 2.3.** *(Belief function)*
*The belief function (Bel) is a mapping from set* $2^\Theta$ *to [0,1] and satisfied:*

$$Bel\left(A\right)=\sum_{B\subseteq A}m\left(B\right) \tag{6}$$

**Definition 2.4.** *(Plausibility function)*
*The plausibility function (Pl):* $2^\Theta\to[0,1]$*, and satisfied:*

$$Pl\left(A\right)=\sum_{B\cap A\neq\phi}m\left(B\right)=1-Bel(\bar{A}) \tag{7}$$

As can be seen from the above, $\forall A\subseteq\Theta$ , $Bel(A)<Pl(A), Bel(A), Pl(A)$ are respectively the lower and upper limits of A, namely [Bel(A), Pl(A)], which indicates uncertain interval for A.

**Definition 2.5.** *(Dempster Combine Rule)*
*There are two BPAs indicated by* $m_1$ *and* $m_2$*, the Dempster combination rule is used to combine them as follows [4]:*

$$m(A)=\left\{\begin{matrix}\frac{1}{K-1}\sum_{B\cap C=A}m_1(B)\times m_2(C)\\0\end{matrix}\right\} \tag{8}$$

*with*

$$K=\sum_{B\cap C=\phi}m_1(B)\times m_2(C) \tag{9}$$

**Definition 2.6.** *(Pignistic Probability Transition )*
*Let m is a BPA under frame of discernment* $\Theta$*, and the pignistic probability tran-*

*sition is defined as follows [26]:*

$$p(B) = \sum_{B \subseteq A} \frac{m(A)}{|A|} \tag{10}$$

*where $|A|$ represents the cardinality of $A$.*

Besides, the pignistic probability transition also named PPT.

*2.2. Euler formula*

**Definition 2.7.** *The Euler formula is defined as follows:*

$$a = e^{j\theta} = cos(\theta) + isin(\theta) \tag{11}$$

*where e is natural logarithm, i is imaginary number, $\theta$ is angle. Besides, the amplitude of a is always 1, namely, $|a| = 1$.*

Besides,

$$|ae^{j\theta}| = a^2 \tag{12}$$

$$|ae^{j\theta 1} + be^{j\theta 2}| = a^2 + b^2 + 2abcos(\theta 1 - \theta 2) \tag{13}$$

where

$$interference = 2abcos(\theta 1 - \theta 2) \tag{14}$$

Interference is the term of quantum mechanics, which is superposition of micro-particles and is similarly with interference of light in classical mechanics. The $Eq$. 13 and 14 has been used in many fields [27, 28]. There are various methods to calculate the interference which has been used many fields [27, 28].

## 3. Quantum Model of Mass Function

Quantum probability has been used in many fields [29]. In order to better explore the connection of different evidence and expand the application of D-S evidence theory, the paper proposed the quantum model of mass function.

**Definition 3.1.** *(Quantum Frame of Discernment)*
*Let $\Theta$ be the set of mutually exclusive and collectively exhaustive events $|A_i>$, namely*

$$|\Theta>= \{|A_1>, |A_2>, \cdots, |A_n>\} \tag{15}$$

*The power set of $|\Theta>$ composed of $2^N$ elements of is indicated by $2^{|\Theta>}$, namely:*

$$2^{|\Theta>} = \{\phi, \{|A_1>\}, \{|A_2>\}, \cdots, \{|A_1, A_2>\}, \cdots, |\Theta>\} \tag{16}$$

**Definition 3.2.** *(Quantum Mass Function)*
*In quantum frame of discernment, the quantum mass function $\mathbb{Q}$ is defined as follows:*

$$\mathbb{Q}(|A>) = \psi_1 e^{j\theta 1} \tag{17}$$

*which is a mapping of $\mathbb{Q}$ from 0 to 1 and satisfies :*

$$\mathbb{Q}(\phi) = 0 \tag{18}$$

$$\sum_{|A>\subseteq|\Theta>} |\mathbb{Q}(|A>)| = 1 \tag{19}$$

*where $|\mathbb{Q}(|A>)| = \psi_1^2$*

The quantum mass function also is called as quantum basic probability assignment(QBPA), where $|\psi_1|^2$ represents the belief degree to $|A>$, namely, evidence supports the proposition or hypothesis $|A>$. The $\theta$ represents phase angle of $|A>$.

**Remark 1** : If the phase angle of quantum mass function equals $0°$, the quantum mass function can degenerate the classical mass function.

**Remark 2** : The quantum mass function does not satisfy additivity, as *Eq*. 20.

$$\mathbb{Q}(A) + \mathbb{Q}(B) \neq |\mathbb{Q}(A)| + |\mathbb{Q}(A)| \tag{20}$$

**Definition 3.3.** *(Quantum Belief Function)*
*In quantum frame of discernment, the quantum belief function QBel is defined as follows:*

$$\begin{aligned} QBel(|A>) &= \sum_{|B>\subseteq|A>} \mathbb{Q}(|B>) \\ &= \psi_{A1}e^{j\theta A1} + \psi_{A2}e^{j\theta A2} + \cdots \end{aligned} \tag{21}$$

**Definition 3.4.** *(Quantum Plausibility Function)*
*In quantum frame of discernment, the quantum plausibility function QPl is defined as follows:*

$$\begin{aligned} QPl(|A>) &= \sum_{|B>\cap|A>\neq\phi} \mathbb{Q}(|B>) \\ &= \psi_{A1}e^{j\theta A1} + \psi_{A2}e^{j\theta A2} + \cdots \end{aligned} \tag{22}$$

Apparently, $|QBel(|A>)|$ represents the lower probability of supporting proposition $|A>$ and $|QPl(|A>)|$ represents the upper probability of supporting proposition $|A>$. Hence, $[|QBel(|A>)|, |QPl(|A>)|]$ is the interval of proposition $|A>$.

**Remark 3** : If the phase angle of quantum mass function equals $0°$ or the interference of different events equals 0, the quantum belief function and plausibility function can degenerate the classical belief and plausibility function.

**Proof** : Suppose there are some QBPAs under a quantum frame of discernment $\Omega = \{|A>, |B>, |C>\}$, the QBPs are given as follows:

$$\mathbb{Q}(|A>) = ae^{j\theta 1}, \mathbb{Q}(|B>) = be^{j\theta 2}, \mathbb{Q}(|A>, |C>) = ce^{j\theta 3}$$

where $a^2 + b^2 + c^2 = 1$. The QBel and QPl of $|A>$ are shown as follows:

$$QBel(|A>, |B>) = ae^{j\theta 1} + be^{j\theta 2}, QPl(|A>, |B>) = ae^{j\theta 1} + be^{j\theta 2} + ce^{j\theta 3}$$

If the all phase angle of QBPAs equal $0^\circ$, the QBel and QPl are given as follows:

$$QBel(|A>, |B>) = a + b, QPl(|A>, |B>) = a + b + c$$

which is equal with classical $Bel(A, B)$ and $Pl(A, B)$.

If the phase angle of QBPAs does not equal $0^\circ$, the $|QBel(|A>, |B>)|$ and $|QPl(|A>, |B>)|$ are given as follows:

$$|QBel(|A>, |B>)| = a^2 + b^2 + 2abcos(\theta 1 - \theta 2)$$

$$|QPl(|A>, |B>)| = a^2 + b^2 + c^2 + 2abcos(\theta 1 - \theta 2) + 2accos(\theta 1 - \theta 3) + 2bccos(\theta 2 - \theta 3)$$

when the interference of $|QBel(|A>, |B>)|$ and $|QPl(|A>, |B>)|$ are equal 0, the $|QBel(|A>, |B>)|$ and $|QPl(|A>, |B>)|$ is similar with classical $Bel(A, B)$ and $Pl(A, B)$.

**Definition 3.5.** *(Quantum Combination Rule)*
*There are two QBPAs* $\mathbb{Q}1$ *and* $\mathbb{Q}2$*, the quantum combination rule is defined as*

*follows:*

$$\mathbb{Q}(|A>) = \left\{ \begin{array}{cc} \frac{1}{1-|\mathbb{K}|} \sum_{|B>\cap|C>=|A>} \mathbb{Q}(|B>) \times \mathbb{Q}(|C>) & |A>\neq \phi \\ 0 & |A>= \phi \end{array} \right\} \tag{23}$$

*After normalization:*

$$|\mathbb{Q}(|A>)| = \frac{|\mathbb{Q}(|A>)|}{|\mathbb{Q}(|A>)| + |\mathbb{Q}(|B>)| + \cdots + |\mathbb{Q}(|A>, |B>)| + \cdots} \tag{24}$$

*where* $\mathbb{K}$ *is defined as follows:*

$$\mathbb{K} = \sum_{|B>\cap|C>=\phi} \mathbb{Q}1(|B>) \times \mathbb{Q}2(|C>) \tag{25}$$

In quantum combination rule, $\mathbb{K}$ is the quantum probability and $|\mathbb{K}|$ represents the conflict degree between the QBPAs $\mathbb{Q}1$ and $\mathbb{Q}2$.

**Remark 4** : The conflict $|\mathbb{K}|$ of quantum combination rule can generate the classical conflict $k$ when the interference of $\mathbb{K}$ is 0, namely the two elements are orthogonal to each other.

**Proof** : Suppose there are the two QBPAs $\mathbb{Q}1$ and $\mathbb{Q}2$ in the quantum frame of discernment $|\Theta >= \{|A>, |B>\}$, and the two QBPAs are given as follows:

$$\mathbb{Q}1(|A>) = a_1 e^{j\theta 11}, \mathbb{Q}1(|A>) = b_1 e^{j\theta 12}, \mathbb{Q}1(|A>, |B>) = c_1 e^{j\theta 13}$$

$$\mathbb{Q}2(|A>) = a_2 e^{j\theta 21}, \mathbb{Q}2(|B>) = b_2 e^{j\theta 22}$$

where QBPAs satisfies that $a_1^2 + b_1^2 + c_1^2 = 1$ and $a_2^2 + b_2^2 = 1$

The $\mathbb{K}$ is calculated as follows:

$$\mathbb{K} = a_1 b_2 e^{j\theta 1} + a_2 b_1 e^{j\theta 2}$$

$$|\mathbb{K}| = (a_1 b_2)^2 + (a_2 b_1)^2 + 2a_1 b_2 a_2 b_1 cos\theta \tag{26}$$

In $Eq.$ 19, the $interference = 2a_1 b_2 a_2 b_1 cos\theta$ represents the interference. If the interference is 0, the $|\mathbb{K}|$ can generate the classical conflict coefficient $k$.

**Remark 5** : The quantum combination rule is only application for QBPAs $\mathbb{Q}1$ and $\mathbb{Q}2$ under the condition $\mathbb{K} \neq 1$

**Remark 6** : The quantum combination rule can generate the classical Dempster's combination rule when quantum evidence theory does not exist interference, namely interference=0.

**Proof** : Using the $Example$ 1, the fusion results are as follows:

$$\mathbb{Q}(|A>) = \frac{a_1 a_2 e^{j\theta 3} + a_2 c_1 e^{j\theta 4}}{1 - |\mathbb{K}|}$$

$$\mathbb{Q}(|B>) = \frac{b_1 b_2 e^{j\theta 5} + b_2 c_1 e^{j\theta 6}}{1 - |\mathbb{K}|}$$

$$|\mathbb{Q}(|A>)| = \frac{(a_1 a_2)^2 + (a_2 c_1)^2 + 2a_1 a_2 a_2 c_1 cos\theta 1}{1 - |\mathbb{K}|}$$

$$\mathbb{Q}(|B>) = \frac{(b_1 b_2)^2 + (b_2 c_1)^2 + 2b_1 b_2 b_2 c_1 cos\theta 2}{1 - |\mathbb{K}|}$$

where $interference1 = 2a_1 a_2 a_2 c_1 cos\theta 1$ and $interference2 = 2b_1 b_2 b_2 c_1 cos\theta 2$. The quantum combination rule can generate the classical Dempster's combination rule when $interference = 0$, $interference1 = 0$ and $interference2 = 0$.

**Definition 3.6.** *(Quantum Pignistic Probability Transformation)*
*Let $\mathbb{Q}(|A>)$ is the QBPAs under the quantum frame of discernment $\Omega$ and $|A>\subseteq \Omega$, the quantum pignistic probability transformation is defined as follows:*

$$\mathbb{P}(|B>) = \sum_{|B>\subseteq|A>} \frac{\mathbb{Q}(|A>)}{||A>|} \tag{27}$$

*where $|A|$ represents the cardinality of $|A>$.*

The quantum pignistic probability transformation assign the multi-elements sets into the single element sets. In $Eq$. 19, $\mathbb{P}(|B>)$ is the quantum probability.

**Remark 7** : If the phase angle of quantum mass function equals $0^\circ$ or the interference of different events equals 0, the quantum belief function and plausibility function can degenerate the classical belief and plausibility function.

## 4. Numerical Example

In this section, there are some examples to explain the effectiveness quantum model of mass function.

**Example 4.1.** *Suppose the quantum frame of discernment is $\Omega = \{|A>, |B>, |C>\}$, the QBPAs is shown as follows:*

$$\mathbb{Q}(|A>) = \sqrt{0.2}e^{j0^\circ}, \mathbb{Q}(|A>, |B>) = \sqrt{0.3}e^{j0^\circ}$$

$$\mathbb{Q}(|B>) = \sqrt{0.2}e^{j0^\circ}, \mathbb{Q}(|A>, |C>) = \sqrt{0.3}e^{j0^\circ}$$

when the phase angles of all QBPAs equal $0^\circ$, the quantum mass function can degenerate the classical mass function.

**Example 4.2.** *Suppose the quantum frame of discernment is $\Omega = \{|A>, |B>, |C>\}$, the QBPAs is shown as follows:*

$$\mathbb{Q}(|A>) = \sqrt{0.2}e^{j\theta 11}, \mathbb{Q}(|A>, |B>) = \sqrt{0.3}e^{j\theta 12}$$

$$\mathbb{Q}(|B>) = \sqrt{0.2}e^{j\theta 13}, \mathbb{Q}(|A>, |C>) = \sqrt{0.3}e^{j\theta 14}$$

The $QBel$ and $QPl$ is calculated as follows:

$$QBel(|A>) = \sqrt{0.2}e^{j\theta 11}, QPl(|A>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 14}$$

$$QBel(|B>) = \sqrt{0.2}e^{j\theta 13}, QPl(|A>) = \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12}$$

$$QBel(|C>) = 0, QPl(|C>) = \sqrt{0.3}e^{j\theta 14}$$

$$QBel(|A>, |B>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12}$$

$$QPl(|A>, |B>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 14}$$

$$QBel(|A>, |C>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.3}e^{j\theta 14}$$

$$QPl(|A>, |C>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 13}$$

$$QBel(|B>, |C>) = \sqrt{0.2}e^{j\theta 13}, QPl(|B>, |C>) = \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 14}$$

$$QBel(|A>, |B>, |C>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 14}$$

$$QPl(|A>, |B>, |C>) = \sqrt{0.2}e^{j\theta 11} + \sqrt{0.2}e^{j\theta 13} + \sqrt{0.3}e^{j\theta 12} + \sqrt{0.3}e^{j\theta 14}$$

If the phase angle of all QBPAs equals $0^\circ$, the $QBel$ and $QPl$ can degenerate the classical $Bel$ and $Pl$. Besides, if there is no interference, the $|QBel|$ and $|QPl|$ is similar with the classical $Bel$ and $Pl$.

**Example 4.3.** *There are two QBPAs* $\mathbb{Q}1$ *and* $\mathbb{Q}2$ *under the quantum frame of discernment* $\Omega = \{|A>, |B>, |C>\}$*, and the two QBPAs* $\mathbb{Q}1$ *and* $\mathbb{Q}2$ *are given as follows:*

$$\mathbb{Q}1(|A>) = \sqrt{0.5}e^{j\theta 11}, \mathbb{Q}1(|B>) = \sqrt{0.2}e^{j\theta 12}, \mathbb{Q}1(|A>, |B>, |C>) = \sqrt{0.3}e^{j\theta 13}$$

$$\mathbb{Q}2(|B>) = \sqrt{0.1}e^{j\theta 21}, \mathbb{Q}2(|A>, |C>) = \sqrt{0.2}e^{j\theta 22}, \mathbb{Q}2(|A>, |B>, |C>) = \sqrt{0.7}e^{j\theta 23}$$

It is easy to verify that the $|\mathbb{Q}1(|A>)| + |\mathbb{Q}1(|B>)| + |\mathbb{Q}1(|A>, |B>, |C>)| = 1$ and $|\mathbb{Q}2(|B>)| + |\mathbb{Q}2(|A>, |C>)| + |\mathbb{Q}2(|A>, |B>, |C>)| = 1$.

According the $Eq$. 18, the conflict between two evidences can be calculated as follows:

$$\mathbb{K} = \mathbb{Q}1(|A>) \times \mathbb{Q}2(|B>) + \mathbb{Q}1(|B>) \times \mathbb{Q}2(|A>, |C>)$$

$$\mathbb{K} = \sqrt{0.5}e^{j\theta 11} \times \sqrt{0.1}e^{j\theta 21} + \sqrt{0.2}e^{j\theta 12} \times \sqrt{0.2}e^{j\theta 22}$$

$$\mathbb{K} = \sqrt{0.05}e^{j\theta 1} + \sqrt{0.04}e^{j\theta 2}$$

$$|\mathbb{K}| = 0.09 + 2 \times \sqrt{0.002} \times cos\theta \tag{28}$$

In $Eq$. 20, $2 \times \sqrt{0.002} \times cos\theta$ represents the interference. The amplitude of $\mathbb{K}$ changes with $\theta$.

By using $Eq$. 17, the fusion results can be got as follows:

$$\mathbb{Q}(|A>) = \frac{\sqrt{0.1}e^{j\theta 3} + \sqrt{0.35}e^{j\theta 4}}{1 - |\mathbb{K}|}, \mathbb{Q}(|B>) = \frac{\sqrt{0.02}e^{j\theta 5} + \sqrt{0.14}e^{j\theta 6} + \sqrt{0.03}e^{j\theta 7}}{1 - |\mathbb{K}|}$$

$$\mathbb{Q}(|A>, |C>) = \frac{\sqrt{0.06}e^{j\theta 8}}{1 - |\mathbb{K}|}, \mathbb{Q}(|A>, |B>, |C>) = \frac{\sqrt{0.21}e^{j\theta 9}}{1 - |\mathbb{K}|}$$

It can be seen that the fusion results is also the QBPA. Next, the amplitude of QBPA is calculated as follows:

$$|\mathbb{Q}(|A>)| = \frac{0.45 + 0.3742cos\theta_1}{1 - |\mathbb{K}|}, |\mathbb{Q}(|B>)| = \frac{0.19 + 0.1058cos\theta_2 + 0.049cos\theta_3 + 0.1296cos\theta_4}{1 - |\mathbb{K}|}$$

$$|\mathbb{Q}(|A>, |C>)| = \frac{0.06}{1 - |\mathbb{K}|}, |\mathbb{Q}(|A>, |B>, |C>)| = \frac{0.21}{1 - |\mathbb{K}|}$$

when $\theta = 90^\circ$, $\theta_1 = 90^\circ$, $\theta_2 = 90^\circ$, $\theta_3 = 90^\circ$ and $\theta_4 = 90^\circ$, the amplitude of QBPA is as follows:

$$|\mathbb{Q}(|A>)| = 0.49, |\mathbb{Q}(|B>)| = 0.21, |\mathbb{Q}(|A>, |C>)| = 0.07, |\mathbb{Q}(|A>, |B>, |C>)| = 0.23$$

The result is the same with classical Dempster's combination rule, showing that the quantum combination rule can degenerate the classical Dempster's combination rule when there is no interference, namely, all element is mutually orthogonal.

Moreover, if there are interferences, namely, $\theta = 45^\circ$, $\theta_1 = 60^\circ$, $\theta_2 = 30^\circ$, $\theta_3 = 90^\circ$ and $\theta_4 = 120^\circ$, the fusion results are given as follows:

$$|\mathbb{Q}(|A>)| = 0.57, |\mathbb{Q}(|B>)| = 0.19, |\mathbb{Q}(|A>, |C>)| = 0.05, |\mathbb{Q}(|A>, |B>, |C>)| = 0.19$$

So, it can be seen that interference can influence the final results.

**Example 4.4.** *Supposing that there are two QBPAs in a quantum frame of discernment, and the two QBPAs are given as follows:*

$$\mathbb{Q}1(|A>) = \sqrt{0.3}e^{j30^\circ}, \mathbb{Q}1(|B>) = \sqrt{0.2}e^{j180^\circ}, \mathbb{Q}1(|A>, |C>) = \sqrt{0.5}e^{j270^\circ}$$

$$\mathbb{Q}2(|A>, |B>) = \sqrt{0.4}e^{j90^\circ}, \mathbb{Q}2(|B>) = \sqrt{0.3}e^{j120^\circ}, \mathbb{Q}2(|A>, |C>) = \sqrt{0.3}e^{j300^\circ}$$

Uing the $Eq.$ $22-24$, the final fusion results are shown as follows:

$$\mathbb{K} = 0.3e^{j150^{\circ}} + 0.2449e^{j120^{\circ}} + 0.3873e^{j30^{\circ}}, |K| = 0.3111$$

$$\mathbb{Q}(|A>) = \frac{0.3464e^{j120^{\circ}} + 0.3e^{j330^{\circ}} + 0.4472e^{j150^{\circ}}}{0.6889}, |\mathbb{Q}(|A>)| = 0.4664$$

$$\mathbb{Q}(|B>) = \frac{0.2828e^{j270^{\circ}} + 0.2449e^{j300^{\circ}}}{0.6889}, |\mathbb{Q}(|B>)| = 0.3774$$

$$\mathbb{Q}(|A>, |C>) = \frac{0.3873e^{j210^{\circ}}}{0.6889}, |\mathbb{Q}(|A>, |C>)| = 0.2177$$

After normalization,

$$|\mathbb{Q}(|A>)| = 0.4394, |\mathbb{Q}(|B>)| = 0.3555, |\mathbb{Q}(|A>, |C>)| = 0.2051$$

After quantum combination rule, the sum of amplitude does not equal 1. The reason is that any operation will change the original state of the system and operation is irreversible.

**Example 4.5.** *There are QBPAs under the quantum frame of discernment* $\Omega = \{|A>, |B>, |C>\}$*. The QBPAs are given as follows:*

$$\mathbb{Q}(|A>, |C>) = \sqrt{0.3}e^{j\theta 1}, \mathbb{Q}(|A>, |B>) = \sqrt{0.3}e^{j\theta 2}, \mathbb{Q}(|A>, |B>, |C>) = \sqrt{0.4}e^{j\theta 3},$$

According the $Eq.$ 21, the results are shown as follows:

$$\mathbb{P}(|A>) = \frac{\sqrt{0.3}e^{j\theta 1}}{2} + \frac{\sqrt{0.3}e^{j\theta 2}}{2} + \frac{\sqrt{0.4}e^{j\theta 3}}{3}$$

$$\mathbb{P}(|B>) = \frac{\sqrt{0.3}e^{j\theta 2}}{2} + \frac{\sqrt{0.4}e^{j\theta 3}}{3}, \mathbb{P}(|C>) = \frac{\sqrt{0.3}e^{j\theta 1}}{2} + \frac{\sqrt{0.4}e^{j\theta 3}}{3}$$

From the results, it can be seen that the classical PPT is the special case of quantum pignistic probability transition.

Analysing the above example, it can be known that the quantum evidence theory can be generate the classical D-S evidence theory when there is no interference between two or multiple elements. More importantly, interference is the unique phenomenon in quantum mechanics, which represents effect of different events before the final decision and is similar with interference of light in classical mechanics. Considering the interference of operations is helpful to analyse the connection of different events.

## 5. Conclusion

Dempster-Shafer (D-S) evidence theory can have $2^n$ sets so that it can handle some imprecise and unknown information very well. Quantum theory attracted more and more researchers attention due to its powerful capabilities of solving the decision making problems. How to establish a bridge between quantum theory and D-S evidence theory is an also open issue. Hence, the paper propose quantum model of mass function. In quantum model of mass function, the paper uses euler formula to represent the quantum mass function, the of quantum mass function represents the probability of occurrence of $|A>$. Meanwhile, the paper also discusses the quantum belief function, quantum plausibility function and quantum combination rule. Quantum evidence theory can degenerate the classical D-S evidence theory if there is no interference or the phase angle is $0^{\circ}$. That is to say, the classical D-S evidence theory is the special case of quantum model of mss function when different elements are orthogonal to each other. Finally, the paper uses some numerical example to explain the difference between classical D-S evidence theory and quantum evidence theory.

## Acknowledgments

The work is partially supported by National Natural Science Foundation of China (Grant Nos. 61573290, 61503237).

## Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## References

[1] M. J. Kochenderfer, Decision making under uncertainty: theory and application, MIT press, 2015.

[2] L. A. Zadeh, Fuzzy sets, Information and control 8 (3) (1965) 338–353.

[3] Z. Pawlak, Rough sets, International journal of computer & information sciences 11 (5) (1982) 341–356.

[4] A. Dempster, Upper and lower probabilities induced by a multivalued mapping, The Annals of Mathematical Statistics 38 (2) (1967) 325–339.

[5] G. Shafer, A mathematical theory of evidence, Vol. 42, Princeton university press, 1976.

[6] R. R. Yager, Generalized dempster–shafer structures, IEEE Transactions on Fuzzy Systems 27 (3) (2019) 428–435.

[7] X. Gao, Y. Deng, The negation of basic probability assignment, IEEE Access 7 (1) (2019) 10.1109/ACCESS.2019.2901932.

[8] P. A. M. Dirac, The principles of quantum mechanics, no. 27, Oxford university press, 1981.

[9] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, J. L. OBrien, Quantum computers, Nature 464 (7285) (2010) 45.

[10] W. Deng, Y. Deng, Entropic methodology for entanglement measures, Physica A: Statistical Mechanics and its Applications 512 (2018) 693–697.

[11] M. A. Nielsen, I. Chuang, Quantum computation and quantum information (2002).

[12] C. H. Bennett, D. P. DiVincenzo, Quantum information and computation, nature 404 (6775) (2000) 247.

[13] A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, P. Grangier, Generation of optical schrödinger cats from photon number states, Nature 448 (7155) (2007) 784.

[14] R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, H. Shtrikman, Phase measurement in a quantum dot via a double-slit interference experiment, Nature 385 (6615) (1997) 417.

[15] S. Dürr, T. Nonn, G. Rempe, Origin of quantum-mechanical complementarity probed by a which-wayexperiment in an atom interferometer, Nature 395 (6697) (1998) 33.

[16] H.-K. Lo, Classical-communication cost in distributed quantum-information processing: a generalization of quantum-communication complexity, Physical Review A 62 (1) (2000) 012313.

[17] J. Eisert, M. Wilkens, M. Lewenstein, Quantum games and quantum strategies, Physical Review Letters 83 (15) (1999) 3077.

[18] J. R. Busemeyer, P. D. Bruza, Quantum models of cognition and decision, Cambridge University Press, 2012.

[19] Z. He, W. Jiang, An evidential dynamical model to predict the interference effect of categorization on decision making, Knowledge-Based Systems 150 (2018) 139–149.

[20] P. D. Kvam, T. J. Pleskac, S. Yu, J. R. Busemeyer, Interference effects of choice on confidence: Quantum characteristics of evidence accumulation, Proceedings of the National Academy of Sciences 112 (34) (2015) 10645–10650.

[21] A. Vourdas, Non-additive probabilities and quantum logic in finite quantum systems, in: Journal of Physics: Conference Series, Vol. 597, IOP Publishing, 2015, p. 012073.

[22] A. Vourdas, Lower and upper probabilities in the distributive lattice of subsystems, Journal of Physics A: Mathematical and Theoretical 47 (34) (2014) 345203.

[23] A. Vourdas, Quantum probabilities as dempster-shafer probabilities in the lattice of subspaces, Journal of Mathematical Physics 55 (8) (2014) 082107.

[24] Y. Deng, Meta mass function, vixra preprint (2017) vixra:1708.0065v1.

[25] F. Xiao, Quantum dynamical mode (qdm): A possible extension of belief function, arXiv preprint (2018) arXiv:1801.05707.

[26] P. Smets, R. Kennes, The transferable belief model, Artificial intelligence 66 (2) (1994) 191–234.

[27] C. Moreira, A. Wichert, Quantum-like bayesian networks for modeling decision making, Frontiers in psychology 7 (2016) 11.

[28] Z. Huang, L. Yang, W. Jiang, Uncertainty measurement with belief entropy on the interference effect in the quantum-like bayesian networks, Applied Mathematics and Computation 347 (2019) 417–428.

[29] I. Pitowsky, Quantum probability-quantum logic, Springer, 1989.