On the various mathematical connections with the Ramanujan's numbers 1729, 728, the Ramanujan's class invariant, some sectors of Particle Physics and some formulae concerning the Supersymmetry

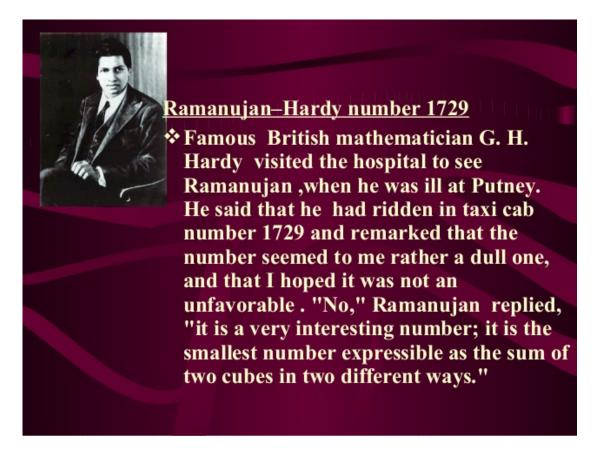
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Abstract

In the present research thesis, we have obtained various and interesting mathematical connections with the Ramanujan's numbers 1728, 1729, 728, 729 and some sectors of Particle Physics and Supersymmetry

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From:

https://www.slideshare.net/SSridhar2/talk-on-ramanujan

From:

Ramanujan's Astonishing Knowledge of 1729 - Published May 12, 2016 - https://thatsmaths.com/2016/05/12/ramanujans-astonishing-knowledge-of-1729/

$$\begin{array}{lll}
\text{Sf} \\
\text{(i)} & \frac{1+53x+9x^{2}}{1-92x-92x^{2}+x^{3}} = \alpha_{0}+\alpha_{1}x+\alpha_{2}x^{2}+\alpha_{3}x^{3}+\cdots \\
& \text{on } \frac{\alpha_{0}}{x} + \frac{\alpha_{1}}{2x} + \frac{\alpha_{1}}{2x} + \cdots \\
& \text{on } \frac{\alpha_{0}}{x} + \frac{\alpha_{1}}{2x} + \alpha_{1}x^{2} + \alpha_{3}x^{3} + \cdots \\
& \text{on } \frac{\alpha_{0}}{x} + \frac{\beta_{1}}{2x} + \beta_{2}x^{2} + \alpha_{3}x^{4} + \cdots \\
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& \text{on } \frac{$$

Page from Ramanujan's Lost Notebook. Image credit: Trinity College Cambridge. Reproduced from Ono, 2015.]

We note the fundamental expressions:

$$9^{3} + 10^{3} = 12^{3} + 1$$
; $729 + 1000 = 1728 + 1$
 $6^{3} + 8^{3} = 9^{3} - 1$; $216 + 512 = 729 - 1$
 $135^{3} + 138^{3} = 172^{3} - 1 = 5088447$; $(5088447)^{1/32} = 1,62024537...$
 $(5088447)^{1/31} = 1,645665103...$; $(5088447)^{1/30} = 1,673219209...$
 $5088447 / 1729 = 2943$;

From: https://www.scienceandnonduality.com/article/the-secrets-of-ramanujans-garden

We have: 8J + 3 and $64J^2 - 24J + 9$

For J = 1, 3, 30, 165, 20010 we have:

$$8J + 3 = 11$$
; $8J + 3 = 27$; $8J + 3 = 243$; $8J + 3 = 1323$; $8J + 3 = 160083$;

11; 27;
$$27 * 3^2 = 243$$
; $27 * 7^2 = 1323$; $27 * 77^2 = 160083$;

$$64J^2 - 24J + 9 = 49$$
; $64J^2 - 24J + 9 = 64*9 - 24*3 + 9 = 576 - 72 + 9 = 513$;

$$64J^2 - 24J + 9 = 64*900 - 24*30 + 9 = 57600 - 720 + 9 = 56889;$$

$$64J^2 - 24J + 9 = 64*27225 - 24*165 + 9 = 1742400 - 3960 + 9 = 1738449;$$

 $64J^2 - 24J + 9 = 64*400400100 - 24*20010 + 9 = 25625606400 - 480240 + 9 = 25625126169;$

Note that $64J^2 - 24J + 9$ if set equal to zero, can be considered a quadratic equation. The quadratic formula for the roots of the general quadratic equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We have: $64J^2 - 24J + 9 = 0$

$$\frac{24 \pm \sqrt{576 - 2304}}{128} = \frac{24 \pm \sqrt{-1728}}{128} = \frac{24}{128} + \frac{\sqrt{-1728}}{128}; \quad \frac{24}{128} - \frac{\sqrt{-1728}}{128};$$

$$\frac{3}{16} + \frac{\sqrt{-1728}}{128};$$
 $\frac{3}{16} - \frac{\sqrt{-1728}}{128};$ $x_1 = 0.512259526$ $x_2 = -0.137259526;$

We observe that the algebraic sum of the roots is: $x_1 + x_2 = 0.375$ and that:

$$\sqrt{\frac{1}{0,375}} = 1,63299316185 \dots$$

Note that $(25625126169)^{1/3} = 2948,1891086...$ value very near to the following charmonium particle:

$$|\underline{\eta_c}(1S)| 2983.4\pm0.5$$

$$7^2 = 49$$
; $27*19 = 513$; $27*7^2*43 = 27*49*43 = 56889$;

$$27 * 31^2 * 67 = 1738449; \quad 27 * 2413^2 * 163 = 25625126169;$$

For J = 1, 3, 30, 165, 20010 we have also:

$$\sqrt[6]{2\sqrt{64J^2 - 24J + 9} - (16J - 3)} = t$$

$$\sqrt[6]{2\sqrt{64 - 24 + 9} - (16 - 3)} = \sqrt[6]{2 \cdot 7 - 13} = 1$$

$$\sqrt[6]{2\sqrt{64 \cdot 9 - 24 \cdot 3 + 9} - (16 \cdot 3 - 3)} = \sqrt[6]{2\sqrt{513} - 45} =$$

$$\sqrt[6]{45,299006611624498183420108841677 - 45} =$$

= 0.81773665470181306492092503966592;

Or:

$$\sqrt[6]{2\sqrt{64 \cdot 900 - 24 \cdot 30 + 9} - (16 \cdot 30 - 3)} = \sqrt[6]{2\sqrt{56889} - 477} = \sqrt[6]{477,02830104722298331495639065536 - 477} = \sqrt[6]{2\sqrt{56889} - 477} = \sqrt[6]{2\sqrt{5689} - 477} = \sqrt[6]{2\sqrt{5689} - 477} = \sqrt[6$$

= 0,55203559829918124633667279829108;

$$\sqrt[6]{2\sqrt{1738449} - 2637} = 0,41514887896093143232307651326475$$

$$\sqrt[6]{2\sqrt{25625126169} - 320157} = 0,18656426483645848306470281669354$$

The sum of the results is:

2.97148539679838422664537716791529

2,971485396

The difference is:

Result

-0.97148539679838422664537716791529

-0.971485396

The algebraic sum between the two results is: 2

 $10^3 (1/\pi * 2,971485396) = 945,853178...$ very near to the mass of proton 938,27231(28)

And, for J = 1, 3, 30, 165, 20010

$$\frac{3\sqrt{3}}{R^6} = \sqrt[2]{8J+3} + \sqrt[2]{2\sqrt{64J^2 - 24J + 9} - 8J + 6} =$$

$$\sqrt[2]{8J+3} + \sqrt[2]{2\sqrt{64J^2 - 24J + 9} - 8J + 6} =$$

3,3166247903553998491149327366707 + 3,4641016151377545870548926830117 =

= 6,7807264054931544361698254196824

$$\sqrt[2]{27} + \sqrt[2]{2\sqrt{64 \cdot 9 - 72 + 9} - 24 + 6} =$$

$$\sqrt[2]{27} + \sqrt[2]{27,299006611624498183420108841677} =$$

= 10,420997550710379532279250221154

$$\sqrt[2]{240+3} + \sqrt[2]{2\sqrt{57600-720+9}-240+6} =$$

= 31,177822266323734711257872601252

$$\sqrt[2]{1320 + 3} + \sqrt[2]{2\sqrt{64 \cdot 165^2 - 24 \cdot 165 + 9} - 1320 + 6} =$$

= 72,746204291996970561556523525202

$$\sqrt[2]{160083} + \sqrt[2]{2\sqrt{64 \cdot 20010^2 - 24 \cdot 20010 + 9} - 8 \cdot 20010 + 6} =$$

= 800,20747314951615853325696603331

The sum of the results is:

Result:

921.3332236640403977745204378006004

921,333223664... an approximation to the mass of the proton 938,27231(28)

and $(921,333223664)^{1/14} = 1,628336104...$

The difference is:

Result:

-907.7717708530540889021807869612356

$$-907,771770853$$
 and $-(907,771770853)^{1/14} = -1,62661228...$

The difference between the two results is: 13,561452811. This value is a good approximation of the energy spectrum of the hydrogen atom which is discrete, and the fundamental level is:

$$E_1 = -rac{E_{ha}}{2} = -13.6 \; eV$$

Now, from:

$$\frac{3\sqrt{3}}{R^6} = \sqrt[2]{8J+3} + \sqrt[2]{2\sqrt{64J^2 - 24J + 9} - 8J + 6}$$

We have:

$$3\sqrt{3} = 5,1961524227066318805823390245176$$

 $R^6 = 5,1961524227066318805823390245176$

6,7807264054931544361698254196824 = 0,76631206;

 $R^6 = 5,1961524227066318805823390245176$

10,420997550710379532279250221154 = 0,498623323;

 $R^6 = 5,1961524227066318805823390245176$

31,177822266323734711257872601252 = 0,166661814;

 $R^6 = 5,1961524227066318805823390245176$

72,746204291996970561556523525202 = 0,0714285023;

 $R^6 = 5,1961524227066318805823390245176$

800,20747314951615853325696603331 = 0,00649350649;

R = 0.95660859082436004061727328369768

R = 0.89048942173962161448423500735182

R = 0.74183277566207698599349771995781

R = 0,64413751080965522991175648552217

R = 0,43192984468327433334089152205382

 $1/R^6 = 1.3049514058280643527912114550305$

 $1/R^6 = 2,0055219117778812765242431309215$

 $1/R^6 = 6,0001747010865968373535163849831$

 $1/R^6 = 14,00001354921311292845069215458$

 $1/R^6 = 154,00000008316000004490640002425$

Note that from $64J^2 - 24J + 9$ we have that (64 * 24 * 9) / 8 = 13824 / 8 = 1728

And
$$154 - 14 + 6 - 2 + 1{,}30 = 145{,}3$$
; $(145{,}3 * 12) - 16 = 1727{,}6$

$$154 - 14 - 6 - 2 - 1.30 = 130.7$$

$$154 + 14 + 6 + 2 + 1,30 = 177,3$$
; $(177,3 * 10) - 48 = 1725$;

And

0,95660859082436004061727328369768 +

0,89048942173962161448423500735182 +

0,74183277566207698599349771995781 +

0,64413751080965522991175648552217 +

0,43192984468327433334089152205382

3,66499814

Result

3.6649981437189882043476540185833

(0.95660859082436004061727328369768 +

0.89048942173962161448423500735182 +

0.74183277566207698599349771995781 +

0.64413751080965522991175648552217 +

0.43192984468327433334089152205382) *(Pi/7)

Result:

1.6448473205325432233594072969452...

$$130.7 + 3.66499814 = 134.36499814$$
; $(134.36499814 * 13) - 18 = 1728.74497582$

(0,76631206 + 0,498623323 + 0,166661814 + 0,0714285023 + 0,00649350649) = 1,50951920579;

1/1,50951920579 = 0,66246258...

 $(0.76631206 + 0.498623323 + 0.166661814 + 0.0714285023 + 0.00649350649)*sqrt((1.085)^18))$

Input interpretation:

$$(0.76631206 + 0.498623323 + 0.166661814 + 0.0714285023 + 0.00649350649)$$

$$\sqrt{1.085^{18}}$$

Result:

3.14562021156455784171215191839771484375 that is a good approximation to π

$$\frac{3\sqrt{3}}{R^6} = \sqrt[2]{8J+3} + \sqrt[2]{2\sqrt{64J^2 - 24J + 9} - 8J + 6}$$

For J = 3

$$\frac{3\sqrt{3}}{R^6} = \sqrt[2]{27} + \sqrt[2]{2\sqrt{64 \cdot 9 - 72 + 9} - 24 + 6} =$$

$$3\sqrt{3} \cdot \frac{128}{64} = \sqrt[2]{27} + \sqrt[2]{27,299006611624498183420108841677} =$$

$$10,3923048454 = 10,4209975507$$

$$\frac{1}{2\pi} \cdot 3\sqrt{3} \cdot \frac{128}{64} = \frac{10,392304845413263761164678049035}{2\pi} = 1,6539866862$$

$$\frac{1}{2\pi} \cdot \sqrt[2]{27} + \sqrt[2]{27,299006611624498183420108841677} =$$

$$=\frac{10,420997550710379532279250221154}{2\pi}=1,658553272144$$

 $1,6539866862 \approx 1,658553272144$

The mean is: 1,656269979172

This result 1,656269 is very near to the fourteenth root of Ramanujan's class invariant 1164,2696 that is 1,65578..., value very near to the mass of proton.

We have further, for J = 1, 3, 30, 165, 20010:

$$\frac{1}{3}\sqrt{1 + \frac{8}{3}J} = 0,6382847 \dots$$

$$\frac{1}{3}\sqrt{1 + \frac{8}{3} \cdot 3} = 1$$

$$\frac{1}{3}\sqrt{1+\frac{8}{3}\cdot 30} = 3$$

$$\frac{1}{3}\sqrt{1 + \frac{8}{3} \cdot 165} = 7$$

$$\frac{1}{3}\sqrt{1+\frac{8}{3}\cdot20010} = 77$$

The sum of the results is:

$$1+3+7+77=88; (88*16)^{1/15}=1,621462255... (88*12)^{1/14}=1,6442808...$$

 $(1408)^{1/15}=1,621462255... (1056)^{1/14}=1,6442808...$

We have also the following two equations:

$$t^2 - 14t - 3 = 0$$
; where $t_1 = 14,2111025509$; $t_2 = -0,2111025509$;

$$t^2 - 26t - 11 = 0$$
; where $t_1 = 26,4164078649$; $t_2 = -0,4164078649$;

we note that the algebraic sum of the two roots is: 14 and 26, where 26 - 14 = 12 and $(12)^{1/5} = 1,64375182951...$

The various results highlighted in blue are good approximations to the electric charge of positron and to the mass of proton.

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the J-function itself, so

$$Z(q) = q^{-1} + 196884q + \dots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is $4\pi=12.57...$ So we are off by just a few percent.

We note that the value that we have obtained 12 is a very good approximation of the value 12,19... that is the black hole entropy obtained from log(196883)

In conclusion, we have the following equation:

$$\frac{e^{\frac{\pi\sqrt{n}}{6}} + 6e^{-\frac{\pi\sqrt{n}}{6}}}{6\sqrt{3}}$$

(2,4766322710964233011331665943154 + 2,4226446816602918287345554659281) / 10,392304845413263761164678049035

The result is:

0.471433144584769818249007360727972738250162080988804214746

0.47143314458476...

 $e^{0.471433144584769818249} = 1,602288860133$

$$e^{(e^{\frac{\pi\sqrt{n}}{6}+6e^{-\frac{\pi\sqrt{n}}{6}})/6\sqrt{3}}} = 1,602288860133$$

value 1,602288 very near to the electric charge of positron.

We have calculate the following integral:

Pi^3/18 * integrate sqrt(((1/((2.4766322710964233011331665943154 + 2.4226446816602918287345554659281)/10.392304845413263761164678049035))) x

$$\frac{\pi^3}{18} \int \sqrt{\frac{1}{\frac{2.4766322710964233011331665943154+2.4226446816602918287345554659281}{10.392304845413263761164678049035}} \; x \; dx$$

Result:

 $1.6725372445167463037334360871954 x^{3/2}$

Indefinite integral assuming all variables are real:

 $0.66901489780669852149337443487817 x^{5/2} + constant$

The result 1,67253724 is very near to the value of the mass of proton.

From: "SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS" MAXIE D. SCHMIDT - https://arxiv.org/abs/1609.02803v2

Corollary 4.7 (Special Values of Ramanujan's φ -Function). For any $k \in \mathbb{R}^+$, the variant of the Ramanujan φ -function, $\varphi\left(e^{-k\pi}\right) \equiv \vartheta_3\left(e^{-k\pi}\right)$, has the integral representation

$$\varphi\left(e^{-k\pi}\right) = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{k\pi} \left(e^{2k\pi} - \cos\left(\sqrt{2\pi kt}\right)\right)}{e^{4k\pi} - 2e^{2k\pi} \cos\left(\sqrt{2\pi kt}\right) + 1} \right] dt. \tag{33}$$

Moreover, the special values of this function corresponding to the particular cases of $k \in \{1, 2, 3, 5\}$ in (33) have the respective integral representations

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_{0}^{\infty} \frac{e^{-t^{2}/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi} \left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi} \cos\left(\sqrt{2\pi}t\right) + 1} \right] dt \qquad (34)$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{2} + 2}}{2} = 1 + \int_{0}^{\infty} \frac{e^{-t^{2}/2}}{\sqrt{2\pi}} \left[\frac{4e^{2\pi} \left(e^{4\pi} - \cos\left(2\sqrt{\pi}t\right)\right)}{e^{8\pi} - 2e^{4\pi} \cos\left(2\sqrt{\pi}t\right) + 1} \right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{3} + 1}}{2^{1/4}3^{3/8}} = 1 + \int_{0}^{\infty} \frac{e^{-t^{2}/2}}{\sqrt{2\pi}} \left[\frac{4e^{3\pi} \left(e^{6\pi} - \cos\left(\sqrt{6\pi}t\right)\right)}{e^{12\pi} - 2e^{6\pi} \cos\left(\sqrt{6\pi}t\right) + 1} \right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{5 + 2\sqrt{5}}}{5^{3/4}} = 1 + \int_{0}^{\infty} \frac{e^{-t^{2}/2}}{\sqrt{2\pi}} \left[\frac{4e^{5\pi} \left(e^{10\pi} - \cos\left(\sqrt{10\pi}t\right)\right)}{e^{20\pi} - 2e^{10\pi} \cos\left(\sqrt{10\pi}t\right) + 1} \right] dt.$$

From the first of (34):

$$\frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}(e^{2\pi} - \cos(\sqrt{2\pi}t))}{e^{4\pi} - 2e^{2\pi}\cos(\sqrt{2\pi}t) + 1} \right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4,44288293815}{3,625609908} = 1,2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1,3313353638}{1,2254167025} = 1,08643481 \dots$$

For the integral, we have calculate as follows:

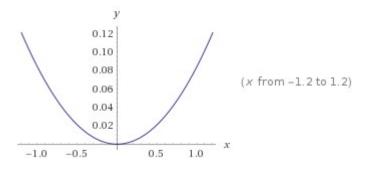
integrate $[(2.71828^{0.89})/(sqrt6.283185307)][4e^{3.14159265}*(e^{6.283185307}-cos((sqrt6.283185307)1.33416))]/[e^{12.56637}-2e^{6.283185307}(cos(sqrt6.283185307)1.33416))+1]x$

Indefinite integral

$$\int \frac{2.71828^{0.89} \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307} \ 1.33416\right)\right)\right) x}{\sqrt{6.283185307} \left(e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)}$$

$$dx = 0.0837798 \ x^2 + \text{constant}$$

Plot of the integral:



Alternate form assuming x is real:

$$0.0837798 x^2 + 0 + constant$$

Thence:
$$1 + 0.0837798 = 1.0837798$$

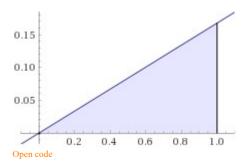
and:

integrate [(2.71828^0.89)/(sqrt6.283185307)][4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)1.33416))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)1.33416))+1] x, [0, 1]

Definite integral:

$$\int_{0}^{1} \frac{2.71828^{0.89} \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307} \right) 1.33416\right)\right) x}{\sqrt{6.283185307} \left(e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)} dx = 0.0837798$$

Visual representation of the integral:



Riemann sums:

left sum
$$0.0837798 - \frac{0.0837798}{n} = 0.0837798 - \frac{0.0837798}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$$

(assuming subintervals of equal length)

Indefinite integral:

$$\int \frac{2.71828^{0.89} \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307} \right) 1.33416\right)\right) x}{\sqrt{6.283185307} \left(e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307}\right) 1.33416\right) + 1\right)}$$

$$dx = 0.0837798 x^2 + \text{constant}$$

Thence: 1 + 0.0837798 = 1.0837798

With regard the integral, from 0 to 0,58438 for t = 2, where $(2.71828^2)/(sqrt6.283185307) = 2,94780$ for t=2, we have:

integrate (2.94780)[4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)2))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)2))+1] x, [0,0.58438]

$$\int_{0}^{0.58438} \frac{2.94780 \left(4 e^{3.14159265} \left(e^{6.283185307} - \cos\left(\sqrt{6.283185307} \ 2\right)\right)\right) x}{e^{12.56637} - \left(2 e^{6.283185307}\right) \left(\cos\left(\sqrt{6.283185307} \ 2\right) + 1\right)} dx = 0.0864364$$

Thence, 1 + 0.0864364 = 1.0864364; $1.08643481 \cong 1.0864364$.

In conclusion, the value of this, defined by us, "New Ramanujan's Constant" is 1.08643.

In this and others our papers, we have used 1,08643 as a new "Ramanujan's constant" and we can see as this constant is fundamental for some results that we have obtained in various equations analyzed and developed.

Search for pair production of higgsinos in final states with at least three b-tagged jets in $\sqrt{s} = 13$ TeV pp collisions using the ATLAS detector

The ATLAS Collaboration

A search for pair production of the supersymmetric partners of the Higgs boson (higgsinos \tilde{H}) in gauge-mediated scenarios is reported. Each higgsino is assumed to decay to a Higgs boson and a gravitino. Two complementary analyses, targeting high- and low-mass signals, are performed to maximize sensitivity. The two analyses utilize LHC pp collision data at a center-of-mass energy $\sqrt{s}=13$ TeV, the former with an integrated luminosity of 36.1 fb⁻¹ and the latter with 24.3 fb⁻¹, collected with the ATLAS detector in 2015 and 2016. The search is performed in events containing missing transverse momentum and several energetic jets, at least three of which must be identified as b-quark jets. No significant excess is found above the predicted background. Limits on the cross-section are set as a function of the mass of the \tilde{H} in simplified models assuming production via mass-degenerate higgsinos decaying to a Higgs boson and a gravitino. Higgsinos with masses between 130 and 230 GeV and between 290 and 880 GeV are excluded at the 95% confidence level. Interpretations of the limits in terms of the branching ratio of the higgsino to a Z boson or a Higgs boson are also presented, and a 45% branching ratio to a Higgs boson is excluded for $m_{\tilde{H}} \approx 400$ GeV.

The signal region (SR) is defined by the requirement

$$X_{hh}^{SR} = \sqrt{\left(\frac{m_{2j}^{\text{lead}} - 120 \text{ GeV}}{0.1 \times m_{2j}^{\text{lead}}}\right)^2 + \left(\frac{m_{2j}^{\text{subl}} - 110 \text{ GeV}}{0.1 \times m_{2j}^{\text{subl}}}\right)^2} < 1.6,$$

where $0.1 \times m_{2j}^{\rm lead}$ and $0.1 \times m_{2j}^{\rm subl}$ represent the approximate mass resolution of the leading and subleading Higgs boson candidates, respectively. The central values for the masses of the Higgs boson candidates of 120 GeV and 110 GeV, as well as the value of the $X_{hh}^{\rm SR}$ cut, were optimized using the data-driven background model described in Section 6.2.2 and simulated signal events.

For
$$m^{lead} = 130$$
 and $m^{subl} = 127$

$$\sqrt{\left(\frac{130 - 120}{13}\right)^2 + \left(\frac{127 - 110}{12.7}\right)^2}$$

1.54387...

To derive the background model and estimate uncertainties in the background prediction, the following regions in the mass plane of the leading and subleading p_T Higgs boson candidates are defined: control region (CR), validation region 1 (VR1) and validation region 2 (VR2), using the variables

$$\begin{split} R_{hh}^{\text{CR}} & \equiv \sqrt{(m_{2j}^{\text{lead}} - 126.0 \text{ GeV})^2 + (m_{2j}^{\text{subl}} - 115.5 \text{ GeV})^2}, \\ X_{hh}^{\text{VR1}} & \equiv \sqrt{\left(\frac{m_{2j}^{\text{lead}} - 96 \text{ GeV}}{0.1 \times m_{2j}^{\text{lead}}}\right)^2 + \left(\frac{m_{2j}^{\text{subl}} - 88 \text{ GeV}}{0.1 \times m_{2j}^{\text{subl}}}\right)^2}, \\ X_{hh}^{\text{VR2}} & \equiv \sqrt{\left(\frac{m_{2j}^{\text{lead}} - 149 \text{ GeV}}{0.1 \times m_{2j}^{\text{lead}}}\right)^2 + \left(\frac{m_{2j}^{\text{subl}} - 137 \text{ GeV}}{0.1 \times m_{2j}^{\text{subl}}}\right)^2}. \end{split}$$

All regions satisfy the same selection criteria as those for the SR, except for the requirement on $X_{hh}^{\rm SR}$. The control region is defined by $R_{hh}^{\rm CR} < 55$ GeV and excludes the SR, $X_{hh}^{\rm SR} > 1.6$. The two validation regions are defined by functional forms similar to that of the SR but are displaced towards lower and higher Higgs boson candidate masses satisfying $X_{hh}^{\rm VR1} < 1.4$ and $X_{hh}^{\rm VR2} < 1.25$, respectively. The CR center (126,115) was set so that the means of the Higgs candidates' mass distributions in the control region are equal to those in the signal region. The VR definitions were optimized to be similar to the SR while retaining sufficient statistical precision to test the background model. The CR and VRs are defined in both the 2-tag and 4-tag samples. Figure 5 shows the distributions of $m_{2j}^{\rm lead}$ versus $m_{2j}^{\rm subl}$ for the 2-tag and the 4-tag data after the event selection with the region definitions superimposed.

Search for electroweak production of supersymmetric states in scenarios with compressed mass spectra at $\sqrt{s} = 13$ TeV with the ATLAS detector

M. Aaboud et al.*
(ATLAS Collaboration)

(Received 21 December 2017; published 27 March 2018)

A search for electroweak production of supersymmetric particles in scenarios with compressed mass spectra in final states with two low-momentum leptons and missing transverse momentum is presented. This search uses proton-proton collision data recorded by the ATLAS detector at the Large Hadron Collider in 2015–2016, corresponding to 36.1 fb⁻¹ of integrated luminosity at $\sqrt{s} = 13$ TeV. Events with same-flavor pairs of electrons or muons with opposite electric charge are selected. The data are found to be consistent with the Standard Model prediction. Results are interpreted using simplified models of *R*-parity-conserving supersymmetry in which there is a small mass difference between the masses of the produced supersymmetric particles and the lightest neutralino. Exclusion limits at 95% confidence level are set on next-to-lightest neutralino masses of up to 145 GeV for Higgsino production and 175 GeV for wino production, and slepton masses of up to 190 GeV for pair production of sleptons. In the compressed mass regime, the exclusion limits extend down to mass splittings of 2.5 GeV for Higgsino production, 2 GeV for wino production, and 1 GeV for slepton production. The results are also interpreted in the context of a radiatively-driven natural supersymmetry model with nonuniversal Higgs boson masses.

For $m^{lead} = 160$ and $m^{subl} = 157,45$

$$\sqrt{(160-126)^2+(157.45-115.5)^2}$$

53.9982...

For $m^{lead} = 103$ and $m^{subl} = 100$

$$\sqrt{\left(\frac{103 - 96}{10.3}\right)^2 + \left(\frac{100 - 88}{10}\right)^2}$$

1.379084...

For $m^{lead} = 157$ and $m^{subl} = 154$

$$\sqrt{\left(\frac{157-149}{15.7}\right)^2 + \left(\frac{154-137}{15.4}\right)^2}$$

1.21583...

Note that, for $m^{lead} = 105$ and $m^{subl} = 102$, we have:

$$\sqrt{\left(\frac{105 - 96}{10.5}\right)^2 + \left(\frac{102 - 88}{10.2}\right)^2}$$

1.61820...

This result 1,61820 is practically the value of the golden ratio 1,61803398...

149 Gev mass = $149 * 9 * 10^{16} = 13410000000000000000$ GeV;

and $1,65578 * 5\phi = 13,395541517022 * 10^{18} = 13395541517022000000 \text{ GeV}$

furthermore: (149 * 12) - 48 - 12 = 1728 (Ramanujna's number 1729 - 1)

From:

DELPHI Collaboration



DELPHI 2000-015 CONF 336 1, March 2000

Search for pair production of supersymmetric particles with R-parity violating LLE couplings at $\sqrt{s}=192~{\rm GeV}$ to 202 GeV

C. Berat, E. Merle ISN Grenoble

Searches for R_p effects in e⁺e⁻ collisions at $\sqrt{s} = 192$ GeV to 202 GeV have been performed with the DELPHI detector. The pair production of neutralinos, charginos and sleptons have been studied under the assumption that the LLE term is responsible for the supersymmetric particle decays into standard particles. No evidence for R-parity violation has been observed, allowing to update the limits previously obtained at $\sqrt{s} = 189$ GeV. A neutralino mass lower than 35.5 GeV/ c^2 and a chargino mass lower than 99 GeV/ c^2 are excluded at 95% C.L

If the sneutrino is the LSP, the present limits are, with $\tan \beta = 1.5$:

- $m_{\tilde{\nu}_e} > 96 \text{ GeV}/c^2 \text{ for } \mu = -150 \text{ GeV}/c^2 \text{and } M_2 = 200 \text{ GeV}/c^2;$
- $m_{\tilde{\nu}_u} > 84 \text{ GeV}/c^2$;
- $m_{\bar{\nu}_{\tau}} > 86 \text{ GeV}/c^2$;

If $\tilde{\chi}_1^0$ is the LSP and the branching fraction $\tilde{\nu}(\tilde{\ell}) \to \nu$ (ℓ) $\tilde{\chi}_1^0$ is equal to 1, taking into account the limit on the neutralino mass at 35.5 GeV/ c^2 , sneutrinos with mass lower than 83 GeV/ c^2 and right-handed sleptons with mass lower than 87 GeV/ c^2 were excluded at 95% C.L.

We have that:

$$m_{\tilde{\nu}_e} > 96 \text{ GeV}/c^2 \text{ for } \mu = -150 \text{ GeV}/c^2 \text{ and } M_2 = 200 \text{ GeV}/c^2$$
;

 $(97200000000000000000)^{1/87} = 1,65290935449971...$ or

 $1164,2696 * \pi \sqrt{7} = 9677,2609156539240463076732725537 * 10^{15} =$

= 9677260915653924046,3

 $(86400000000000000000)^{1/87} = 1,650673113624964$

 $1164.2696 * e^2 = 8602,84181522190464 * 10^{15} = 8602841815221904640 \text{ GeV}$

where 1164.2696 is the Ramanujan's class invariant and 1,6529 1,65067 are very near to the fourteenth root of 1164,2696 and to the mass of proton.

From:

Formulae for Supersymmetry | MSSM and more |

Toru Goto - KEK Theory Center, IPNS, KEK - Tsukuba, Ibaraki, 305-0801 JAPAN Last Modified: March 31, 2019

We have that:

$$E^{(3)} = \left(\frac{1331}{2} - 121n_l + \frac{22}{3}n_l^2 - \frac{4}{27}n_l^3\right) L_{\mu}^3$$

$$+ \left(\frac{4521}{2} - \frac{10955}{24}n_l + \frac{1027}{36}n_l^2 - \frac{5}{9}n_l^3\right) L_{\mu}^2$$

$$+ \left[\frac{247675}{96} + \frac{32087}{108}\pi^2 - \frac{99}{16}\pi^4 + \frac{3025}{2}\zeta_3\right]$$

$$- \left(\frac{166309}{288} + \frac{5095}{162}\pi^2 - \frac{3}{8}\pi^4 + \frac{902}{3}\zeta_3\right) n_l$$

$$+ \left(\frac{10351}{288} + \frac{11}{9}\pi^2 + \frac{158}{9}\zeta_3\right) n_l^2 - \left(\frac{50}{81} + \frac{2}{81}\pi^2 + \frac{8}{27}\zeta_3\right) n_l^3\right] L_{\mu}$$

$$- \frac{865}{18}\pi^2 L_{\alpha_s} + \frac{1267919}{1728} + \left(\frac{14286253}{38880} - \frac{7225}{162}\log 2\right) \pi^2$$

$$- \frac{723119}{51840}\pi^4 + \left(\frac{114917}{48} - \frac{1331}{8}\pi^2\right) \zeta_3 + \frac{3993}{2}\zeta_5 - \frac{128}{81}\pi^2 L_1^E + \frac{a_3}{32}$$

$$- \left[\frac{52033}{288} + \frac{397591}{7776}\pi^2 - \frac{59677}{77760}\pi^4 + \left(\frac{8797}{18} - \frac{121}{4}\pi^2\right)\zeta_3 + 363\zeta_5\right] n_l$$

$$+ \left[\frac{3073}{288} + \frac{905}{432}\pi^2 + \frac{11}{1080}\pi^4 + \left(\frac{3239}{108} - \frac{11}{6}\pi^2\right)\zeta_3 + 22\zeta_5\right] n_l^2$$

$$- \left[\frac{98}{729} + \frac{19}{486}\pi^2 + \frac{1}{4860}\pi^4 + \left(\frac{44}{81} - \frac{1}{27}\pi^2\right)\zeta_3 + \frac{4}{9}\zeta_5\right] n_l^3. \tag{9.1.64c}$$

We have calculated and simplified the above expression. We have obtained:

Input interpretation:

 $551.6851 + 1832.0138 + 4702.7364 + 878.1284 + 310.40515 - 36.52840 + \\ 53.4965 + 12.0628 - 0.669886 + 1208.0387 + 2208.6181376 - 1358.7647 \\ \frac{1}{2} Open code$

Enlarge Data Customize A Plaintext Interactive

10361.2220016

Input interpretation:

752.04873 + 1696.5 - 15.5964118 + 0.03125 - 1163.7182131 + 66.23464189 - 1.1624358988 Open code

Enlarge Data Customize A Plaintext Interactive Result:

1334.3375610912

Input interpretation: 10 361.2220016 + 1334.337561 Open code

Enlarge Data Customize A Plaintext Interactive

11695.5595626

11695,5595626.

Or, for $\alpha_s \approx 5$, multiplying for 5, we obtain: 58477,797813

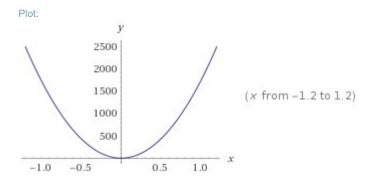
We have integrated the result 11695,5595626:

Pi^3 * 1/(1.6770424^9) integrate [10361.2220016+1334.337561]x

$$\pi^3 \times \frac{1}{1.6770424^9} \int (10\,361.2220016 + 1334.337561) \, x \, dx$$

Result: 1728. x²

The result is the Ramanujan's number 1729 – 1



Alternate form assuming x is real:

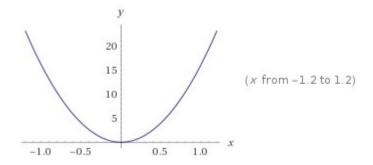
1728.
$$x^2 + 0$$

Indefinite integral assuming all variables are real:

Pi^3/(27*4) * 1/(1.6770424^9) integrate [10361.2220016+1334.337561]x

$$\frac{\pi^3}{27 \times 4} \times \frac{1}{1.6770424^9} \int (10\,361.2220016 + 1334.337561) \, x \, dx$$

Plot:



Alternate form assuming x is real:

$$16. x^2 + 0$$

Indefinite integral assuming all variables are real:

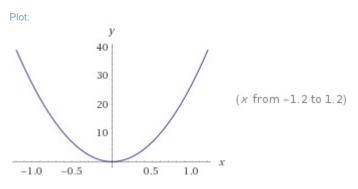
Note that $(5,33334)^{1/21} = 1,08297645043...$ very near to the Ramanujan's new constant.

And

Pi^3/(64) * 1/(1.6770424^9) integrate [10361.2220016+1334.337561]x

$$\frac{\pi^3}{64} \times \frac{1}{1.6770424^9} \int (10361.2220016 + 1334.337561) x dx$$

Result: $27. x^2$



Alternate form assuming x is real:

$$27. x^2 + 0$$

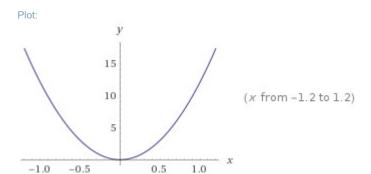
Indefinite integral assuming all variables are real:

$$9.00001 \, x^3 + \text{constant}$$

Pi^3/(8) * 1/(2*9) * 1/(1.6770424^9) integrate [10361.2220016+1334.337561]x

$$\frac{\pi^3}{8} \times \frac{1}{2 \times 9} \times \frac{1}{1.6770424^{\circ}} \int (10361.2220016 + 1334.337561) x \, dx$$

Result: $12. x^2$



Alternate form assuming x is real:

$$12. x^2 + 0$$

Indefinite integral assuming all variables are real:

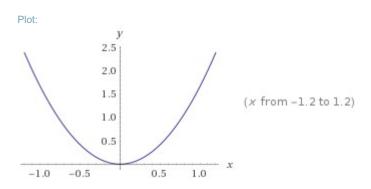
$$4. x^3 + constant$$

The result 12 is a good approximation to the value the black hole entropy (12,19)

6.620/(1728*4) * Pi^3 * 1/(1.6770424^9) integrate [10361.2220016+1334.337561]x

$$\frac{6.62}{1728 \times 4} \, \pi^3 \times \frac{1}{1.6770424^9} \int (10\,361.2220016 + 1334.337561) \, x \, dx$$

Result: $1.655 x^2$



Alternate form assuming x is real:

$$1.655 x^2 + 0$$

Indefinite integral assuming all variables are real:

$$0.551667 x^3 + constant$$

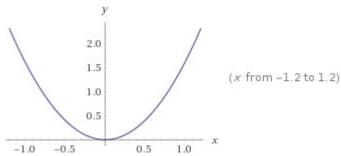
6.58/(1723*4) * Pi^3 * 1/(27*4) integrate [10361.22+1334.337]x

$$\frac{6.58}{1723 \times 4} \, \pi^3 \times \frac{1}{27 \times 4} \, \int (10\,361.22 + 1334.337) \, x \, dx$$

Result:

 $1.60287 x^2$

Plot:



Alternate form assuming x is real:

$$1.60287 x^2 + 0$$

Indefinite integral assuming all variables are real:

The results 1.655 and 1.602 are very near to the fourteenth root of Ramanujan's class invariant 1164.2696 and to the mass of proton and the electric charge of positron.

We have, for $n_1 = 5$, that:

$$E^{(1)} = \left(11 - \frac{2}{3}n_l\right)L_{\mu} + \frac{97}{6} - \frac{11}{9}n_l$$

And obtain: 17,72.... or, for $n_1 = 1$: 25,27

And

$$E^{(2)} = \left(\frac{363}{4} - 11n_l + \frac{1}{3}n_l^2\right) L_{\mu}^2 + \left(\frac{927}{4} - \frac{193}{6}n_l + n_l^2\right) L_{\mu}$$

$$+ \frac{1793}{12} + \frac{2917}{216}\pi^2 - \frac{9}{32}\pi^4 + \frac{275}{4}\zeta_3$$

$$- \left(\frac{1693}{72} + \frac{11}{18}\pi^2 + \frac{19}{2}\zeta_3\right) n_l + \left(\frac{77}{108} + \frac{1}{54}\pi^2 + \frac{2}{9}\zeta_3\right) n_l^2,$$

And obtain: 512833.4435 / 1728 = 296,778034 or, for $n_1 = 1$:

979421.79734 / 1728 = 566,795

We have that:

9.1.5 1S quarkonium mass

The mass of a 1S quarkonium is decomposed into perturbative and nonperturbative contributions

$$M(1S) = 2m_{q,OS} + \Delta E^{p} + \Delta E^{np}$$
 (9.1.57)

The perturbative correction $\Delta E^{\rm p}$ is given in N³LO as follows [1014] (see also [1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023]):

$$\Delta E^{\rm p} = -\frac{C_F^2 \alpha_s^2 m_{q, \rm OS}}{4} \left\{ 1 + \frac{\alpha_s}{\pi} E^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 E^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 E^{(3)} + \cdots \right\}, \qquad (9.1.58a)$$

For $\alpha_s = 5{,}13$ we have that:

$$\left\{1 + \frac{\alpha_s}{\pi} E^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 E^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 E^{(3)} + \cdots\right\}$$

= 316959,3707073....

And

$$\Delta E^{\rm p} = -\frac{C_F^2 \alpha_s^2 m_{q,\rm OS}}{4} \left\{ 1 + \frac{\alpha_s}{\pi} E^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 E^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 E^{(3)} + \cdots \right\}$$

= 2085347,015742...

For $\alpha_s = 1$, and $n_l = 1$, we have that:

443,671771948.... and 110,91794298.... or, for $\alpha_s = 5,13$ and the result of $E^{(3)} = 11695,5595626$:

52477,714003.... and 345262,68791....

We have also that:

Nonperturbative contribution [1032, 1033]

$$\Delta E^{\rm np} \ = \ \frac{\pi^2 m_q}{(C_F \alpha_s m_q)^4} \frac{624}{425} \left<0\right| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \left|0\right> \,, \label{eq:deltaEnp}$$

where the gluon condensate is evaluated as [1034, 1035, 1036]

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \right| 0 \right\rangle \approx 0.012 \,\mathrm{GeV}^4$$
.

The mass of a 1S quarkonium is:

$$M(1S) = 2m_{a,OS} + \Delta E^p + \Delta E^{np}$$

 $\Delta E^{np} = 2,5107715019136191675645344294841e-4$

Thence: M(1S) = 2085349,0159 or 345264,688161 or 113,0918338

We have that:

At this stage, all the factors, which are required for the evaluation of the quark masses at $(QCD, \overline{MS}, n_f = 6, \mu = \mu_W)$, are determined, as well as some byproducts such as the quark masses in various schemes, and low energy values of α_s . The quark masses $m_q(QCD, \overline{MS}, n_f = 6, \mu = \mu_W)$ are written as:

$$m_t^{(6)}(\mu_W) = \frac{m_q^{(6)}(\mu_W)}{m_q^{(6)}(\mu_t)} m_t^{(6)}(\mu_t),$$
 (3.2.2a)

$$m_b^{(6)}(\mu_W) = \frac{m_q^{(6)}(\mu_W)}{m_q^{(5)}(\mu_W)} \frac{m_q^{(5)}(\mu_W)}{m_q^{(5)}(\mu_b)} m_b^{(5)}(\mu_b),$$
 (3.2.2b)

$$m_c^{(6)}(\mu_W) = \frac{m_q^{(6)}(\mu_W)}{m_q^{(5)}(\mu_W)} \frac{m_q^{(5)}(\mu_W)}{m_q^{(5)}(\mu_b)} \frac{m_q^{(5)}(\mu_b)}{m_q^{(4)}(\mu_b)} \frac{m_q^{(4)}(\mu_b)}{m_q^{(4)}(\mu_c)} m_c^{(4)}(\mu_c), \qquad (3.2.2c)$$

$$m_{d,u,s}^{(6)}(\mu_W) = \frac{m_q^{(6)}(\mu_W)}{m_q^{(5)}(\mu_W)} \frac{m_q^{(5)}(\mu_W)}{m_q^{(5)}(\mu_b)} \frac{m_q^{(5)}(\mu_b)}{m_q^{(4)}(\mu_b)} \frac{m_q^{(4)}(\mu_b)}{m_q^{(4)}(\mu_L)} m_{d,u,s}^{(4)}(\mu_L) \quad \text{for } \mu_L > \mu_c. \quad (3.2.2d)$$

 $m_q(\text{QCD}, \overline{\text{DR}}, n_f = 6, \mu_W)$ are obtained by (9.1.92a) with $n_f = 6$.

9.1.6 $\overline{\rm DR}$ scheme

The leading order relation between the coupling constants in $\overline{\text{MS}}$ and $\overline{\text{DR}}$ schemes is given as [772, 1043]:

$$a_{\overline{DR}}(\mu)^{-1} = a_{\overline{MS}}(\mu)^{-1} - 1.$$
 (9.1.90)

The $O(\alpha_s^2)$ relation between the pole mass and the $\overline{\rm DR}$ running mass is given in Ref. [1001]:

$$\frac{m_{\text{pole}}}{m_{\overline{\text{DR}}}(\mu)} = 1 + a_{\overline{\text{DR}}}(\mu) \left[\frac{20}{3} - 4L_{\overline{\text{DR}}} \right]
+ a_{\overline{\text{DR}}}^2(\mu) \left[\frac{3043}{18} + \frac{32}{3} (2 + \log 2)\zeta_2 - \frac{8}{3}\zeta_3 - \left(\frac{74}{9} + \frac{16}{3}\zeta_2 \right) n_f + \frac{64}{3} \sum_{j=1}^{n_f} \Delta \left(\frac{m_j}{m} \right) \right]
- \left(\frac{350}{3} - \frac{52}{9} n_f \right) L_{\overline{\text{DR}}} + \left(30 - \frac{4}{3} n_f \right) L_{\overline{\text{DR}}}^2 \right],$$
(9.1.91)

where $L_{\overline{DR}} = \log \left(m_{\overline{DR}}^2(\mu)/\mu^2 \right) = L_{\overline{MS}} - \frac{8}{3}a(\mu) + O(a^2)$. The relation between the \overline{MS} and the \overline{DR} masses is derived from (9.1.21) and (9.1.91) as [1001, 1044]:

$$\frac{m_{\overline{\rm DR}}(\mu)}{m_{\overline{\rm MS}}(\mu)} = 1 - \frac{4}{3} a_{\overline{\rm MS}}(\mu) + \left(-\frac{73}{9} + \frac{n_f}{3}\right) a_{\overline{\rm MS}}^2(\mu) + O(a^3) \qquad (9.1.92a)$$

$$= 1 - \frac{4}{3} a_{\overline{\rm DR}}(\mu) + \left(-\frac{61}{9} + \frac{n_f}{3}\right) a_{\overline{\rm DR}}^2(\mu) + O(a^3) . \qquad (9.1.92b)$$

For $n_f = 6$, we calculate m_q :

$$1 - \frac{4}{3} + \left(-\frac{73}{9} + \frac{6}{3}\right) = -\frac{58}{9} = -6, \overline{4} = -6,444 \dots$$

From:

$$\Delta E^{\rm p} = -\frac{C_F^2 \alpha_s^2 m_{q, \rm OS}}{4} \left\{ 1 + \frac{\alpha_s}{\pi} E^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 E^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 E^{(3)} + \cdots \right\}$$

$$\Delta E^{\rm np} = \frac{\pi^2 m_q}{(C_F \alpha_s m_q)^4} \frac{624}{425} \left\langle 0 \middle| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \middle| 0 \right\rangle$$

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \right| 0 \right\rangle \approx 0.012 \,\text{GeV}^4$$

We obtain:

-(-6,4 / 4) * 443,671771948 = 709,8748351168; 472,98302563428555096212907908526 / 713031,68 = 6,63340828e-4

Thence:

$$M(1S) = 2m_{q,OS} + \Delta E^{p} + \Delta E^{np}$$

$$M(1S) = 2 * -6.4 + 6.63340828 * 10^{-5} + 709,8748351168 = -722,67908$$

For the value: 52477,714003 (for $\alpha_s = 5,13$), we obtain:

$$-(-6.4 * 5.13^2 / 4) * 52477,714003 = 42,10704 * 52477,71403 = 2209681,203769;$$

$$M(1S) = 2 * -6.4 + 6.63340828 * 10^{-5} + 2209681,203769 = -2209694,004;$$

$$-2209694 / 1278 = -1729,025; 1278 = 142 * 9; (142 * 12) + 24 = 1728$$

Note that 2209694,004 / 1728 * 100 = 12,7875810416666...

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the *J*-function itself, so

$$Z(q) = q^{-1} + 196884q + \dots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is $4\pi=12.57...$ So we are off by just a few percent.

We note that the value that we have obtained 12,7875... is a very good approximation of the value 12,57... that is the classical entropy of a black hole with k = 1 and mass 2

From:

Breaking SU(3) Symmetry and Baryon Masses

Kai-Wai Wong, Gisela A. M. Dreschhoff, Högne J. N. Jungner

Department of Physics and Astronomy, University of Kansas, Lawrence, USA Radiocarbon Dating Lab, University of Helsinki, Helsinki, Finland Email: kww88ng@gmail.com - Received 16 July 2015; accepted 13 September 2015; published 16 September 2015

In the recent papers [3]-[6] we have shown how the meson masses are calculated, including the pion gluon potential pairs of intermediate quark currents, u or t and d. We explicitly obtained $U(\pi)$ as 121 MeV. Similarly, the proton gluon U can also be calculated with the gauge loop parameter r_0 already determined [4] [6], and get U(p) = 934.6 MeV instead of the number fitting U(p) = 928 MeV we gave in Ref. [1]. It was also shown in ref. [4]-[6] that the inter quark interactions within hadrons are divided into 2 body for mesons and 3 body for baryons. The 3-body problem obeys the equilateral structure, meaning that all 3 pairs of relative distances are equal.

From this paper we observe that the proton gluon U can be of value in a range of 928-934.6 MeV

Note that:

From:

http://quantumpulse.com/page1.php - Physics Beyond the Standard Model

```
      u = mass of up quark
      u = mass of up quark

      d = mass of down quark
      d = mass of down quark

      u = 2.2431 (46) MeV
      u = 2.15 (15) MeV

      d = 4.8310 (46) MeV
      d = 4.70 (20) MeV

      \frac{u}{d} = .4644 (14)
      \frac{u}{d} = .46 (5)

      d - u = 2.5867 (92) MeV
      d - u = 2.55 (35) MeV
```

¹2012 Particle Data Group Update-Lawrence Berkeley Nation Laboratory A.V. Manohar (University of California, San Diego) and C.T. Sachrajda (University of Southampton)

We note that the mass of up quark is very near to the result of the expression, i.e. 2209694 (2,209694)

We calculate the following integral:

(Pi^3/(1.65578)^6) * integrate [2209694]x where 1,65578 is the fourteenth root of the following Ramanujan's class invariant:

$$\sqrt[14]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,65578 \dots$$

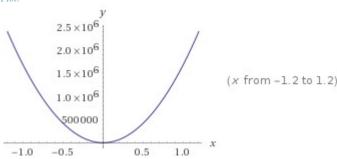
we obtain:

$$\frac{\pi^3}{1.65578^6} \int 2209694 \, x \, dx$$

Result

$$1.6624 \times 10^6 x^2$$

Plot:

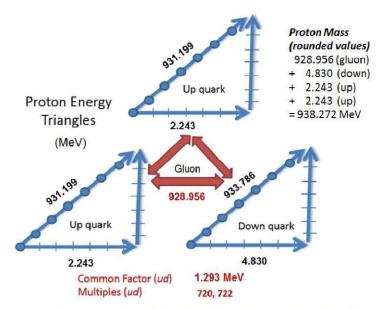


The result 1,6624 is very near to the 1.65578 and to the mass of proton.

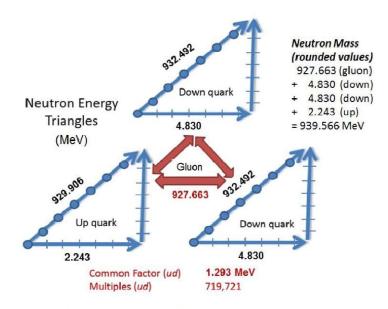
Now:

The gluon energy in the proton and neutron is shared between quarks; therefore the total energy in each quark is simply the gluon energy plus the quark mass. In the proton (see figure below), the total energy of the up quark equals its mass (2.243 MeV) plus the proton gluon energy (928.956 MeV) for a total of 931.199 MeV. The total energy of the down quark is 933.786 MeV (mass 4.830 MeV + gluon 928.956 MeV). A wide search range (0.1-35 MeV) of possible quark mass values were tested in order to find the highest common factor between all quark total energies using this simple model.

The highest common factor was 1.29333217 MeV, which is exactly equal to the mass difference between the neutron and the proton meaning this is also the highest possible common factor. The multiples of this factor in the proton are 720 in the up quark (720 X 1.29333217 = 931.199 MeV) and 722 in the down quark (722 X 1.29333217 = 933.706 MeV).



The quark mass search was concurrently run using the neutron model (see below). Again, the highest common factor was again 1.29333217 MeV, the difference in mass energy between the neutron and proton. The multiples of this factor in the neutron are 719 in the up quark (719 X 1.29333217 - 929.906 MeV) and 721 in the down quark (721 X 1.29333217 - 932.492 MeV).



While concurrently searching for high common factors between the proton and neutron quark energy wavelengths, often times there was found a high common factor in one composite particle and not the other. Not only did the highest possible common factor (1.29333 MeV) occur in one particle, it occurred in both at a up-down quark mass difference value within QCD predicted values (2.5867MeV).

We have that:

Proton mass: 938,27231 MeV/c²

Neutron mass: 939,56564217 MeV/c²

Difference: 939,56564217 - 938,27231 = 1,29333217;

Now:

$$720 * 1,29333217 = 931,1991624 = 931,199$$

In this computation the gluon value is 928,956. Adding two time the value of quark up and the value of quark down, we have:

$$928,956 + 4,830 + 2,243 + 2,243 = 938,272 \text{ MeV}$$

And:

Proton mass: 938,27231 MeV/c²

Neutron mass: 939,56564217 MeV/c²

Difference: 939,56564217 - 938,27231 = 1,29333217;

Now:

In this computation the gluon value is 927,663. Adding two time the value of quark down and the value of quark up, we have:

$$927,663 + 4,830 + 4,830 + 2,243 = 939,566 \text{ MeV}$$

To summarize, the proton and neutron are composite particles that form at the exact energy which creates the maximum possible common factor between their total quark energies (mass plus kinetic energy). These multiples range from 719 to 722 X the mass difference between the neutron and the proton (1.29333MeV). This occurred at a down-up mass difference of 2.5866MeV, right where QCD predicted it should be.

We note that the values 719, 720, 721 and 722 are very near to the value 728. Indeed:

$$719 + 9 = 728$$
; $720 + 8 = 728$; $721 + 7 = 728$; $722 + 6 = 728$; note that:

6+7+8+9=30=24+6 where 24 and 6 are divisible for 1728.

From:

http://quantumpulse.com/page1.php - Physics Beyond the Standard Model

With regard the usual Ramanujan invariant class:

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164,2696$$

and the numbers 728 and 1728, it is possible to obtain some interesting mathematical connections with various values of particles' masses. We have the following gluon level:

$$U(d) = U_0 + 4E_0 = 934.6 + 178 = 1112.6 \text{ MeV}.$$

$$1112 = 1728 - 576 - 36 - 4$$
;

Should U(d) represent the gluon potential for Λ^0 , then it's mass is given by (see Ch. 8 of Ref. [1] for more details on the hadron mass formula)

$$M(\Lambda^0) = \{1112.6^2 + 86^2\}^{0.5} = 1115.9 \text{ MeV}.$$
 (2.4)

1115.9 = 1728 - 576 - 36 = 1116;

$$U(s) = 1183 \text{ MeV}.$$
 (2.7)

Finally, we get

$$M(\Sigma^{0}) = \{1183^{2} + 146.9^{2}\}^{0.5} = 1192.5 \text{ MeV}.$$
 (2.8)

$$1183 = 1728 - 288 - 144 - 72 - 32 - 9$$
;

$$1192 = 1728 - 288 - 144 - 72 - 32$$
:

$$M(\Sigma^+) = \{1183^2 + 118^2\}^{0.5} = 1189 \text{ MeV}.$$

$$1189 = 1728 - 288 - 144 - 72 - 32 - 3$$
:

$$\begin{split} \mathbf{M}(\Sigma^-) &= \left\{ 1188.9^2 + 98.2^2 \right\}^{0.5} = 1192.9 \, \text{MeV}. \\ 1193 &= 1728 - 288 - 144 - 64 - 36 - 3; \\ \mathbf{U}(\Xi^0) &= 44.5 \left\{ 16f + 2 \times 4f^2 + 2f^2 + 1 \right\} = 1301 \, \text{MeV} \\ 1301 &= 1728 - 288 - 108 - 27 - 4; \\ 1301 &= 1164 + 108 + 27 + 2; \\ \mathbf{U}(\Xi^-) &= 44.5 \left\{ 16f + 2 \times 4f^2 + 2f^2 + 1 + 3 \times (f - 1)^2 \right\} = 1312.2 \, \text{MeV} \\ 1312 &= 1728 - 288 - 64 - 36 - 16 - 12; \\ 1312 &= 1164 + 144 + 4; \\ \mathbf{M}(\Xi^-) &= \left\{ 1312.9^2 + 139.8^2 \right\}^{0.5} = 1320.4 \, \text{MeV}. \\ 1320 &= 1728 - 288 - 64 - 54 - 2; \\ 1320 &= 1164 + 144 + 12; \\ \mathbf{M}(\Omega^-) &= 1672 \, \text{MeV}. \\ 1672 &= 1728 - 54 - 2; \\ 1672 &= 1164 + 288 + 144 + 64 + 12; \\ \mathbf{M}(\Lambda) &= \left\{ 2248^2 + 111^2 \right\}^{0.5} = 2257 \, \text{MeV}. \\ 2257 &= 1164 + 576 + 288 + 108 + 54 + 64 + 3; \\ 2257 &= 1164 + 728 + 288 + 54 + 16 + 4 + 3; \\ \mathbf{U}(\mathbf{c}) &= 2.0945 \times 1112.6 + (1 - 2.0945) \times 44.5 = 2330.3 - 48.7 = 2281.6 \, \text{MeV}. \\ 2282 &= 1164 + 728 + 288 + 64 + 36 + 2; \\ \end{split}$$

Now, from: Formulae for Supersymmetry | MSSM and more | Toru Goto - KEK Theory Center, IPNS, KEK - Tsukuba, Ibaraki, 305-0801 JAPAN - Last Modified: March 31, 2019

we have:

$$\begin{split} \Delta_{\overline{\text{MS}}.4}^{(0)} &= \frac{291716893}{6123600} - \frac{2362581983}{87091200} \zeta_3 - \frac{76940219}{2177280} \zeta_4 + \frac{1389}{256} \zeta_5 \\ &+ \frac{3031309}{54432} \widetilde{a}_4 + \frac{121}{36} \widetilde{a}_5 - \frac{151369}{2177280} X_0 \\ &= \frac{134805853579559}{43342154956800} - \frac{18233772727}{783820800} \zeta_3 - \frac{254709337}{8709120} \zeta_4 + \frac{4330717}{207360} \zeta_5 \\ &+ \frac{9869857}{272160} \widetilde{a}_4 - \frac{121}{36} \widetilde{a}_5 - \frac{151369}{11612160} \widetilde{T}_{62,2} + \frac{82037}{30965760} T_{54,3}, \end{split} \tag{9.1.28b}$$

$$\widetilde{a}_4 = \text{Li}_4(\frac{1}{2}) + \frac{1}{24}(\log 2)^4 - \frac{1}{4}\zeta_2(\log 2)^2,$$
(9.1.29a)

$$\widetilde{a}_5 = \operatorname{Li}_5(\frac{1}{2}) - \frac{1}{120}(\log 2)^5 + \frac{1}{12}\zeta_2(\log 2)^3 + \frac{17}{16}\zeta_4\log 2,$$
(9.1.29b)

$$\tilde{T}_{62,2} = T_{62,2} - \frac{64}{3}\zeta_3^2 + 360\zeta_6.$$
 (9.1.29c)

The expressions (9.1.28a) and (9.1.28b) are found in Ref. [1010] and [1003], respectively. The constants X_0 , $T_{62,2}$ and $T_{54,3}$ are numerically obtained [1011, 1012]:

$$X_0 = 1.80887954620833474$$
, (9.1.30a)

$$T_{62,2} = -4553.4004372195263$$
, $T_{54,3} = -8445.8046390310298$. (9.1.30b)

Also a relation among X_0 (= $T_{91,0}$ in the notation in [1003, 1012]), $T_{62,2}$ and $T_{54,3}$ is available in Ref. [1012]:

$$X_0 = \frac{5511907345}{7962624} - \frac{4103}{36}\zeta_3 + \frac{89}{4}\zeta_4 - \frac{273}{2}\zeta_5 + 176\tilde{a}_4 - \frac{9}{256}T_{54,3} + \frac{3}{16}\tilde{T}_{62,2}. (9.1.31)$$

 $T_{54,3}$ is solved with use of (9.1.28a), (9.1.28b) and (9.1.31) as

$$T_{54,3} = -\frac{4908181487}{279936} + \frac{1602496}{81}\zeta_3 - \frac{335104}{9}\zeta_4 - \frac{87296}{3}\zeta_5 + \frac{315392}{9}\widetilde{a}_4 + 32768\widetilde{a}_5.(9.1.32)$$

Substituting (9.1.32), one obtains alternative expressions

$$\Delta_{\overline{\text{MS}},4}^{(0)} = -\frac{4852990063}{111974400} + \frac{2538746237}{87091200} \zeta_3 - \frac{1113800801}{8709120} \zeta_4 - \frac{27194483}{483840} \zeta_5 + \frac{35137253}{272160} \widetilde{\alpha}_4 + \frac{315443}{3780} \widetilde{\alpha}_5 - \frac{151369}{11612160} \widetilde{T}_{62,2}, \qquad (9.1.33)$$

$$X_0 = \frac{10469}{8} - \frac{1619}{2}\zeta_3 + \frac{5325}{4}\zeta_4 + \frac{1773}{2}\zeta_5 - 1056\widetilde{a}_4 - 1152\widetilde{a}_5 + \frac{3}{16}\widetilde{T}_{62,2}.(9.1.34)$$

We have that:

$$X_0 = \frac{5511907345}{7962624} - \frac{4103}{36}\zeta_3 + \frac{89}{4}\zeta_4 - \frac{273}{2}\zeta_5 + 176\,\widetilde{a}_4 - \frac{9}{256}T_{54,3} + \frac{3}{16}\widetilde{T}_{62,2}$$

is equal to -47,232625 for a = 0,4082;

(-47,232625 * 37) - 18 = 1747,607125 - 18 = 1729,607125

$$\Delta_{\overline{\rm MS},4}^{(0)} \ = \ -\frac{4852990063}{111974400} + \frac{2538746237}{87091200} \zeta_3 - \frac{1113800801}{8709120} \zeta_4 - \frac{27194483}{483840} \zeta_5 \\ + \frac{35137253}{272160} \widetilde{a}_4 + \frac{315443}{3780} \widetilde{a}_5 - \frac{151369}{11612160} \widetilde{T}_{62,2} \,,$$

is equal to -58,8742714; (-58,8742714 * 30) - 36 = 1766,228 - 36 = 1730,228; Note that 1728 is divisible for 48 and 54.

$$\Delta_{\text{OS,4}}^{(0)} = \Delta_{\overline{\text{MS}},4}^{(0)} - \frac{7478339}{139968} - \left[\frac{697121}{19440} - \frac{1027}{162} \log 2 + \frac{11}{9} (\log 2)^2 \right] \zeta_2
- \left[\frac{341}{648} - \frac{1439}{216} \zeta_2 \right] \zeta_3 + \frac{3475}{1296} \zeta_4 - \frac{1975}{648} \zeta_5 + \frac{220}{81} \widetilde{a}_4$$

$$= -\frac{141841753}{24494400} - \left[\frac{697121}{19440} - \frac{1027}{162} \log 2 + \frac{11}{9} (\log 2)^2 \right] \zeta_2
- \left[\frac{2408412383}{8709120} - \frac{1439}{216} \zeta_2 \right] \zeta_3 - \frac{71102219}{2177280} \zeta_4 + \frac{49309}{20736} \zeta_5$$
(9.1.39a)

$$+\frac{3179149}{54432}\tilde{a}_4 + \frac{121}{36}\tilde{a}_5 - \frac{151369}{2177280}X_0. \tag{9.1.39b}$$

That is equal to:

$$-58,514996647845 - 319,36083504 - 4,38127424 = -382,257106$$

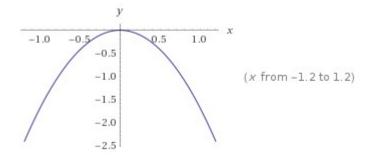
We have the following integral:

1/48 * (728)/1728 integrate [- 382.257106]x

$$\frac{1}{48} \times \frac{728}{1728} \int -382.257106 \, x \, dx$$

Result:
$$-1.67754 \, x^2$$

Plot:



The result -1.67754 is very near to the value of the mass of the neutron with minus sign (antineutron)

We now note that:

8 integrate
$$[1/(((1+x^{(1728))})(1+x^{2}))]$$
 [0, 1]

Input

$$8\int_0^1 \frac{1}{(1+x^{1728})(1+x^2)} \, dx$$

Computation result:

$$8\int_0^1 \frac{1}{(1+x^{1728})(1+x^2)} dx = 6.28319$$

Decimal approximation:

6.281580800516077977125464725985681029862111334280455979192...

and

$$(64*3^2)*1/(6Pi)$$
 integrate $[1/(((1+x^{(1728))}(1+x^2)))]$ $[0, 1]$

$$(64 \times 3^2) \times \frac{1}{6\pi} \int_0^1 \frac{1}{(1+x^{1728})(1+x^2)} dx$$

$$\frac{64 \times 3^2}{6 \pi} \int_0^1 \frac{1}{\left(1 + x^{1728}\right) \left(1 + x^2\right)} \, dx = 24.$$

The result 6,28158.... is practically the length of a circle or radius equal to 1, while 24 is connected with the dimension of bosonic string (D - 2 = 26 - 2 = 24 that are the physical degrees of freedom of the bosonic string).

From:

Heterotic String

David J. Gross, Jeffrey A. Harvey, Emil Martinec, and Ryan Rohm Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544 (Received 21 November 1984)

A new type of superstring theory is constructed as a chiral combination of the closed D-26 bosonic and D=10 fermionic strings. The theory is supersymmetric, Lorentz invariant, and free of tachyons. Consistency requires the gauge group to be Spin(32)/ Z_2 or $E_8 \times E_8$.

The construction of the heterotic string is based on the observation that the states of the first quantized type-II closed strings, fermionic or bosonic, are essentially direct products of left- and right-moving modes. The physical degrees of freedom of the bosonic string are the 24 transverse coordinates $X^{i}(\tau - \sigma)$ and $\tilde{X}^{l}(\tau + \sigma)$ which describe right- (left-) moving twodimensional free fields, with periodic boundary conditions on the circle $0 \le \sigma \le \pi$. The fermionic string contains eight transverse coordinates as well as eight right- and left-moving two-dimensional real fermions, $S^{a}(\tau - \sigma)$ and $\tilde{S}^{a}(\tau + \sigma)$ (a = 1, ..., 8) which are Majorana-Weyl ten-dimensional light-cone spinors.1 The right- and left-handed components of the string are tied together by the constraint that the total momentum and position of each component be identical. Thus the bosonic coordinates are given by the operators (we choose units in which the slope parame-

Now, with 0,527 that is 1/5 of 2,634547 (the Vertex solid angle of Icosahedron), thence 0,5269094 we calculate the following integrals:

$$0.527 * integrate 4 * [1/(((1+x^{24494400/1728)))(1+x^2))] [0, 1]$$

$$0.527 \int_0^1 4 \times \frac{1}{\left(1 + x^{24494400/1728}\right) \left(1 + x^2\right)} \, dx$$

$$0.527 \int_0^1 \frac{4}{\left(1 + x^{24494400/1728}\right)\left(1 + x^2\right)} \, dx = 1.65562$$

Result:

1.655567788609469639295445753091313466816342693599461489257...

$$0.527 * integrate 4 * [1/(((1+x^{(8709120/1728))})(1+x^{2}))] [0, 1]$$

$$0.527 \int_0^1 4 \times \frac{1}{\left(1 + x^{8709120/1728}\right) \left(1 + x^2\right)} \, dx$$

$$0.527 \int_0^1 \frac{4}{\left(1 + x^{8709120/1728}\right) \left(1 + x^2\right)} \, dx = 1.65562$$

Result:

1.655474372669816656712990119960242818134017862946834713495...

$$0.527 * integrate 4 * [1/(((1+x^{2177280/1728)))(1+x^2))] [0, 1]$$

Input:

$$0.527 \int_0^1 4 \times \frac{1}{\left(1 + x^{2177280/1728}\right) \left(1 + x^2\right)} dx$$

Computation result:

$$0.527 \int_0^1 \frac{4}{\left(1+x^{2177280/1728}\right)\left(1+x^2\right)} \, dx = 1.65562$$

Result

1.655039505799136616108981534605007078142164652554115100193...

Where 24494400, 8709120 and 2177280 are all divisible for 1728:

The three results 1.655 are practically equal to fourteenth root of Ramanujan's class invariant and very near to the mass of proton.

$$d_{\overline{MS},4}^{\prime(0)} = \frac{484}{27} - 256\Delta_{\overline{MS},4}^{(0)} + \left[\frac{4770941}{8748} - \frac{3645913}{3888}\zeta_3 + \frac{541549}{648}\zeta_4 - \frac{460}{9}\zeta_5 - \frac{2740}{81}\tilde{a}_4\right]n_l + \left[\frac{271883}{17496} - \frac{668}{81}\zeta_3\right]n_l^2, \qquad (9.1.41e)$$

That is equal to

$$\begin{aligned} &\frac{484}{27} - 256 \times (-58.8742714) + \\ &\left(\frac{4770941}{8748} - \frac{3645913 \times 1.2025}{3888} + \frac{541549 \times 1.0823}{648} - \frac{460 \times 1.0362}{9} - \\ &\frac{2740 \times 0.4082}{81}\right) \times 5 + \left(\frac{271883}{17496} - \frac{668 \times 1.20205}{81}\right) \times 25 \end{aligned}$$

Result:

16507.81833198081847279378143575674439871970736168267032464...

16507,8183

We have that:

sqrt((Pi^2/(492*288))*1/(1728)) integrate [[484/27-256*(-58.8742714)+[4770941/8748-(3645913*1.2025)/3888+(541549*1.0823)/648-(460*1.0362)/9-(2740*0.4082)/81]*5+[271883/17496-(688*1.20205)/81]*25]]x

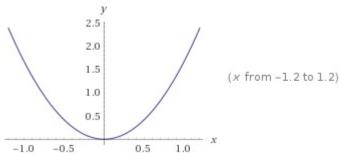
$$\sqrt{\frac{\pi^2}{492 \times 288} \times \frac{1}{1728}}$$

$$\int \left(\frac{484}{27} - 256 \times (-58.8742714) + \left(\frac{4770941}{8748} - \frac{3645913 \times 1.2025}{3888} + \frac{541549 \times 1.0823}{648} - \frac{460 \times 1.0362}{9} - \frac{2740 \times 0.4082}{81}\right) \times 5 + \left(\frac{271883}{17496} - \frac{688 \times 1.20205}{81}\right) \times 25\right) x \, dx$$

Result:

 $1.65639 x^2$





The result 1.65639 is practically equal to the fourteenth root of Ramanujan's class invariant and very near to the mass of proton.

$$d_{\text{OS,4}}^{\prime(0)} = \frac{196}{3} - 256\Delta_{\text{OS,4}}^{(0)} - \left[\frac{1773073}{2916} + \left(\frac{71296}{81} + \frac{5632}{81}\log 2 - \frac{512}{27}(\log 2)^2\right)\zeta_2 + \frac{4756441}{3888}\zeta_3 - \frac{44653376}{4147}\zeta_4 + \frac{460}{9}\zeta_5 + \frac{692}{81}\tilde{a}_4\right]n_l + \left[\frac{140825}{5832} + \frac{1664}{81}\zeta_2 + \frac{76}{27}\zeta_3\right]n_l^2,$$

$$(9.1.43e)$$

$$\begin{aligned} &\frac{196}{3} - 256 \times (-382.257106) - \\ &\left(\frac{1773\,073}{2916} + \left(\frac{71\,296}{81} + 69.53\log(2) - 18.96 \times 0.48\right) \times 1.6449 + \frac{4756\,441 \times 1.202}{3888} - \frac{44\,653\,376 \times 1.0823}{4147} + \frac{460 \times 1.03692}{9} + 692 \times \frac{0.4082}{81}\right) \times 5 + \\ &\left(\frac{140\,825}{5832} + \frac{1664 \times 1.6449}{81} + 2.8148 \times 1.202\right) \times 25 \end{aligned}$$

Result

 $1.39489... \times 10^{5}$

139489

$$\widetilde{d}_{\text{OS},4}^{(0)} = 256\Delta_{\text{OS},4}^{(0)} - \frac{392}{9} + \left[\frac{1773073}{2916} + \left(\frac{71296}{81} + \frac{5632}{81} \log 2 - \frac{512}{27} (\log 2)^2 \right) \zeta_2 + \frac{4756441}{3888} \zeta_3 - \frac{44653376}{4147} \zeta_4 + \frac{460}{9} \zeta_5 + \frac{692}{81} \widetilde{a}_4 \right] n_l - \left[\frac{140825}{5832} + \frac{1664}{81} \zeta_2 + \frac{76}{27} \zeta_3 \right] n_l^2, \tag{9.1.47d}$$

$$256 \times (-382.257106) - \frac{392}{9} + \left(\frac{1773073}{2916} + \left(\frac{71296}{81} + 69.53\log(2) - 18.96 \times 0.48\right) \times 1.6449 + \frac{4756441 \times 1.202}{3888} - \frac{44653376 \times 1.0823}{4147} + \frac{460 \times 1.03692}{9} + 692 \times \frac{0.4082}{81}\right) \times 5 - \left(\frac{140825}{5832} + \frac{1664 \times 1.6449}{81} + 2.8148 \times 1.202\right) \times 25$$

Result:

$$-1.39468... \times 10^{5}$$

-139468

Now, we have that:

Pi/(1728) integrate [[-97857.82-392/9+[608.0497+(71296/81+69.53ln2-18.96*0.480)*1.645+(4756441*1.202)/3888-11653.8+(460*1.03692)/9+692*0.4082/81]*5-[24.1469+(1664*1.6449)/81+2.8148*1.202]*25]]x

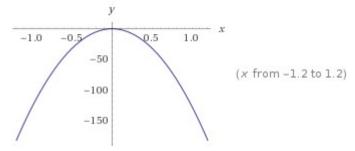
$$\frac{\pi}{1728}$$

$$\int \left(-97857.82 - \frac{392}{9} + \left(608.0497 + \left(\frac{71296}{81} + 69.53 \log(2) - 18.96 \times 0.48\right) \times 1.645 + \frac{4756441 \times 1.202}{3888} - 11653.8 + \frac{460 \times 1.03692}{9} + 692 \times \frac{0.4082}{81}\right) \times 5 - \left(24.1469 + \frac{1664 \times 1.6449}{81} + 2.8148 \times 1.202\right) \times 25\right) x \, dx$$

Result:

$$-126.779 x^2$$

Plot:



This result -126.779 is very near to the mass of Higgs boson (125,09 \pm 0,24) with minus sign (Higgs antiboson)

Now, we take the sum of the various results that we have obtained:

$$-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349$$

-108304,546; we note that -108304.546 / (1728*10) = 6,2676241898148... a value very near to 2π . Thence, we have a length of a circle $C = 2\pi r$ with r = 1728*10 or precisely $C = 2\pi r = 108303,26513$ with r = 1723,7*10

And: 108304,546 / (5*1728) = 12,535248379629

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the *J*-function itself, so

$$Z(q) = q^{-1} + 196884q + \dots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is $4\pi=12.57...$ So we are off by just a few percent.

We note that the value that we have obtained 12,535... is a very good approximation of the value 12,57... that is the classical entropy of a black hole with k = 1 and mass 2.

Further: $(\ln 108304,546)^{1/5} = 1,632438908...$

and

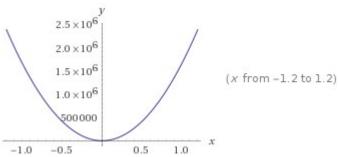
Pi/178 * 1728 integrate [108304.546]x where 178 = 144 + 34 that are Fibonacci's numbers

$$\frac{\pi}{178} \times 1728 \int 108\,304.546\,x\,dx$$

Result:

 $1.65154 \times 10^6 x^2$

Plot:



The results 1,6324 and 1.65154 are very near to the fourteenth root of Ramanujan's class invariant and to the mass of the proton

Furthermore:

 $108304,546 \approx [1164,27 * (27*3+12)+27] = 108304,11$ where 27 = 1728 / 64; 12 = 1728 / 144 and 1164.27 is equal to:

$$\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3 = 1164,2696 = 1164,27$$

Now, we have:

9.1.3 Pole mass

The relation between the $\overline{\text{MS}}$ running mass and the pole mass, assuming $(n_f - 1)$ massless quarks and one massive quark, is written as a power series of $a^{[n_f]}(\mu)$ and $L_{\text{OS}} = \log(m_{\text{OS}}^2/\mu^2)$.

$$\frac{m_{\overline{MS}}^{[n_f]}(\mu)}{m_{OS}} = 1 + \sum_{k=1}^{\infty} a^k(\mu) \left[\sum_{m=0}^k C_k^{(m)} L_{OS}^m \right], \qquad (9.1.19a)$$

$$C_1^{(0)} = -\frac{16}{3},$$
 (9.1.19b)

$$C_2^{(0)} = -\frac{3161}{18} - \frac{16}{3}(7 + 2\log 2)\zeta_2 + \frac{8}{3}\zeta_3 + \left[\frac{71}{9} + \frac{16}{3}\zeta_2\right]n_f, \qquad (9.1.19c)$$

$$C_3^{(0)} = -\frac{1163813}{162} + \frac{608}{27} (\log 2)^4 + \left(-\frac{2815124}{405} + \frac{36160}{27} \log 2 + \frac{1024}{9} (\log 2)^2 \right) \zeta_2$$

$$+ \left(-\frac{3304}{9} + \frac{11512}{9} \zeta_2 \right) \zeta_3 + \frac{6260}{9} \zeta_4 - \frac{15800}{27} \zeta_5 + \frac{4864}{9} \operatorname{Li}_4(\frac{1}{2})$$

$$+ \left[\frac{167566}{243} - \frac{64}{81} (\log 2)^4 + \left(\frac{16304}{27} + \frac{1408}{27} \log 2 - \frac{256}{27} (\log 2)^2 \right) \zeta_2$$

$$+ \frac{6232}{27} \zeta_3 - \frac{4880}{27} \zeta_4 - \frac{512}{27} \operatorname{Li}_4(\frac{1}{2}) \right] n_f$$

$$+ \left[-\frac{4706}{729} - \frac{416}{27} \zeta_2 - \frac{224}{27} \zeta_3 \right] n_f^2, \qquad (9.1.19d)$$

where $\text{Li}_4(x)$ is the polylogarithm (11.5.23) of the fourth order and $\text{Li}_4(1/2) = 0.51747906 \cdots$. $C_k^{(m)}$ for $m \geq 1$ are written in terms of $C_{k'}^{(0)}$ (k' < k) with the coefficients in the RGEs (9.1.1) and (9.1.11) as follows.

$$C_1^{(1)} = \frac{1}{2}\gamma_m^{(0)} = 4,$$
 (9.1.20a)

$$C_2^{(1)} = \frac{1}{2} \left[\gamma_m^{(1)} + \left(\gamma_m^{(0)} - 2\beta^{(0)} \right) C_1^{(0)} \right] = \frac{314}{3} - \frac{52}{9} n_f, \qquad (9.1.20b)$$

$$C_2^{(2)} = \frac{1}{8} \left(\gamma_m^{(0)} - 2\beta^{(0)} \right) \gamma_m^{(0)} = -14 + \frac{4}{3} n_f,$$
 (9.1.20c)

$$C_{3}^{(1)} - \frac{1}{2} \left[\gamma_{m}^{(2)} + \left(\gamma_{m}^{(1)} - 2\beta^{(1)} \right) C_{1}^{(0)} + \left(\gamma_{m}^{(0)} - 4\beta^{(0)} \right) C_{2}^{(0)} \right]$$

$$= \frac{41354}{9} + \left(672 + 192 \log 2 \right) \zeta_{2} - 48\zeta_{3}$$

$$- \left[\frac{13876}{27} + \left(\frac{1312}{9} + \frac{128}{9} \log 2 \right) \zeta_{2} + \frac{448}{9} \zeta_{3} \right] n_{f} + \left[\frac{712}{81} + \frac{64}{9} \zeta_{2} \right] n_{f}^{2} (9.1.20e)$$

$$C_3^{(2)} = \frac{1}{8} \left[-2\beta^{(1)} \gamma_m^{(0)} + \left(\gamma_m^{(0)} - 2\beta^{(0)} \right) \left(2\gamma_m^{(1)} + \left(\gamma_m^{(0)} - 4\beta^{(0)} \right) C_1^{(0)} \right) \right]$$
(9.1.20f)

$$= -\frac{3034}{3} + \frac{428}{3}n_f - \frac{104}{27}n_f^2, \tag{9.1.20g}$$

$$C_3^{(3)} = \frac{1}{48} \left(\gamma_m^{(0)} - 4\beta^{(0)} \right) \left(\gamma_m^{(0)} - 2\beta^{(0)} \right) \gamma_m^{(0)} = 84 - \frac{128}{9} n_f + \frac{16}{27} n_f^2. \tag{9.1.20h}$$

 $O(\alpha_s^2)$ terms are given in Ref. [996, 997, 990] and $O(\alpha_s^3)$ terms in Ref. [998, 999, 1000]. Inversion formulae of (9.1.19) is obtained as

We obtain:

-5,333;

$$-\frac{3161}{18} - \frac{16}{3} (7 \times 1.6449 + 2 \times 0.69314718 \times 1.6449) + \frac{8}{3} \times 1.20205 + \left(\frac{71}{9} + \frac{16}{3} \times 1.6449\right) \times 4$$

-179,33017...

$$\left(-\frac{1163813}{162} + \frac{608}{27} \times 0.69314718^4 + \left(-\frac{2815124}{405} + \frac{36160 \times 0.69314718}{27} + \frac{1}{9}\left(1024 \times 0.69314718^2\right)\right)\right) \times 1.6449$$

Result

More digits

-21625.1509257993171384286028538183763154299259259259259259...

-21625.1509...

$$\left(-\frac{3304}{9} + \frac{11512 \times 1.6449}{9} \right) \times 1.20205 + \\ \frac{6260 \times 1.0823}{9} - \frac{15800 \times 1.03692}{27} + \frac{4864 \times 0.517479062}{9} \right)$$

2513.5173888453 (period 1)

2513,51738...

$$\begin{aligned} &\left(\frac{167566}{243} - \frac{1}{81} \left(64 \times 0.69314718^4\right) + \\ & \left(\frac{16304}{27} + \frac{1408 \times 0.69314718}{27} - \frac{1}{27} \left(256 \times 0.69314718^2\right)\right) \times 1.6449\right) \times 4 \end{aligned}$$

6938.517876498652546258577447346465416954732510288065843621... (period 27)

6938.51787

$$\Big(\frac{6232 \times 1.20205}{27} - \frac{4880 \times 1.0823}{27} - \frac{512 \times 0.517479062}{27}\Big) \times 4$$

288.089232630518 (period 3)

288.08923

$$\left(-\frac{4706}{729}-\frac{416\times1.6449}{27}-\frac{224\times1.20205}{27}\right)\!\!\times\!16$$

-668.346012620027434842249657064471879286694101508916323731... (period 81)

-668.3460126...

Result:

-12553.3724326

-21625.1509 + 2513.51738 + 6938.51787 + 288.08923 - 668.3460126 = -12553.3724326 - 12553.3724326 - 179.33017 - 5.33333

Result:

-12738.0359326

Final result

$$-12738,0359326$$
 about $(728+21) * 17 + 5 = 12738$; $12738 / 1158 = 11$; $1158 = 1164 - 6 = 1164 - 1728/288$

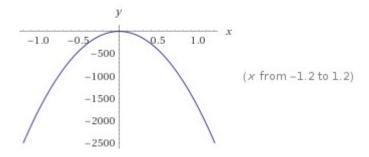
We have calculate the following integrals:

32 [(Pi^2/1164)] integrate [-12738]x

Indefinite integral:

$$\frac{32\,\pi^2}{1164}\int -12\,738\,x\,dx \approx \text{constant} + -1728.1\,x^2$$

Plot:

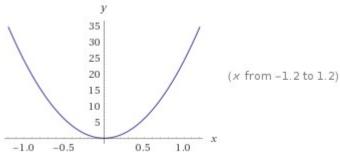


The result 1728 is the Ramanujan's number (1729 - 1)

Indefinite integral:

$$\int \frac{2((762\pi^2)\sqrt[3]{12738} x)}{(2\pi)1164.27} dx = 24.0098 x^2 + \text{constant}$$

Plot of the integral:



Note that 762 * 4 * 0.56706789 = 1728.42 where 0.56706789 is about the Infinite power tower of 1/e (Omega constant) 0.567143...

$$\left(\frac{1}{e}\right)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)}^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)^{\!(1/e)$$

 $0.5670678983907883690903972468241867067062310076575123455907248\dots \\$

and

0.0680174 * integrate ((([2*[((762/(2*Pi)) * Pi^2/(1164.27)*[12738])))^1/3 x

0.0680174761587831693972779 that is $1/10 * (\pi \sqrt{3})/8$ i.e. 1/10 of "Body-centered cubic (bcc)"

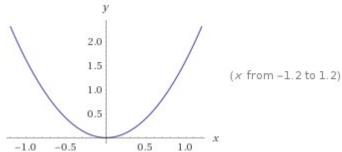
Input interpretation:

$$0.0680174 \int 2 \left(\sqrt[3]{\frac{762}{2\pi} \times \frac{\pi^2}{1164.27} \times 12738} \right) dx$$

Result:

 $1.60322 x^2$

Plot:



The result 1.60322 is very near to the value of the electric charge of the positron.

We have:

4;

$$314/3 - (52*4)/9 = 81,5555...$$

$$-14 + 16/3 = -8,6666...$$

$$\frac{41\,354}{9} + (672 + 192 \times 0.69314718) \times 1.6449 - 48 \times 1.20205$$

5861.4735857942328 (period 1)

5861,47358...

$$-\bigg(\frac{13\,876}{27} + \bigg(\frac{1312}{9} + \frac{128\times0.69314718}{9}\bigg)\times1.6449 + \frac{448\times1.20205}{9}\bigg)\times4 + \bigg(\frac{712}{81} + \frac{64\times1.6449}{9}\bigg)\times16$$

-2991.271949700348839506172 (period 9)

-2991.2719497

5861,47358 - 2991,2719497 = 2870,2016303

$$-\frac{3034}{3} + \frac{428 \times 4}{3} - \frac{104 \times 16}{27}$$

Exact result:

$$-\frac{13562}{27}$$

-502.296 (period 3)

-502,296

$$84 - \frac{128 \times 4}{9} + \frac{16 \times 16}{27}$$

Exact result:

988 27

36.592 (period 3)

36,592

2481,396

We have that: -12738,0359326 + 2481,396 = -10256,6399326

-10256,639 / 6 = -1709,4398333...

-10256,639 / 14 = 732,61707

-12738,0359326 - 2481,396 = -15219,4319326

-15219,4319326 / 8,8 = -1729,4809014318 where 8,8 = 0,55 * 16;

(-15219,4319326 / 108) - 3,141592653 * 12 = -1728,7471043471

The mean of the two values obtained is: -1729,114 a value very near to the Ramanujan's number 1729

From Wikipedia

In particle physics, **quarkonium** (from quark and -onium, pl. **quarkonia**) is a flavorless meson whose constituents are a heavy quark and its own antiquark, making it a neutral particle and the antiparticle of themselves. We take the following charmonium particle;

Term symbol n ^{2S+1} L _J	$\underline{\mathbf{I}}^{\mathrm{G}}(\underline{\mathbf{J}}^{\mathrm{PC}})$	Particle	mass (MeV/c ²)
$1^{1}S_{0}$	0+(0-+)	<u>η</u> _c (1S)	2983.4±0.5

Now, we take the mass equal to 2983,9 thence: $2983,9 * 9 = 2,68551 * 10^{19}$ MeV is the correspondent energy and calculate the following integral:

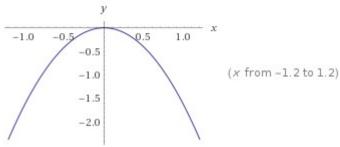
1/(1728)* 1/(2.68551) integrate [-15219.4319326]x

$$\frac{1}{1728} \times \frac{1}{2.68551} \int -15219.4319326 \, x \, dx$$

Result:

$$-1.63983 x^2$$

Plot:



Alternate form assuming x is real:

$$0 - 1.63983 x^2$$

Indefinite integral assuming all variables are real:

$$-0.546609 x^3 + constant$$

$$\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3 = 1164,2696 = 1164,27$$

Note that:

$$\sqrt{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,6398339 \dots$$

Note that $(1,0866246503513631746138436141496)^{32} = 14,274$ where 1,08662465... is a very good approximation to the Ramanujan's new constant, i.e. 1,08643...

Furthermore:

$$\sqrt{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} \cdot 10^{-36} =$$

 $= 6,6317902080152356725056810692356 \cdot 10^{-34}$

The result 1,6393 and 6,631790208 \cdot 10⁻³⁴ are very near to the value of the mass of proton and to the Planck's constant 6,626 * 10⁻³⁴

With regard the -15219,4319326 / 8,8 = -1729,4809014318 we observe that 8,8 is the $\Lambda(1520)$ branching ratios.

From Wikipedia, the free encyclopedia

In particle physics and nuclear physics, the **branching fraction** (or **branching ratio**) for a decay is the fraction of particles which decay by an individual decay mode with respect to the total number of particles which decay. It is equal to the ratio of the **partial decay constant** to the overall decay constant. Sometimes a **partial half-life** is given, but this term is misleading; due to competing modes it is not true that half of the particles will decay through a particular decay mode after its partial half-life.

$$\Lambda(1520) \ 3/2^-$$

$$I(J^P) = O(\frac{3}{2})$$
 Status: ***

Discovered by FERRO-LUZZI 62; the elaboration in WATSON 63 is the classic paper on the Breit-Wigner analysis of a multichannel resonance.

The measurements of the mass, width, and elasticity published before 1975 are now obsolete and have been omitted. They were last listed in our 1982 edition Physics Letters **111B** 1 (1982).

Production and formation experiments agree quite well, so they are listed together here.

Λ(1520) MASS

VALUE (I	MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1519.5	±1.0	OUR ESTIMATE				
1519.53	±0.19	OUR AVERAGE				
1520.4	± 0.6	± 1.5	1 QIANG	10	SPEC	$e p \rightarrow e' K^+ X$ (fit to X)
1517.3	± 1.5	300	BARBER	80D		$\gamma p \rightarrow \Lambda(1520) K^+$
1517.8	± 1.2	5k	BARLAG	79	HBC	K- p 4.2 GeV/c
1520.0	± 0.5		ALSTON	78	DPWA	$\overline{K}N \to \overline{K}N$
1519.7	± 0.3	4k	CAMERON	77	HBC	$K^- p 0.96-1.36 \text{ GeV}/c$
1519	± 1		GOPAL	77	DPWA	KN multichannel
1519.4	±0.3	2000	CORDEN	75	DBC	$K^- d 1.4 - 1.8 \text{ GeV}/c$
1 QIA	NG 10	gets 1518.8 MeV	for the pole mas	s (no	errors gi	ven).

$\Gamma(\Lambda\gamma)/\Gamma_{\text{total}}$					Г8/Г
VALUE (units 10^{-3})	EVT5	DOCUMENT ID)	TECN	COMMENT
8.5±1.5 OUR E 8.8±1.1 OUR F 8.8±1.1 OUR A	IT	65	å	14 T	
$10.7 \pm 2.9^{+1.5}_{0.4}$	32	TAYLOR	05	CLAS	$\gamma p \rightarrow K^+ \Lambda \gamma$
$10.2 \pm 2.1 \pm 1.5$ 8.0 ± 1.4	290 238	ANTIPOV MAST			$\rho N(C) \rightarrow \Lambda(1520) K^+ N(C)$ Using $\Gamma(NK)/\Gamma_{\text{total}} = 0.45$

We see as the value 8.8 ± 1.1 are present in the above table. We have that the value 15219,43 / 1729 that is the Ramanujan's taxicab number, is equal to 8,8024465 that is practically equal to the value of branching ratios

From Wikipedia

The **Lambda baryons** are a family of subatomic hadron particles containing one up quark, one down quark, and a third quark from a higher flavour generation, in a combination where the quantum wave function changes sign upon the flavour of any two quarks being swapped (thus differing from a Sigma baryon). They are thus baryons, with total isospin of 0, and have either neutral electric charge or the elementary charge +1.

Lambda baryons	Lam	bda	baryons
----------------	-----	-----	---------

Particle name	Symbol	Quark content	Rest mass (MeV/c²)	1	<u>j</u> P	Q (e)	<u>s</u>	c	<u>B'</u>	Ī	Mean lifetime (s)	Commonly decays to
Lambda ^[6]	^°	<u>uds</u>	1 115.683 ±0.006	0	<u>1</u> +	0	-1	0	0	0	(2.631 ± 0.020) × 10 ⁻¹⁰	$\frac{\underline{p}^+ + \underline{\pi}^- \text{ or }}{\underline{n}^0 + \underline{\pi}^0}$
charmed Lambda ^[15]	∧ _c +	udc	2 286.46 ±0.14	0	1/2	+1	0	+1	0	0	$(2.00 \pm 0.06) \times 10^{-13}$	See Action decay modes
bottom Lambda ^[16]	∧ ^B	udb	5 620.2 ± 1.6	0	1/2 +	0	0	0	-1	0	$1.409 + 0.055 \times 10^{-12}$	See Λ_b^0 decay modes
top Lambda [†]	∧ _t +	udt	_	0	1/2 +	+1	0	0	0	+1	_	_

We remember that:

The relation between the $\overline{\text{MS}}$ running mass and the pole mass, assuming $(n_f - 1)$ massless quarks and one massive quark, is written as a power series of $a^{[n_j]}(\mu)$ and $L_{\text{OS}} = \log (m_{\text{OS}}^2/\mu^2)$.

From the equations concerning this argument, we have obtained the following result -15219,43

From Wikipedia

In quantum field theory, the **pole mass** of an elementary particle corresponds to the concept of rest mass in the special theory of relativity.

In particle physics, the **invariant mass** m_0 is equal to the mass in the rest frame of the particle, and can be calculated by the particle's energy E and its momentum \mathbf{p} as measured in *any* frame, by the energy–momentum relation:

$$m_0^2c^2=\left(rac{E}{c}
ight)^2-\left\|\mathbf{p}
ight\|^2$$

or in natural units where c = 1

$$m_0^2 = E^2 - \|\mathbf{p}\|^2$$
.

This invariant mass is the same in all frames of reference (see also special relativity).

In quantum field theory, quantities like coupling constant and mass "run" with the energy scale of high energy physics. The running mass of a fermion or massive boson depends on the energy scale at which the observation occurs, in a way described by a renormalization group equation (RGE) and calculated by a renormalization scheme such as the on-shell scheme or the minimal subtraction scheme. The running mass refers to a Lagrangian parameter whose value changes with the energy scale at which the renormalization scheme is applied. A calculation, typically done by a computerized algorithm intractable by paper calculations, relates the running mass to the pole mass. The algorithm typically relies on a perturbative calculation of the self energy.

We note that dividing 15219,43 / 5620,2 = 2,70798726; it is interesting observe that the square root of this value is: $\sqrt{2,70798726} = 1,64559632...$ a good approximation to the mean of the two values obtained from the Ramanujan's class invariant. Indeed, we have:

$$(1,6398339 + 1,6557845) / 2 = 1,6478092$$

With regard 5620,2 we calculate the following integral:

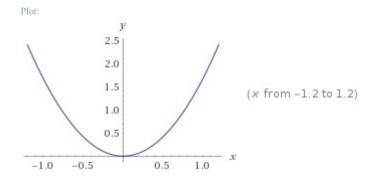
 $Pi^2/(9*1729)$ * integrate [5260.2]x

Input interpretation:

$$\frac{\pi^2}{9 \times 1729} \int 5260.2 \, x \, dx$$

Result:

$1.66815 x^2$



The mean between 1,64559 and 1,66815 is 1,65687 a value very near to the Ramanujan's class invariant and to the value of the mass of proton.

Now, we have:

 $O(\alpha_s^2)$ terms are given in Ref. [996, 997, 990] and $O(\alpha_s^3)$ terms in Ref. [998, 999, 1000]. Inversion formulae of (9.1.19) is obtained as

$$\frac{m_{\text{OS}}}{m_{\overline{\text{MS}}}^{[n_f]}(\mu)} = 1 + \sum_{k=1}^{\infty} a^k(\mu) \left[\sum_{m=0}^k C_k^{\prime(m)} L_{\overline{\text{MS}}}^m \right], \qquad L_{\overline{\text{MS}}} = \log \frac{m_{\overline{\text{MS}}}^{[n_f]2}(\mu)}{\mu^2}, \qquad (9.1.21a)$$

$$C_1^{\prime(0)} = -C_1^{(0)} = \frac{16}{3}, C_1^{\prime(1)} = -C_1^{(1)} = -4, (9.1.21b)$$

$$C_2^{\prime(0)} = -C_2^{(0)} + 2C_1^{(1)}C_1^{(0)} + C_1^{(0)2}$$

$$= \frac{2905}{18} + \frac{16}{3}(7 + 2\log 2)\zeta_2 - \frac{8}{3}\zeta_3 - \left[\frac{71}{9} + \frac{16}{3}\zeta_2\right]n_f, \qquad (9.1.21c)$$

$$C_2^{\prime(1)} = -C_2^{(1)} + 2C_1^{(1)2} + 2C_1^{(1)}C_1^{(0)} = -\frac{346}{3} + \frac{52}{9}n_f,$$
 (9.1.21d)

$$C_2^{\prime(2)} = -C_2^{(2)} - C_2^{(1)}C_1^{(1)2} + 2C_1^{(1)2} + 2C_1^{(1)}C_1^{(0)} = 30 - \frac{4}{3}n_f,$$
 (9.1.21e)

$$C_3^{\prime(0)} = -C_3^{(0)} + 2C_2^{(1)}C_1^{(0)} + 2C_2^{(0)}C_1^{(1)} + 2C_2^{(0)}C_1^{(0)}
-4C_1^{(1)2}C_1^{(0)} - 5C_1^{(1)}C_1^{(0)2} - C_1^{(0)3}
= \frac{1046525}{162} - \frac{608}{27}(\log 2)^4 + \left(\frac{2855444}{405} - \frac{35392}{27}\log 2 - \frac{1024}{9}(\log 2)^2\right)\zeta_2
+ \left(360 - \frac{11512}{9}\zeta_2\right)\zeta_3 - \frac{6260}{9}\zeta_4 + \frac{15800}{27}\zeta_5 - \frac{4864}{9}\operatorname{Li}_4(\frac{1}{2})
+ \left[-\frac{157702}{243} + \frac{64}{81}(\log 2)^4 + \left(-\frac{16688}{27} - \frac{1408}{27}\log 2 + \frac{256}{27}(\log 2)^2 \right)\zeta_2
- \frac{6232}{27}\zeta_3 + \frac{4880}{27}\zeta_4 + \frac{512}{27}\operatorname{Li}_4(\frac{1}{2}) \right]n_f$$

$$+ \left[\frac{4706}{729} + \frac{416}{27} \zeta_2 + \frac{224}{27} \zeta_3 \right] n_f^2,$$

$$(9.1.21f)$$

$$C_3^{\prime (1)} = -C_3^{(1)} + 4C_2^{(2)} C_1^{(0)} + 4C_2^{(1)} C_1^{(1)} + 2C_2^{(1)} C_1^{(0)}$$

$$+ 2C_2^{(0)} C_1^{(1)} - 4C_1^{(1)3} - 10C_1^{(1)2} C_1^{(0)} - 3C_1^{(1)} C_1^{(0)2}$$

$$= -\frac{43982}{9} - \left(\frac{2912}{3} + \frac{832}{3} \log 2 \right) \zeta_2 + \frac{208}{3} \zeta_3$$

$$+ \left[\frac{4660}{9} + \left(\frac{1696}{9} + \frac{128}{9} \log 2 \right) \zeta_2 + \frac{448}{9} \zeta_3 \right] n_f - \left[\frac{712}{81} + \frac{64}{9} \zeta_2 \right] n_f^2, (9.1.21g)$$

$$C_3^{\prime (2)} = -C_3^{(2)} + 6C_2^{(2)} C_1^{(1)} + 2C_2^{(2)} C_1^{(0)} + 2C_2^{(1)} C_1^{(1)} - 5C_1^{(1)3} - 3C_1^{(1)2} C_1^{(0)}$$

$$= 1598 - \frac{1540}{9} n_f + \frac{104}{27} n_f^2,$$

$$(9.1.21h)$$

$$C_3^{\prime (3)} = -C_3^{(3)} + 2C_2^{(2)} C_1^{(1)} - C_1^{(1)3} = -260 + \frac{224}{9} n_f - \frac{16}{27} n_f^2.$$

$$(9.1.21i)$$

We have that:

$$16/3 = 5,3333333333; -4;$$

$$\frac{2905}{18} + \frac{16}{3} \left(7 \times 1.6449 + 2 \times 0.69314718 \times 1.6449\right) - \\ \frac{8 \times 1.20205}{3} - \frac{71 \times 4}{9} - \frac{1}{3} \left(16 \times 4 \times 1.64499\right)$$

165.10602982807466

$$-\frac{346}{3} + \frac{52 \times 4}{9}$$

-92.22222222222

$$30 - \frac{4 \times 4}{3}$$

24.66666666666666

$$\begin{aligned} &\frac{1\,046\,525}{162} - \frac{1}{27} \left(608 \times 0.69314718^4\right) + \\ &\left(\frac{2\,855\,444}{405} - \frac{35\,392 \times 0.69314718}{27} - \frac{1}{9} \left(1024 \times 0.69314718^2\right)\right) \times 1.6449 \end{aligned}$$

16467.7117416566723149

$$\frac{360 \times 1.20205 - \frac{1}{9} (11512 \times 1.6449 \times 1.20205) - \\ \frac{6260 \times 1.0823}{9} + \frac{15800 \times 1.03692}{27} - \frac{4864 \times 0.517479062}{9}$$

-2522.0652999564444

$$4\left(-\frac{157702}{243}+\frac{1}{81}\left(64\times0.69314718^4\right)+\right.\\ \left.\left(-\frac{16688}{27}-\frac{1408\times0.69314718}{27}+\frac{1}{27}\left(256\times0.69314718^2\right)\right)\!\!\times\!1.6449-\right.\\ \left.\frac{6232\times1.20205}{27}+\frac{4880\times1.0823}{27}+\frac{512\times0.517479062}{27}\right)$$

59

-7157.81327209213402

$$4\left(\frac{4706}{729} + \frac{416 \times 1.6449}{27} + \frac{224 \times 1.20205}{27}\right)$$

167.0865031550068587

5.333333333 - 4 + 165.10602982807466 - 92.2222222222222 + 24.666666666666666 + 16467.7117416566723149 - 2522.0652999564444 - 7157.81327209213402 + 167.0865031550068587

7053.8034803689198596 total partial

$$\left(-\frac{43\,982}{9} - \frac{2912 \times 1.6449}{3} + \frac{1}{3}\,(832 \times 0.69314718 \times 1.6449) + \frac{208 \times 1.20205}{3} + \\ \left(\frac{4660}{9} + \frac{1696 \times 1.6449}{9} + \frac{1}{9}\,(128 \times 0.69314718 \times 1.6449) + \frac{448 \times 1.20205}{9} \right) \times 4 \right) - \\ \left(\frac{712}{81} + \frac{64 \times 1.6449}{9} \right) \times 16$$

-2796.5836362511913086

$$1598 - \frac{1540 \times 4}{9} + \frac{104 \times 16}{27}$$

975.185185185185185

$$-260 + \frac{224 \times 4}{9} - \frac{16 \times 16}{27}$$

-169.925925925925925

7053.8034803689198596 - 2796.5836362511913086 - 975.185185185185185185185 - 169.925925925925925925

3112.10873300661743989 Final Result

3112 = 389 * 8

Note that $3112.108733 / 16^2 = 12,156674738307...$

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten_Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the J-function itself, so

$$Z(q) = q^{-1} + 196884q + \dots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is $4\pi=12.57...$ So we are off by just a few percent.

We note that the value that we have obtained 12,156... is a very good approximation of the value 12,19... that is the black hole entropy obtained from log(196883)

We have that:

$$\sqrt[16]{3112.108733} = 1,6531639364667...$$

$$\sqrt{\left(\frac{3112.108733}{1164.2696}\right)} = \sqrt{2,6730138217 \dots} = 1,6349354182 \dots$$

The result 1,63493 is very near to the fourteenth root of the Ramanujan's class invariant and to the mass of proton.

We note that: 3112,108733 - 1728 = 1384,108733

With regard the baryon sigma, we have that (from: Citation: C. Amsler *et al.* (Particle Data Group), PL **B667**, 1 (2008) (URL: http://pdg.lbl.gov)

Σ(1385)0 MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1383.7±1.0 OUF	RAVERAGE	Error includes scal	e fact	or of 1.	4. See the ideogram below.
1384.1 ± 0.8	5722	AGUII AR	81D	HBC	$K^-p \rightarrow \Lambda 3\pi 4.2 \text{ GeV}/c$
1380 ±2	3100	5 BORENSTEIN	74	HBC	$K^-p \rightarrow \Lambda 3\pi \ 2.18$ GeV/c
1385.1 ± 2.5	240	⁴ THOMAS	73	HBC	$\pi^- p \rightarrow \Lambda \pi^0 K^0$
• • • We do not	use the follo	wing data for avera	ges, f	its, limit	ts, etc. • • •
1389 ±3	500	6 BAUBILLIER	79B	HBC	K [−] p 8.25 GeV/c

We observe that:

1383.7±1.0 OUR AVERAGE
1384.1±0.8 5722
$$K^-p \rightarrow \Lambda 3\pi 4.2$$
 GeV/c
1380±2 3100 $K^-p \rightarrow \Lambda 3\pi 2.18$ GeV/c

Indeed: 1384.1 ± 0.8 is very near to the our result:

$$3112,108733 - 1728 = 1384,108733$$

From:

A Study of Excited Charm-Strange Baryons with Evidence for new Baryons Ξ_c $(3055)^+$ and Ξ_c $(3123)^+$ - https://arxiv.org/pdf/0710.5763.pdf

Note that Ξ_c (3123) $(1.6 \pm 0.6 \pm 0.2)$ fb < 1.4fb;

 $\Xi_{\rm c} (3123)^{+}$ Mass Resolution $\pm 0.3 \pm 1.5 \pm 5.0$ NA NA

Background Shape $\pm 0.2 \pm 0.6 \pm 6.9$ NA NA

Phase-Space Thresh. $\pm 0.1 \pm 0.5 \pm 3.0$ NA NA

Mass Scale ±0.1 NA NA NA NA

Total $\pm 0.3 \pm 1.7 \pm 8.9$ NA NA

In quoting upper limits for Ξ_c (3077)⁰ and Ξ_c (3077)⁺, we consider the integrated yield up to 3093MeV/c² and 3089MeV/c², respectively (\approx 30MeV/c² above threshold in each case).

We note that: 3123 - 0.3 - 1.7 - 8.9 = 3112,10 that is perfectly the obtained result!

Furthermore, we have calculate the following integrals:

1728/(728+288) * integrate [3112.10873300661743989]x

Input interpretation:

$$\frac{1728}{728 + 288} \int 3112.10873300661743989 \, x \, dx$$

Result:

2646.51766271428884652 x²

Plot:

y

4000

3000

2000

1000

(x from -1.2 to 1.2)

-1.0 -0.5 0.5 1.0 x

1728/(728+226) * integrate [3112.10873300661743989]x

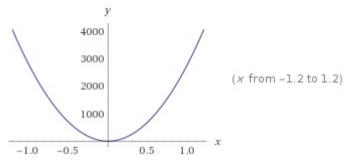
Input interpretation:

$$\frac{1728}{728 + 226} \int 3112.10873300661743989 \, x \, dx$$

Result:

2818.51356951542711537 x²

Plot:



Indefinite integral assuming all variables are real:

939.504523171809038457 x3 + constant

1728/(728+236) * integrate [3112.10873300661743989]x

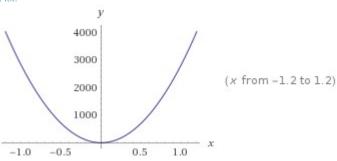
Input interpretation:

$$\frac{1728}{728 + 236} \int 3112.10873300661743989 \, x \, dx$$

Result

2789.27587688559903326 x²

Plot:



Indefinite integral assuming all variables are real:

929.758625628533011087 x3 + constant

1728/(728+316) * integrate [3112.10873300661743989]x

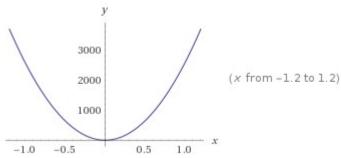
Input interpretation:

$$\frac{1728}{728 + 316} \int 3112.10873300661743989 \, x \, dx$$

Result

2575.53826179857995025 x²





1728/(728+360) * integrate [3112.10873300661743989]x

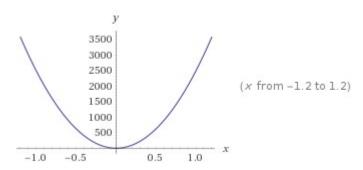
Input interpretation:

$$\frac{1728}{728 + 360} \int 3112.10873300661743989 \, x \, dx$$

Result:

2471.38046444643149638 x²

Plot:



results that are very good approximations of the values of the mass of the charmed baryons.

We have other very significant connections between the number 1728 and its factors and all the mass of the charmed baryons:

$$1728 + 576 + 144 + 96 + 36 + 16 = 2596$$

$$1728 + 576 + 288 + 144 + 64 + 16 = 2816$$

$$1728 + 576 + 288 + 144 + 48 + 6 = 2790$$

$$1728 + 576 + 288 + 48 + 6 = 2646$$

$$1728 + 576 + 144 + 64 + 48 + 16 = 2576$$

$$1728 + 576 + 144 + 24 = 2472$$

$$1728 + 576 + 144 + 108 + 64 + 8 = 2628$$

and other...!

further, we have that 1728 + 728 + 288 + 144 + 64 + 16 = 2968

Note that 3112.108733 + 2698 = 5810.108733 value very near to the Sigma bottom that is 5807.8 ± 2.7

From the Ramanujan's equation above analyzed

$$64J^2 - 24J + 9 = 64*400400100 - 24*20010 + 9 = 25625606400 - 480240 + 9 = 25625126169;$$

we have that $(25625126169)^{1/3} = 2948,1891086...$

and
$$3112,10-728=2384,108733$$
; $2384+144+108+24=2660$;

$$3112 - 144 - 27 = 2941$$
; $3112 - 36 = 3076$;

$$3112 - 108 - 24 = 2980$$
 $3112 - 144 - 64 - 24 = 2880$

$$2384 + 64 + 72 = 2520$$
; $2384 + 288 + 96 = 2768$;

$$2384 + 576 + 16 = 2976$$
 $2384 + 288 + 144 + 64 = 2880$

$$2384 + 288 + 144 + 96 + 27 = 2939$$

TABLE II: Mass spectra and decay widths (in units of MeV) of charmed baryons. Experimental values are taken from the Particle Data Group [3] except $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Xi_c(2980)^{+,0}$, $\Xi_c(3077)^{+,0}$ and $\Omega_c(2768)$ for which we use the most recent available BaBar and Belle measurements.

State	quark content	J^P	Mass	Width
Λ_c^+	udc	1+	2286.46 ± 0.14	
$\Lambda_c(2593)^+$	udc	$\frac{1}{2}$	2595.4 ± 0.6	$3.6^{+2.0}_{-1.3}$
$\Lambda_c(2625)^{+}$	udc	$\frac{\frac{1}{2}^{+}}{\frac{1}{2}^{-}}$ $\frac{3}{2}^{-}$	2628.1 ± 0.6	< 1.9
$\Lambda_c(2765)^{+}$	udc	??	2766.6 ± 2.4	50
$\Lambda_c(2880)^+$	udc	$\frac{5}{2}$	2881.5 ± 0.3	5.5 ± 0.6
$\Lambda_c(2940)^+$	udc	??	2938.8 ± 1.1	13.0 ± 5.0
$\Sigma_c(2455)^{++}$	uuc	$\frac{1}{2}^{+}$	2454.02 ± 0.18	2.23 ± 0.30
$\Sigma_c(2455)^+$	udc	$\frac{1}{2}^{+}$	2452.9 ± 0.4	< 4.6
$\Sigma_c(2455)^0$	ddc	$\frac{1}{2}^{+}$	2453.76 ± 0.18	2.2 ± 0.4
$\Sigma_c(2520)^{++}$	uuc	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $\frac{3}{2}$	2518.4 ± 0.6	14.9 ± 1.9
$\Sigma_c(2520)^+$	udc	$\frac{3}{2}^{+}$	2517.5 ± 2.3	< 17
$\Sigma_c(2520)^0$	ddc	$\frac{3}{2}$ +	2518.0 ± 0.5	16.1 ± 2.1
$\Sigma_c(2800)^{++}$	uuc	??	2801^{+4}_{-6}	75^{+22}_{-17}
$\Sigma_c(2800)^+$	udc	??	2792^{+14}_{-5}	62^{+60}_{-40}
$\Sigma_c(2800)^0$	ddc	??	2802_{-7}^{+4}	61^{+28}_{-18}
Ξ_c^+	usc	$\frac{1}{2}^{+}$	2467.9 ± 0.4	2
Ξ_c^0	dsc	$\frac{\frac{1}{2}^+}{\frac{1}{2}^+}$	2471.0 ± 0.4	
<u></u>	usc	$\frac{1}{2}^{+}$	2575.7 ± 3.1	
$\Xi_c^{\prime 0}$	dsc	$\frac{1}{2}^{+}$	2578.0 ± 2.9	
$\Xi_c(2645)^+$	usc	$\frac{3}{2}^{+}$	2646.6 ± 1.4	< 3.1
$\Xi_c(2645)^0$	dsc	$\frac{3}{2}^{+}$	2646.1 ± 1.2	< 5.5
$\Xi_c(2790)^+$	usc	$\frac{1}{2}^{-}$	2789.2 ± 3.2	< 1 5
$\Xi_c(2790)^0$	dsc	$\frac{1}{2}$	2791.9 ± 3.3	< 12
$\Xi_c(2815)^+$	usc	$\frac{3}{2}$	2816.5 ± 1.2	< 3.5
$\Xi_c(2815)^0$	dsc	$ \begin{array}{r} \hline \hline $	2818.2 ± 2.1	< 6.5
$\Xi_c(2980)^+$	u_{sc}	??	2971.1 ± 1.7	25.2 ± 3.0
$\Xi_c(2980)^0$	dsc	??	2977.1 ± 9.5	43.5
$\Xi_c(3077)^+$	usc	??	3076.5 ± 0.6	6.2 ± 1.1
$\Xi_c(3077)^0$	dsc	??	3082.8 ± 2.3	5.2 ± 3.6
Ω_c^0	ssc	$\frac{1}{2}^{+}$	2697.5 ± 2.6	
$\Omega_c(2768)^0$	ssc	$\frac{3}{2}^{+}$	2768.3 ± 3.0	

We take the precedent values: 10256,639 15219,4319 108304,546 2209694 and we have that:

Values of some charmed baryons

From the following calculations:

$$2557,5 + 24 = 2581.25$$
 $2564,159 + 12 = 2576.159$ $2536,57 - 18 = 2518.57$

$$2256,34 + 32 = 2288.34$$
 $3008,459 - 32 = 2976.459$

we obtain the following very good approximations:

$$2576.159 \approx 2575.7$$
 $2581.25 \approx 2578$ $2518.57 \approx 2517.5 - 2518.4$

$$2288.34 \approx 2286.46$$
 $2976.459 \approx 2977.1$

Now, we have:

 $\delta_c^{(k)}$ are coefficients in the $\overline{\rm MS}$ -pole conversion formula for the charm quark mass.

$$\frac{m_{c,OS}}{m_{c,\overline{MS}}(\mu_c)} = 1 + \epsilon \delta_c^{(1)} + \epsilon^2 \delta_c^{(2)} + \epsilon^3 \delta_c^{(3)} + \cdots,$$
 (9.1.78a)

$$\delta_c^{(1)} = \frac{\alpha_s^{[4]}(\mu)}{\pi} \left[\frac{4}{3} - L_{c,\overline{MS}} \right], \tag{9.1.78b}$$

$$\begin{split} \delta_c^{(2)} &= \frac{\alpha_s^{[4]^2(\mu)}}{\pi^2} \left[\frac{779}{96} + \frac{1}{6}\pi^2 + \frac{1}{9}\pi^2 \log 2 - \frac{1}{6}\zeta_3 - \frac{25}{9}L_{c,\overline{\text{MS}}}^{[\mu]} \right] \\ &- \left(\frac{215}{72} - \frac{25}{12}L_{c,\overline{\text{MS}}}^{[\mu]} \right) L_{c,\overline{\text{MS}}} - \frac{13}{24}L_{c,\overline{\text{MS}}}^2 \right] , \qquad (9.1.78c) \\ \delta_c^{(3)} &= \frac{\alpha_s^{[4]^3(\mu)}}{\pi^3} \left[\frac{5784469}{93312} + \frac{488501}{38880}\pi^2 + \frac{37}{7776}\pi^4 - \frac{49}{162}(\log 2)^4 - \frac{641}{162}\pi^2 \log 2 \right. \\ &- \frac{16}{81}\pi^2 (\log 2)^2 - \frac{1453}{216}\zeta_3 - \frac{1439}{432}\pi^2\zeta_3 + \frac{1975}{216}\zeta_5 - \frac{196}{27}\operatorname{Li}_4(\frac{1}{2}) \\ &+ \left(-\frac{7313}{192} - \frac{25}{36}\pi^2 - \frac{25}{54}\pi^2 \log 2 + \frac{25}{36}\zeta_3 \right) L_{c,\overline{\text{MS}}}^{[\mu]} + \frac{625}{108}L_{c,\overline{\text{MS}}}^{[\mu]^2} \\ &+ \left(-\frac{42019}{5184} - \frac{1}{6}\pi^2 - \frac{1}{9}\pi^2 \log 2 + \frac{7}{2}\zeta_3 + \frac{6761}{432}L_{c,\overline{\text{MS}}}^{[\mu]} - \frac{625}{144}L_{c,\overline{\text{MS}}}^{[\mu]^2} \right) L_{c,\overline{\text{MS}}} \\ &+ \left(-\frac{5357}{864} + \frac{325}{288}L_{c,\overline{\text{MS}}}^{[\mu]} \right) L_{c,\overline{\text{MS}}}^2 - \frac{247}{432}L_{c,\overline{\text{MS}}}^3 \right] , \qquad (9.1.78d) \end{split}$$

where

$$L_{c,\overline{\rm MS}} = \log \frac{m_{c,\overline{\rm MS}}^2(\mu_c)}{\mu_c^2}, \qquad L_{c,\overline{\rm MS}}^{[\mu]} = \log \frac{m_{c,\overline{\rm MS}}^2(\mu_c)}{\mu^2}.$$
 (9.1.79)

Eq. (9.1.78) is obtained by setting $n_f = 4$ in Eq. (9.1.22). $\Delta'_2(L_\mu, a)$ and $\Delta'_3(L_\mu, a)$ are expanded

$$\frac{5.13\times2}{\pi}\left(\frac{4}{3}-4.852\right)$$

-11.4915...

-11.4915

$$\frac{5.13\times2}{\pi^2}\left(\frac{779}{96}+\frac{\pi^2}{6}+\frac{1}{9}\left(\pi^2\times0.69314718\right)-\frac{1.20205}{6}-\frac{25\times5.7693668}{9}-\frac{215\times4.852}{72}+\frac{1}{12}\left(25\times5.7693668\times4.852\right)-\frac{1}{24}\left(13\times5.7693668^2\right)\right)$$

20.8885...

20.8885

0.330901 *

$$\frac{5784469}{93312} + \frac{488501\,\pi^2}{38\,880} + \frac{37\,\pi^4}{7776} - \frac{1}{162} \left(49 \times 0.69314718^4\right) - \frac{1}{162} \left(641\,\pi^2 \times 0.69314718\right)$$

159.320475... 159.320475

Integral representations

$$\frac{5784469}{93312} + \frac{488501\pi^2}{38880} + \frac{37\pi^4}{7776} - \frac{49 \times 0.693147^4}{162} - \frac{641\pi^2 \ 0.693147}{162} = 0.0761317 \left(1.58097 + \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2\right) \left(514.456 + \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2\right)$$

$$5784469 - 488501\pi^2 - 37\pi^4 - 49 \times 0.693147^4 - 641\pi^2 \ 0.693147$$

$$\frac{5784469}{93312} + \frac{488501\pi^2}{38880} + \frac{37\pi^4}{7776} - \frac{49\times0.693147^4}{162} - \frac{641\pi^2\ 0.693147}{162} = 0.0761317 \left(1.58097 + \left(\int_0^\infty \frac{\sin(t)}{t}\ dt\right)^2\right) \left(514.456 + \left(\int_0^\infty \frac{\sin(t)}{t}\ dt\right)^2\right)$$

$$\frac{5\,784\,469}{93\,312} + \frac{488\,501\,\pi^2}{38\,880} + \frac{37\,\pi^4}{7776} - \frac{49\times0.693147^4}{162} - \frac{641\,\pi^2\,0.693147}{162} = \\ 1.21811 \left(0.395242 + \left(\int_0^1 \sqrt{1-t^2}\ dt\right)^2\right) \left(128.614 + \left(\int_0^1 \sqrt{1-t^2}\ dt\right)^2\right)$$

$$-\frac{1}{81} \left(16 \, \pi ^2 \times 0.69314718^2\right)-\frac{1453 \times 1.20205}{216}-\\\frac{1}{432} \left(1439 \, \pi ^2 \times 1.20205\right)+\frac{1975 \times 1.036929}{216}-\frac{196 \times 0.517479062}{27}$$

-42.8164...

-42.8164

Integral representations:

$$\begin{split} &-\frac{1}{81} \left(16 \, \pi^2 \, 0.693147^2\right) - \frac{1453 \times 1.20205}{216} - \frac{1439 \, \pi^2 \, 1.20205}{432} + \\ &-\frac{1975 \times 1.03693}{216} - \frac{196 \times 0.517479}{27} = -2.36135 - 16.3958 \left(\int_0^\infty \frac{1}{1+t^2} \, dt\right)^2 \\ &-\frac{1}{81} \left(16 \, \pi^2 \, 0.693147^2\right) - \frac{1453 \times 1.20205}{216} - \frac{1439 \, \pi^2 \, 1.20205}{432} + \\ &-\frac{1975 \times 1.03693}{216} - \frac{196 \times 0.517479}{27} = -2.36135 - 65.5833 \left(\int_0^1 \sqrt{1-t^2} \, dt\right)^2 \end{split}$$

$$-\frac{1}{81} \left(16 \, \pi ^2 \, 0.693147^2\right) - \frac{1453 \times 1.20205}{216} - \frac{1439 \, \pi ^2 \, 1.20205}{432} + \\ \frac{1975 \times 1.03693}{216} - \frac{196 \times 0.517479}{27} = -2.36135 - 16.3958 \left(\int_0^\infty \frac{\sin(t)}{t} \, dt\right)^2$$

$$\left(-\frac{7313}{192} - \frac{1}{36}\left(25\,\pi^2\right) - \frac{1}{54}\left(25\,\pi^2 \times 0.69314718\right) + \frac{25 \times 1.20205}{36}\right) \times 5.76936 + \frac{1}{108}\left(625 \times 5.76936^2\right)$$

-80.1211...

-80.1211

Integral representation

 $-22.306 - 93.7263 \left(\int_{0}^{1} \sqrt{1-t^2} dt \right)^2$

$$\left(-\frac{7313}{192} - \frac{25 \pi^2}{36} - \frac{25 \pi^2 0.693147}{54} + \frac{25 \times 1.20205}{36} \right) 5.76936 + \frac{625 \times 5.76936^2}{108} =$$

$$-22.306 - 23.4316 \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\left(-\frac{7313}{192} - \frac{25 \pi^2}{36} - \frac{25 \pi^2 0.693147}{54} + \frac{25 \times 1.20205}{36} \right) 5.76936 + \frac{625 \times 5.76936^2}{108} =$$

$$\left(-\frac{7313}{192} - \frac{25 \pi^2}{36} - \frac{25 \pi^2 \cdot 0.693147}{54} + \frac{25 \times 1.20205}{36}\right) 5.76936 + \frac{625 \times 5.76936^2}{108} = -22.306 - 23.4316 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2$$

$$\left(-\frac{42\,019}{5184} - \frac{\pi^2}{6} - \frac{1}{9} \left(\pi^2 \times 0.69314718 \right) + \\ \frac{7 \times 1.20205}{2} + \frac{6761 \times 5.76936}{432} - \frac{1}{144} \left(625 \times 5.76936^2 \right) \right) \times 4.852$$

-293.442...

-293.442

Integral representations:

$$\left(-\frac{42\,019}{5184} - \frac{\pi^2}{6} - \frac{\pi^2\,0.693147}{9} + \frac{7\times1.20205}{2} + \frac{6761\times5.76936}{432} - \frac{625\times5.76936^2}{144} \right) \\ 4.852 = -281.773 - 4.7294 \left(\int_0^\infty \frac{1}{1+t^2} \,dt \right)^2$$

$$\left(-\frac{42019}{5184} - \frac{\pi^2}{6} - \frac{\pi^2 \cdot 0.693147}{9} + \frac{7 \times 1.20205}{2} + \frac{6761 \times 5.76936}{432} - \frac{625 \times 5.76936^2}{144}\right)$$

$$4.852 = -281.773 - 18.9176 \left(\int_0^1 \sqrt{1 - t^2} \ dt\right)^2$$

$$\left(-\frac{42\,019}{5184} - \frac{\pi^2}{6} - \frac{\pi^2\,0.693147}{9} + \frac{7\times1.20205}{2} + \frac{6761\times5.76936}{432} - \frac{625\times5.76936^2}{144} \right)$$

$$4.852 = -281.773 - 4.7294 \left(\int_0^\infty \frac{\sin(t)}{t} \, dt \right)^2$$

$$\left(-\frac{5357}{864}+\frac{325\times5.76936}{288}\right)\times4.852^2-\frac{1}{432}\left(247\times4.852^3\right)$$

-58.003600281074 (period 3)

-58.003600281074

1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-293.442-58.0036)

Result:

-93.857537675125

Final result: -93.8575

We calculate the following integral:

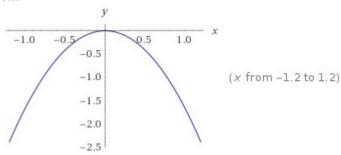
1/(9Pi) integrate [1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-293.442-58.0036)]x

$$\frac{1}{9\pi} \int (1-11.4915 + 20.8885 + 0.330901 (159.320475 - 42.8164 - 80.1211 - 293.442 - 58.0036)) x dx$$

Result:

$$-1.65977 x^2$$

Plot:



1/29 integrate [1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-293.442-58.0036)]x

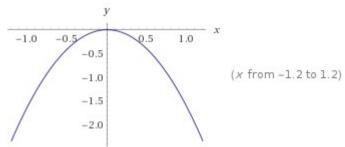
Input interpretation:

$$\frac{1}{29} \int (1 - 11.4915 + 20.8885 + 0.330901 (159.320475 - 42.8164 - 80.1211 - 293.442 - 58.0036)) x dx$$

Result:

$$-1.61823 x^2$$

Plot:



The results -1.6597 and -1.61823 are very near to fourteenth root of Ramanujan's class invariant and to the mass of proton with minus sign and to the electric charge of the electron (1.61823 \approx golden ratio).

We note that:

From:

Citation: J. Beringer et al. (Particle Data Group), PR D86, 010001 (2012) and 2013 partial update for the 2014 edition (URL: http://pdg.lbl.gov)

The parity of the Λ_c^+ is defined to be positive (as are the parities of the proton, neutron, and Λ). The quark content is $u\,d\,c$. Results of an analysis of $p\,K^-\pi^+$ decays (JEZABEK 92) are consistent with J=1/2. Nobody doubts that the spin is indeed 1/2.

A+ BRANCHING RATIOS

Hadronic modes with a p: S = -1 final states =

$$\Gamma(\Lambda \rho^{+})/\Gamma(\rho K^{-}\pi^{+})$$

VALUE

 $CL\%$
 $CL\%$
 $OCCUMENT ID$
 $TECN$
 $COMMENT$
 $COMMENT$
 $OLE2$
 $e^{+}e^{-} \approx \Upsilon(3S), \Upsilon(4S)$
 $CL\%$
 $OLE2$
 $OLE2$
 $OLE2$
 $OLE2$
 $OLE2$
 $OLE2$
 $OLE2$
 $OLE3$
 $OLE3$

$$0.98 - 0.05 = 0.93$$
; $0.977 - 0.015 - 0.051 = 0.911$; < 0.95

LINK

750

Between 0.93 and 0.95 there is the value 0.94 very near to the value 0.938575 Now from 93,8575 we obtain:

 $\sqrt[9]{93,8575} = 1,65639236$... value that is also very near to the fourteenth root of Ramanujan's class invariant

05F FOCS γ nucleus, $\overline{E}_{\gamma} \approx 180 \; \mathrm{GeV}$

$$\sqrt{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,65578454 \dots$$

Further:

$$(94 * 8) - 24 = 728; (728 + 24) / 8 = 94;$$

Now, we have:

 $1.09 \pm 0.11 \pm 0.19$

The relation between the 1S and the $\overline{\rm MS}$ masses of the bottom quark is derived by combining 1S–pole and $\overline{\rm MS}$ –pole mass relations. The latter is written as

$$\begin{split} \frac{m_{b,\text{OS}}}{m_{b,\overline{\text{MS}}}(\mu_b)} &= 1 + \epsilon \delta_b^{(1)} + \epsilon^2 \delta_b^{(2)} + \epsilon^3 \delta_b^{(3)} + \cdots, \qquad \delta_b^{(k)} &= \delta_{b,0}^{(k)} + \delta_{b,m}^{(k)}, \\ \delta_b^{(1)} &= \frac{\alpha_s^{[4]}(\mu)}{\pi} \left[\frac{4}{3} - L_{b,\overline{\text{MS}}} \right], & (9.1.84a) \\ \delta_{b,0}^{(2)} &= \frac{\alpha_s^{[4]2}(\mu)}{\pi^2} \left[\frac{2195}{288} + \frac{1}{9} \pi^2 + \frac{1}{9} \pi^2 \log 2 - \frac{1}{6} \zeta_3 - \frac{25}{9} L_{b,\overline{\text{MS}}}^{[\mu]} \right. \\ & - \left(\frac{205}{72} - \frac{25}{12} L_{b,\overline{\text{MS}}}^{[\mu]} \right) L_{b,\overline{\text{MS}}} - \frac{11}{24} L_{b,\overline{\text{MS}}}^2 \right], & (9.1.84b) \\ \delta_{b,0}^{(3)} &= \frac{\alpha_s^{[4]3}(\mu)}{\pi^3} \left[\frac{4903957}{93312} + \frac{439961}{38880} \pi^2 + \frac{281}{7776} \pi^4 - \frac{47}{162} (\log 2)^4 - \frac{221}{54} \pi^2 \log 2 \right. \\ & - \frac{14}{81} \pi^2 (\log 2)^2 - \frac{55}{6} \zeta_3 - \frac{1439}{432} \pi^2 \zeta_3 + \frac{1975}{216} \zeta_5 - \frac{188}{27} \text{Li}_4(\frac{1}{2}) \\ & + \left(-\frac{62267}{1728} - \frac{25}{54} \pi^2 - \frac{25}{54} \pi^2 \log 2 + \frac{25}{36} \zeta_3 \right) L_{b,\overline{\text{MS}}}^{[\mu]} + \frac{625}{108} L_{b,\overline{\text{MS}}}^{[\mu]2} \end{split}$$

$$+\left(-\frac{54859}{5184} - \frac{1}{9}\pi^2 - \frac{1}{9}\pi^2 \log 2 + \frac{13}{3}\zeta_3 + \frac{6511}{432}L_{b,MS}^{[\mu]} - \frac{625}{144}L_{b,MS}^{[\mu]2}\right)L_{b,\overline{MS}}$$

$$+\left(\frac{841}{864} + \frac{275}{144}L_{b,\overline{MS}}^{[\mu]}\right)L_{b,\overline{MS}}^2 - \frac{1231}{432}L_{b,\overline{MS}}^3\right], \qquad (9.1.84c)$$

$$\delta_{b,m}^{(1)} = 0, \qquad \delta_{b,m}^{(2)} = \frac{\alpha_s^{[4]2}(\mu)}{\pi^2}\frac{4}{3}\Delta(\rho_{\overline{MS}}), \qquad (9.1.84d)$$

$$\delta_{b,m}^{(3)} \approx \frac{\alpha_s^{[4]3}(\mu)}{\pi^3}\frac{\pi^2}{12}\rho_{\overline{MS}}\left[\beta^{(0)[4]}\left(-L_{c,\overline{MS}}^{[\mu]} - 4\log 2 + \frac{14}{3}\right)\right]$$

$$-\frac{4}{3}\left(\frac{29}{15} + 2\log 2\right) + \frac{76}{3\pi}\left(c_1c_2 + d_1d_2\right) + 2\left(L_{b,\overline{MS}} - L_{c,\overline{MS}}\right)\right], \qquad (9.1.84e)$$

$$L_{b,\overline{MS}} = \log\frac{m_{b,\overline{MS}}^2(\mu_b)}{\mu^2}, \quad L_{b,\overline{MS}}^{[\mu]} = \log\frac{m_{b,\overline{MS}}^2(\mu_b)}{\mu^2}, \quad \rho_{\overline{MS}} = \frac{m_{c,\overline{MS}}(\mu_c)}{m_{c,\overline{MS}}(\mu_c)}. \quad (9.1.84f)$$

 $\delta_b^{(1)}$, $\delta_{b,0}^{(2)}$ and $\delta_{b,0}^{(3)}$ are obtained by setting $n_f = 5$ in Eq. (9.1.22) and re-expanding $\alpha_s^{[5]}$ in terms of $\alpha_s^{[4]}$ with use of Eqs. (9.1.40) and (9.1.41). $\delta_{b,m}^{(2)}$ is the term corresponding to Eq. (9.1.23).

$$\frac{\alpha_s^{[4]}(\mu)}{\pi} \left[\frac{4}{3} - L_{b,\overline{\rm MS}} \right]$$

-11,49147072

 $5.33291891559407 * [[[(2195/288+((Pi^2)/9)+((Pi^2)/9)*0.693147)-(1.20205/6)-((25*5.769)/9)-((205*4.852)/72-((25*5.769*4.852)/12)-((11*4.852^2)/24)]]]$

$$5.33291891559407 \left(\left(\frac{2195}{288} + \frac{\pi^2}{9} + \frac{\pi^2}{9} \times 0.693147 \right) - \frac{1.20205}{6} - \frac{25 \times 5.769}{9} - \left(\frac{205 \times 4.852}{72} - \frac{1}{12} \left(25 \times 5.769 \times 4.852 \right) - \frac{1}{24} \left(11 \times 4.852^2 \right) \right) \right)$$

Result:

258.877...

Integral representations:

$$\begin{split} 5.332918915594070000 \left(\left(\frac{2195}{288} + \frac{\pi^2}{9} + \frac{0.693147 \, \pi^2}{9} \right) - \frac{1.20205}{6} - \\ \frac{25 \times 5.769}{9} - \left(\frac{205 \times 4.852}{72} - \frac{25 \times 5.769 \times 4.852}{12} - \frac{11 \times 4.852^2}{24} \right) \right) = \\ 248.975 + 4.01307 \left(\int_0^\infty \frac{1}{1 + t^2} \, dt \right)^2 \end{split}$$

$$5.332918915594070000 \left(\left(\frac{2195}{288} + \frac{\pi^2}{9} + \frac{0.693147 \pi^2}{9} \right) - \frac{1.20205}{6} - \frac{25 \times 5.769}{9} - \left(\frac{205 \times 4.852}{72} - \frac{25 \times 5.769 \times 4.852}{12} - \frac{11 \times 4.852^2}{24} \right) \right) = 248.975 + 16.0523 \left(\int_0^1 \sqrt{1 - t^2} \ dt \right)^2$$

$$\begin{split} 5.332918915594070000 \left(\left(\frac{2195}{288} + \frac{\pi^2}{9} + \frac{0.693147 \, \pi^2}{9} \right) - \frac{1.20205}{6} - \\ \frac{25 \times 5.769}{9} - \left(\frac{205 \times 4.852}{72} - \frac{25 \times 5.769 \times 4.852}{12} - \frac{11 \times 4.852^2}{24} \right) \right) = \\ 248.975 + 4.01307 \left(\int_0^\infty \frac{\sin(t)}{t} \, dt \right)^2 \end{split}$$

8.7082817709 (49.2811243840744-55.0963-357.286)

49.2811243840744

-55.0963...

-357.286...

8.7082817709 (49.2811243840744-55.0963-357.286)

Input interpretation:

8.7082817709 (49.2811243840744 - 55.0963 - 357.286)

Result:

-3161.98734860852448221064504

-3161.9873486

This value is a good approximation to the value of rest mass of vector meson J/Psi that is 3096.916±0.011

Note that:

(864+1728) - 8.7082817709 (49.2811243840744-55.0963-357.286)

Input interpretation:

 $(864 + 1728) + (49.2811243840744 - 55.0963 - 357.286) \times (-8.7082817709)$

Result:

5753.98734860852448221064504

5753.987

This result is a very good approximation to the value of the rest mass of bottom Xi baryon, that is $5787.8\pm5.0\pm1.3$; 5791.1 ± 2.2

Now we calculate the following integrals:

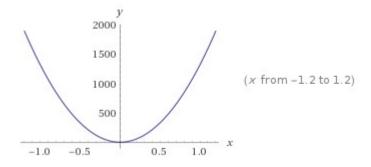
integrate (1728+729+64)/(Pi^7) [-8.7082817709 (49.2811243840744-55.0963-357.286)]x

Indefinite integral:

$$\int \frac{(1728 + 729 + 64)(-8.7082817709(49.2811243840744 - 55.0963 - 357.286))x}{\pi^7}$$

$$dx = 1319.64x^2 + \text{constant}$$

Plot of the integral:



Alternate form assuming x is real: $1319.64 x^2 + 0 + \text{constant}$

The result 1319.64 is a good approximation to the value of rest mass of baryon Xi 1314.86±0.20 1321.71±0.07

Now:

integrate sqrt [[1/(1164.2696) [- 8.7082817709 (49.2811243840744-55.0963-357.286)]]]

$$\int \sqrt{-\frac{8.7082817709 (49.2811243840744 - 55.0963 - 357.286)}{1164.2696}} \ dx = \frac{1.64799 x + constant}{1164.2696}$$

The result 1,64799 is very near to the fourteenth root of Ramanujan's class invariant and to the mass of proton

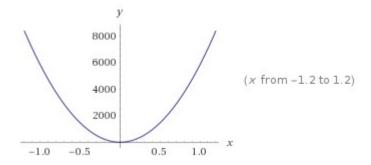
and

integrate [(1729/(1.08643^2)-24) ln [- 8.7082817709 (49.2811243840744-55.0963-357.286)]x

Indefinite integral:
$$\int \left(\frac{1729}{1.08643^2} - 24\right) \log(-8.7082817709) (49.2811243840744 - 55.0963 - 357.286)) x$$

$$dx = 5805.85 x^2 + \text{constant}$$

Plot of the integral:



The result 5805.85 is practically equal to the rest mass of the baryon bottom Sigma, that is $5811.3 \pm 1.7 + (0.9 - 0.8)$ $5815.5 \pm 1.7 + (0.6 - 0.5)$

In conclusion, we remember that:

Nonperturbative contribution [1032, 1033]

$$\Delta E^{\rm np} = \frac{\pi^2 m_q}{(C_F \alpha_s m_q)^4} \frac{624}{425} \left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu a} G^a_{\mu\nu} \right| 0 \right\rangle , \qquad (9.1.65)$$

where the gluon condensate is evaluated as [1034, 1035, 1036]

$$\left\langle 0 \left| \frac{\alpha_s}{\pi} G^{\mu\nu c} G^a_{\mu\nu} \right| 0 \right\rangle \approx 0.012 \,\text{GeV}^4 \,.$$
 (9.1.66)

and

These quantities are evaluated at a hadronic energy scale $\mu_{\rm H} \sim 1$ GeV. In Ref. [582, 584, 576], $\mu_{\rm H}$ is chosen such that the strong coupling constant satisfies $g_3(\mu_{\rm H}) = 4\pi/\sqrt{6}$. See Sec. 9.3.6 for the QCD correction factors due to the running effect between the EW scale and $\mu_{\rm H}$.

 $\beta^{(k)}$ are evaluated with $n_f = n_l$ in (9.1.1). The SU(3) color factors are

$$C_A = 3, \quad C_F = \frac{4}{3}, \quad T_F = \frac{1}{2}.$$
 (9.1.60)

 $\alpha_s = 4\pi/\sqrt{6} = 5.130199 = 5.13$

For $C_F = 4/3$ $\alpha_s = 5.13$ and $m_q = 4.776483$ MeV/ $c^2 = 0.004776483$ GeV/ c^2 (the mass of quark down is $4.8 \pm 0.5 \pm 0.3 = 4.776483$ MeV/ c^2), we obtain:

 $[Pi^2*(0.004776483)*624*0.012]/[425*(((4/3)*5.13*(0.004776483))^4))]$

Input interpretation:

$$\frac{\pi^2 \times 0.004776483 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^4}$$

Result:

729.000...

729

And for m_q = 4.77867 MeV/c² = 0.00477867 GeV/c² , we obtain $[Pi^2*(0.00477867)*624*0.012]/[425*(((4/3)*5.13*(0.00477867))^4))]$

Input interpretation:

$$\frac{\pi^2 \times 0.00477867 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.00477867\right)^4}$$

Result:

728.000... 728

We know that:

Add Lu
$$= 94^{(3)(62)}$$
 (62)

(1) $\frac{1+53x+9x^{1-}}{1-92x-92x^{1-}+x^{3}} = a_{0}+a_{1}x+a_{1}x^{1}+a_{2}x^{3}+\cdots$

(1) $\frac{2+26x-12x^{1-}}{1-92x-92x^{1-}+x^{3}} = l_{0}+l_{1}x+l_{1}x^{1}+l_{2}x^{1}+\cdots$

(1) $\frac{2+9x-10x^{1-}}{1-92x-92x^{1-}+x^{3}} = c_{0}+c_{1}x+l_{1}x^{1}+l_{2}x^{1}+\cdots$

(1) $\frac{2+9x-10x^{1-}}{1-92x-92x^{1-}+x^{3}} = c_{0}+c_{1}x+c_{1}x^{1}+l_{2}x^{1}+\cdots$

(2) $\frac{2+9x-10x^{1-}}{1-92x-92x^{1-}+x^{3}} = c_{0}+c_{1}x+c_{1}x^{1}+l_{2}x^{1}+\cdots$

(3) $\frac{2+9x-10x^{1-}}{1-92x-92x^{1-}+x^{3}} = c_{0}+c_{1}x+c_{1}x^{1}+l_{2}x^{1}+\cdots$

(4) $\frac{2+9x-10x^{1-}}{1-92x-92x^{1}+x^{3}} = c_{0}+c_{1}x+c_{1}x^{1}+l_{2}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}x^{1}+c_{1}$

$$9^3 + 10^3 = 12^3 + 1 = 1729$$
; $6^3 + 8^3 = 9^3 - 1 = 728$;

Practically, the result of the above expression concerning the nonperturbative contributions of the mass of a 1S quarkonium, is equal to a fundamental Ramanujan's numbers: 728 and 729. Furthermore, from the same formula, we obtain the number 1729. Indeed:

$$10^3 + [Pi^2*(0.004776483)*624*0.012]/[425*(((4/3)*5.13*(0.004776483))^4))]$$

Input interpretation:

$$10^{3} + \frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$$

Result:

1729.00...

1729

Further, 1729 is also the fundamental number that is in the range of the mass of the candidate "glueball" $f_0(1710)$:

fo(1710) MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1723 + 6 OU	R AVERAGE	Error includes scale	factor	of 1.6. See the ideogram below.
1720±10 ±	= 10	9 BALTRUSAIT		MRK3 $J/\psi \rightarrow \gamma K^+ K^-$
1742 ± 15		⁸ WILLIAMS	84	MPSF $200 \pi^- N \rightarrow 2K_S^0 X$
1726± 7	74	13 CHEKANOV	04	ZEUS $ep \rightarrow K_S^0 K_S^0 X$
1732+15		¹⁴ ANISOVICH	03	RVUE
1726± 7	74	13 CHEKANOV	04	ZEUS $ep \rightarrow K_S^0 K_S^0 X$
1732+15		¹⁴ ANISOVICH	03	RVUE
1744±15		22 ALDE	92D G	AM2 38 $\pi^- p \rightarrow \eta \eta n$
$1730 + 2 \\ -10$	2	⁷ LONGACRE 86	5 RV	UE $22 \pi^- p \rightarrow n2K_S^0$

Indeed, in we take the various masses, we have the following means: 1729, 1731, 1729, 1729, 1744 – 15 = 1729; 1730 + 2 = 1732, with a partial mean of 1729,83. The mean adding the number with the minus sign is 1726,22, while with the sign positive is 1740,77 that less the algebraic sum of the difference – 69 + 77 = 8 is equal to 1732.77. The final mean is 1729,6

The complete develop of the two above expressions is:

$$[Pi^2*(0.004776483)*624*0.012]/[425*(((4/3)*5.13*(0.004776483))^4))]$$

$$\frac{\pi^2 \times 0.004776483 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^4}$$

Result:

729.000...

Alternative representations:

$$\frac{\pi^2 \cdot (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = \frac{0.0357663 \cdot (180 \, ^\circ)^2}{425 \times 0.0326711^4}$$

$$\frac{\pi^2 (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = \frac{0.0357663 (-i \log(-1))^2}{425 \times 0.0326711^4}$$

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = \frac{0.214598 \, \zeta(2)}{425 \times 0.0326711^4}$$

Series representations:

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 1181.81 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}\right)^2$$

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 295.453 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2$$

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 73.8631 \left(\sum_{k=0}^{\infty} \frac{2^{-k} \left(-6 + 50 \, k\right)}{\binom{3 \, k}{k}}\right)^2$$

Integral representations:

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 295.453 \left(\int_0^\infty \frac{1}{1+t^2} \, dt\right)^2$$

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 1181.81 \left(\int_0^1 \sqrt{1-t^2} \ dt\right)^2$$

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 295.453 \left(\int_0^\infty \frac{\sin(t)}{t} \, dt\right)^2$$

and

$$10^3 + [Pi^2*(0.004776483)*624*0.012]/[425*(((4/3)*5.13*(0.004776483))^4))]$$

$$10^{3} + \frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425 \left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$$

Result:

1729.00...

Alternative representations

$$10^3 + \frac{\pi^2 \ (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 10^3 + \frac{0.0357663 \ (180 \, ^\circ)^2}{425 \times 0.0326711^4}$$

$$10^3 + \frac{\pi^2 \ (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 10^3 + \frac{0.0357663 \ (-i \log(-1))^2}{425 \times 0.0326711^4}$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 10^{3} + \frac{0.214598 \, \zeta(2)}{425 \times 0.0326711^{4}}$$

Series representations:

$$10^3 + \frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 1000 + 1181.81 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2 k}\right)^2$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 1000 + 295.453 \left[-1 + \sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right]^{2}$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 1000 + 73.8631 \left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50 k)}{\binom{3 k}{k}}\right)^{2}$$

Integral representations:

$$10^3 + \frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^4} = 1000 + 295.453 \left(\int_0^\infty \frac{1}{1+t^2} \, dt\right)^2$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 1000 + 1181.81 \left(\int_{0}^{1} \sqrt{1 - t^{2}} \ dt\right)^{2}$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 1000 + 295.453 \left(\int_{0}^{\infty} \frac{\sin(t)}{t} dt\right)^{2}$$

Now, we have:

$$\frac{\pi^2 \left(0.00477648 \times 624 \times 0.012\right)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{2}\right)^4} = 1181.81 \left(\int_0^1 \sqrt{1-t^2} \ dt\right)^2$$

$$10^{3} + \frac{\pi^{2} (0.00477648 \times 624 \times 0.012)}{425 \left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}} = 1000 + 1181.81 \left(\int_{0}^{1} \sqrt{1 - t^{2}} \ dt\right)^{2}$$

From the following integral representations of 729 and 1729, we can to obtain the value 1181,81 a number very near to 1164.2696 that is the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696$

$$\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3 = 1164,269601267364$$

For the value 1181,81 we have that:

$$\sqrt[14]{1181,81} = 1,657554016$$

and

$$\sqrt{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,65578 \dots$$

We note that $1,65755 \approx 1,65578$. The values are also very near to the mass of the proton.

From Ramanujan's Notebook part II:

Integrals and asymptotic expansions:

Entry 9. If

$$\varphi(m) = \int_0^\infty \frac{e^{-m^2x^2}}{1+x^2} dx$$

and if $|m| \ge |n|$, where m and n are real, then

$$\int_0^\infty \frac{e^{-m^2x^2}}{1+x^2} \cos(2mnx) \, dx = \frac{e^{-n^2}}{2} \{ \varphi(m+n) + \varphi(m-n) \}. \tag{9.1}$$

We have that, for m = 32 and n = 27:

integrate $[(e^{-32x^2})/(1+x^2)) \cos 1728] \times [0...infinity]$

Definite integral:

$$\int_0^\infty \frac{\left(e^{-32x^2}\cos(1728^\circ)\right)x}{1+x^2} dx = -\frac{1}{8}\left(\sqrt{5} - 1\right)e^{32} \operatorname{Ei}(-32) \approx 0.00468615$$

Ei(x) is the exponential integral Ei

Indefinite integral:

$$\int \frac{\left(e^{-32x^2}\cos(1728^\circ)\right)x}{1+x^2} dx = \frac{1}{8}\left(\sqrt{5}-1\right)e^{32}\operatorname{Ei}\left(-32\left(x^2+1\right)\right) + \operatorname{constant}$$

Integral representations

$$\begin{split} &\frac{1}{8} \left(-1 + \sqrt{5}\right) (-1) \, e^{32} \, \operatorname{Ei}(-32) = -\frac{1}{8} \left(-1 + \sqrt{5}\right) e^{32} \left(\gamma + \int_0^{-32} \frac{-1 + e^t}{t} \, dt + \log(32)\right) \\ &\frac{1}{8} \left(-1 + \sqrt{5}\right) (-1) \, e^{32} \, \operatorname{Ei}(-32) = -\frac{1}{8} \left(-1 + \sqrt{5}\right) e^{32} \, \mathcal{P} \int_{-\infty}^{-32} \frac{e^t}{t} \, dt \\ &\frac{1}{8} \left(-1 + \sqrt{5}\right) (-1) \, e^{32} \, \operatorname{Ei}(-32) = \frac{1}{8} \left(-1 + \sqrt{5}\right) e^{32} \, \mathcal{P} \int_{32}^{\infty} \frac{e^{-t}}{t} \, dt \end{split}$$

And 1/0.00468615 = 213.394791

[(df.(1285))/F...

 $0.193 \pm 0.013 \pm 0.029$

We have, with regard the meson particles:

(φ/1(1200))/ tota					52/1
VALUE (units 10 ⁻⁴)	EVTS	DOCUMENT ID		TECN	COMMENT
2.6±0.5 OUR AVERA	GE Erro	r includes scale fact	or of	1.1.	
$3.2 \pm 0.6 \pm 0.4$		JOUSSET	90	DM ₂	$J/\psi \rightarrow \phi 2(\pi^{+}\pi^{-})$
$2.1 \pm 0.5 \pm 0.4$	25	74 JOUSSET	90	DM ₂	$J/\psi \to \phi \eta \pi^+ \pi^-$
$\Gamma(\rho\eta)/\Gamma_{\text{total}}$					Γ ₅₄ /Γ
VALUE (units 10 ⁻³)	EVTS	DOCUMENT ID		TECN	COMMENT
0.193±0.023 OUR AVE	RAGE			101111	
$0.194 \pm 0.017 \pm 0.029$	299	JOUSSET	90	DM ₂	$J/\psi ightarrow ext{hadrons}$

 $\Gamma_{\rm Fo}/\Gamma$

MRK3 $e^+e^- \rightarrow \pi^+\pi^-n$

Note that 2.6 - 0.5 = 2.1 or 3.2 - 0.6 - 0.4 = 2.2 and 0.193 + 0.023 = 0.216 are value very near to 213,394791 that is the result of the integral.

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COFFMAN

From:

In Ramanujan's famous last letter to Hardy in 1920, he gives 17 examples of mock theta functions, without giving any complete definition of this term. A typical example (Ramanujan's second mock theta function of "order 7" — a notion that he also does not define) is

$$\mathcal{F}_7(\tau) = -q^{-25/168} \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1-q^n)\cdots(1-q^{2n-1})}$$
$$= -q^{143/168} \left(1+q+q^2+2q^3+\cdots\right).$$

For $q = e^{2\pi i \tau}$ for $i\tau > 0$ (we take $i\tau = 1$), we obtain:

$$(-e^{2\pi})^{143/168} (1 + (e^{2\pi}) + (e^{2\pi})^2 + 2(e^{2\pi})^3 + ...) =$$

= -210,2269147(1+535,49165+286751,313+307105870,79+...) =
= -64622315330,61;

We have: $64622315330,61 / 1728^3 = 12,5242376$

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture%20Notes-Problems/Witten_Threedimgravity.pdf)

Let us give an example. If k = 1, the partition function is simply the J-function itself, so

$$Z(q) = q^{-1} + 196884q + \dots$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore log(196883)=12.19... The classical entropy of a black hole with k=1 and mass 2 is $4\pi=12.57...$ So we are off by just a few percent.

We note that the value that we have obtained 12,524... is a very good approximation of the value 12,57... that is the classical entropy of a black hole with k = 1 and mass 2.

Note that
$$-\sqrt[49]{64622315330,61} = 1,661958...$$

 $-\sqrt[50]{64622315330,61} = 1,645158...$

The results 1,661958 and 1,645158 are very near to the fourteenth root of Ramanujan class invariant and to the mass of proton.

From
$$-\ln(64622315330,61) = 24,8918256...$$

We have, with regard the meson particle:

$$\Gamma(2(π^+π^-)K^+K^-)$$
 × $\Gamma(e^+e^-)/\Gamma_{total}$ $\Gamma_{102}\Gamma_3/\Gamma_3/\Gamma_3$
 $VALUE~(10^{-2}~keV)$ $EVTS$ $DOCUMENT~ID$ $TECN$ $COMMENT$
2.75±0.23±0.17 205 AUBERT 06D BABR $10.6~e^+e^- \rightarrow K^+K^-2(π^+π^-)γ$

 $\Gamma(\phi K^*(892)\overline{K} + \text{c.c.})/\Gamma_{\text{total}}$ Γ_{27}/Γ VALUE (units 10-4) DOCUMENT ID TECN COMMENT 21.8 ± 2.3 OUR AVERAGE 08E BES2 $J/\psi \rightarrow \phi K_S^0 K^{\pm} \pi^{\mp}$ 08E BES2 $J/\psi \rightarrow \phi K^+ K^- \pi^0$ $20.8 \pm 2.7 \pm 3.9$ 195 ± 25 ABLIKIM $29.6 \pm 3.7 \pm 4.7$ 238 ± 30 ABLIKIM 88 DM2 $J/\psi \rightarrow$ hadrons $20.7 \pm 2.4 \pm 3.0$ FALVARD 87 MRK3 $e^+e^- \rightarrow hadrons$ $20 \pm 3 \pm 3$ 155 ± 20 BECKER $\Gamma(p\overline{p}\phi)/\Gamma_{\text{total}}$ Γ_{98}/Γ VALUE (units 10-4) TECN COMMENT $0.45 \pm 0.13 \pm 0.07$ 88 DM2 $J/\psi \rightarrow \text{hadrons}$ FALVARD $\Gamma(2(\pi^+\pi^-)\eta)/\Gamma_{\text{total}}$ Γ_{88}/Γ VALUE (units 10⁻³) EVTS DOCUMENT ID TECN COMMENT 2.29 ± 0.24 OUR AVERAGE 100 AUBERT 07AU BABR 10.6 $e^+e^- \rightarrow 2(\pi^+\pi^-)\eta\gamma$ $2.35\pm0.39\pm0.20$ ABLIKIM 05C BES2 $e^+e^- \rightarrow 2(\pi^+\pi^-)n$ $2.26\pm0.08\pm0.27$ 4839 100 AUBERT 07AU quotes $\Gamma_{ee}^{J/\psi}\cdot \mathrm{B}(J/\psi \to 2(\pi^+\pi^-)\eta)\cdot \mathrm{B}(\eta \to \gamma\gamma) = 5.16\pm0.85\pm0.85$ 0.39 eV.

We have: 2.75 - 0.23 - 0.17 = 2.35 21.8 + 2.3 = 24.1 0.45 - 0.13 - 0.07 = 0.25

2.29 + 2.24 = 2.53 values very near to the result of the ln of Ramanujan's second mock theta function of "order 7" that is 24,8918256

and for the following values of lambda charmed baryon:

$$\Gamma(\Sigma(1385)^+\eta)/\Gamma(\rho K^-\pi^+)$$
 Unseen decay modes of the $\Sigma(1385)^+$ and η are included.
VALUE POSSIBLE DOCUMENT ID TECN COMMENT TO 17±0.04±0.03 54 AMMAR 95 CLE2 $e^+e^-\approx \Upsilon(4S)$

$$\Gamma(\Sigma^0\pi^+)/\Gamma(pK^-\pi^+)$$

VALUE

0.210 ± 0.018 OUR FIT

0.20 ± 0.04 OUR AVERAGE

0.21 ± 0.02 ± 0.04 196 AVERY
0.17 ± 0.06 ± 0.04 AVERY

ALBRECHT

92 ARG $e^+e^-\approx 10.4 \text{ GeV}$

$$\Gamma(\Sigma^{+}\pi^{0})/\Gamma(pK^{-}\pi^{+})$$
 $VALUE$
 $VALUE$

we have: 0.21 + 0.03 + 0.02 = 0.26; 0.17 + 0.04 + 0.03 = 0.24; 0.20 + 0.04 = 0.24; 0.20 + 0.03 = 0.26 all values very near to the value **24,89**18...

We have the following formulae:

$$F(\tau) = \frac{1}{\eta^{24}(\tau)} = \sum_{n=-1}^{\infty} c(n)q^n$$

$$d(n) := c(n) = p_{24}(n+1)$$

where $\eta(au)$ is the familiar Dedekind eta function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)$$
 with $q := e^{2\pi i \tau}$

$$\psi_m^{\mathrm{P}} := \frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{s \in \mathbb{Z}} \frac{q^{ms^2+s} y^{2ms+1}}{(1-q^s y)^2}$$

$$\tau_2^{3/2} \frac{\partial}{\partial \bar{\tau}} \widehat{\psi_m^F}(\tau, z) = \sqrt{\frac{m}{8\pi i}} \frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{\ell \mod 2m} \overline{\vartheta_{m,\ell}(\tau)} \, \vartheta_{m,\ell}(\tau, z)$$

We have that for $i\tau = 1$, n = 6, c(n) = d(n) = -12; $\eta(\tau) = 30634746108626862,17$ ms = 4; y = 3

$$\frac{p_{24}(m+1)}{\eta^{24}(\tau)}$$
 is equal to -1,678766 * 10⁻¹⁶

$$\frac{q^{ms^2+s}y^{2ms+1}}{(1-q^sy)^2}$$
 is equal to = 3,4864841910330477 * 10⁴¹

$$\psi_{m}^{P} := \frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{s \in \mathbb{Z}} \frac{q^{ms^{2}+s} y^{2ms+1}}{(1-q^{s}y)^{2}}$$

 $= -5.8529911194437853551382000 * 10^{25}$

We have that:

 $\psi_{\it m}^{\rm P}$ is the counting function of multi-centered black holes

And 5852,9 is very near to the value of 5832.1 \pm 0.7 and 5835.1 \pm 0.6 that is the mass of bottom Sigma baryons $\underline{\Sigma_{*_b}}^+$ and $\underline{\Sigma_{*_b}}^-$.

From Ramanujan's Notebook part II:

Corollary (ii). If n is a positive integer, as x tends to ∞ ,

$$\sum_{k=0}^{\infty} \left(\frac{x^k}{k!} \right)^n \sim \frac{\exp\left\{ nx + \frac{n^2 - 1}{24} \left(\frac{1}{nx} + \frac{1}{2n^2 x^2} + \cdots \right) \right\}}{\sqrt{n(2\pi x)^{(n-1)/2}}}.$$
 (10.23)

For
$$x = 3$$
, $n = \sqrt{1729}$

$$\exp\!\left(\!\sqrt{1729}\times\!3+\frac{1}{24}\left(\sqrt{1729}^2-1\right)\!\right)\!\left(\!\frac{1}{\sqrt{1729}\times\!3}+\frac{1}{2\sqrt{1729}^2\times\!3^2}\right)$$

$$\left(\frac{1}{31\,122} + \frac{1}{3\,\sqrt{1729}} \right) e^{72 + 3\,\sqrt{1729}}$$

Decimal approximation:

 $2.2409800073806500716620423861317428352710513537297875...\times 10^{83}$

Property:

$$\left(\frac{1}{31122} + \frac{1}{3\sqrt{1729}}\right)e^{72+3\sqrt{1729}}$$
 is a transcendental number

Alternate forms:

$$\frac{\left(1+6\sqrt{1729}\right)e^{72+3\sqrt{1729}}}{31\,122}$$

$$\frac{\left(10\,374+\sqrt{1729}\right)e^{72+3\sqrt{1729}}}{31\,122\sqrt{1729}}$$

$$\frac{e^{72+3\sqrt{1729}}}{31\,122}+\frac{e^{72+3\sqrt{1729}}}{3\sqrt{1729}}$$

Comparison:

≈ 2200 × the number of atoms in the visible universe (≈10⁸⁰)

Series representations:

$$\begin{split} \exp\!\left(\!\sqrt{\,1729\,\,}^3 + \frac{1}{24} \left(\sqrt{\,1729}^{\,2} - 1\right)\!\right)\! \left(\!\frac{1}{\sqrt{\,1729}\,\,}^3 + \frac{1}{2\,\sqrt{\,1729}^{\,2}\,\,}^2 3^2\right) &= \\ \left(\!\exp\!\left(\!\frac{1}{24} \left(\!-1 + 72\,\sqrt{\,1728}\,\sum_{k=0}^\infty 1728^{-k} \left(\!\frac{\frac{1}{2}}{k}\right)\!+ \sqrt{\,1728}^{\,2} \left(\!\sum_{k=0}^\infty 1728^{-k} \left(\!\frac{\frac{1}{2}}{k}\right)\!\right)\!\right)\!\right) \\ \left(\!1 + 6\,\sqrt{\,1728}\,\sum_{k=0}^\infty 1728^{-k} \left(\!\frac{\frac{1}{2}}{k}\right)\!\right)\!\right) / \left(\!18\,\sqrt{\,1728}^{\,2} \left(\!\sum_{k=0}^\infty 1728^{-k} \left(\!\frac{\frac{1}{2}}{k}\right)\!\right)\!\right)\!\right) \end{split}$$

$$\begin{split} \exp\!\left(\!\sqrt{\,1729\,\,}^{\,3} + \frac{1}{24} \left(\sqrt{\,1729}^{\,2} - 1\right)\!\right)\! \left(\!\frac{1}{\sqrt{\,1729}\,\,}^{\,3} + \frac{1}{2\,\sqrt{\,1729}^{\,2}\,\,}^{\,2} \right) &= \\ \left(\!\exp\!\left(\!\frac{1}{24} \left(\!-1 + 72\,\sqrt{\,1728}\,\sum_{k=0}^{\infty} \frac{\left(\!-\frac{1}{1728}\right)^{\!k} \left(\!-\frac{1}{2}\right)_{\!k}}{k!} + \sqrt{\,1728}^{\,2} \left(\!\sum_{k=0}^{\infty} \frac{\left(\!-\frac{1}{1728}\right)^{\!k} \left(\!-\frac{1}{2}\right)_{\!k}}{k!}\right)\!\right)\!\right) \\ \left(\!1 + 6\,\sqrt{\,1728}\,\sum_{k=0}^{\infty} \frac{\left(\!-\frac{1}{1728}\right)^{\!k} \left(\!-\frac{1}{2}\right)_{\!k}}{k!}\right)\!\right)\! / \left(\!18\,\sqrt{\,1728}^{\,2} \left(\!\sum_{k=0}^{\infty} \frac{\left(\!-\frac{1}{1728}\right)^{\!k} \left(\!-\frac{1}{2}\right)_{\!k}}{k!}\right)\!\right)\!\right) \end{split}$$

Open code

$$\begin{split} \exp\!\left(\!\sqrt{\,1729\,\,}\,3 + \frac{1}{24}\left(\sqrt{\,1729}^{\,2} - 1\right)\!\right)\!\left(\!\frac{1}{\sqrt{\,1729}\,\,}\,3 + \frac{1}{2\,\sqrt{\,1729}^{\,2}\,\,}3^2\right) &= \\ \left(\!2\exp\!\left(\!\frac{1}{96\,\sqrt{\pi}^{\,2}}\!\left(\!-4\,\sqrt{\pi}^{\,2} + 144\,\sqrt{\pi}\,\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,1728^{-s}\,\,\Gamma\!\left(\!-\frac{1}{2}-s\right)\!\Gamma\!(s)\right) + \\ \left(\!\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,1728^{-s}\,\,\Gamma\!\left(\!-\frac{1}{2}-s\right)\!\Gamma\!(s)\!\right)\!\right)\!\right) \\ \sqrt{\pi}\left(\!\sqrt{\pi}\,+3\,\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,1728^{-s}\,\,\Gamma\!\left(\!-\frac{1}{2}-s\right)\!\Gamma\!(s)\!\right)\!\right)\!/ \\ \left(\!9\left(\!\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,1728^{-s}\,\,\Gamma\!\left(\!-\frac{1}{2}-s\right)\!\Gamma\!(s)\!\right)\!\right)\!\right) \end{split}$$

1/((sqrt(sqrt(1729))*(2Pi*3)^20.2906225)))

$$\frac{1}{\sqrt{\sqrt{1729}} (2 \pi \times 3)^{20.2906225}}$$

Result:

 $2.060144... \times 10^{-27}$

Series representations:

$$\frac{1}{\sqrt{\sqrt{1729}} \ (2\,\pi\,3)^{20.2906}} = \frac{1.6249\times 10^{-16}}{\pi^{20.2906}\,\sqrt{-1+\sqrt{1729}}} \, \sum_{k=0}^{\infty} \left(\frac{\frac{1}{2}}{k}\right) \! \left(-1+\sqrt{1729}\,\right)^{\!-\!k}}$$

$$\frac{1}{\sqrt{\sqrt{1729}}} \frac{1}{(2\,\pi\,3)^{20.2906}} = \frac{1.6249\times 10^{-16}}{\pi^{20.2906}\,\sqrt{-1+\sqrt{1729}}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+\sqrt{1729}\right)^{-k}}{k!}}$$

$$\frac{1}{\sqrt{\sqrt{1729}}} \frac{1}{(2\,\pi\,3)^{20.2906}} = \frac{3.2498\times 10^{-16}\,\sqrt{\pi}}{\pi^{20.2906}\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,\Gamma\!\left(-\frac{1}{2}-s\right)\!\Gamma\!\left(s\right)\left(-1+\sqrt{1729}\,\right)^{-s}}$$

$$\exp\left(\sqrt{1729 \times 3} + \frac{1}{24} \left(\sqrt{1729}^2 - 1\right)\right)$$
$$\left(\frac{1}{\sqrt{1729 \times 3}} + \frac{1}{2\sqrt{1729}^2 \times 3^2}\right) \times 2.060144 \times 10^{-27}$$

Result:

$$4.616742... \times 10^{56}$$

Comparisons:

≈ 570 × the size of the Monster group (≈8.1×10⁵³)

 $\approx 8.8 \times 10^6 \times \text{the number of chess positions} \ (\approx 5.2 \times 10^{49})$

Input interpretation:

$$\frac{1}{1728000^8} \times \frac{1}{1728 + 576} \left(exp \left(\sqrt{1729} \times 3 + \frac{1}{24} \left(\sqrt{1729}^2 - 1 \right) \right) \right.$$
$$\left. \left(\frac{1}{\sqrt{1729} \times 3} + \frac{1}{2\sqrt{1729}^2 \times 3^2} \right) \times 2.060144 \times 10^{-27} \right)$$

Result:

2520.596...

Note that:

Baryons are <u>composite particles</u> made of three <u>quarks</u>, as opposed to <u>mesons</u>, which are composite particles made of one quark and one antiquark. <u>Baryons</u> and mesons are both <u>hadrons</u>, which are particles composed solely of quarks or both quarks and antiquarks.

$J^{P} = \frac{3}{2}$ baryons											
Particle n	ame Symbol	Quark content	Rest mass (MeV/c²)	i	J٤	Q (e)	<u>s</u>	<u>c</u>	<u>B'</u>	Mean lifetime (s)	Commonly decays to
harmed igma ^[31]	Σ****(2520)	uuc	2517.9±0.6	1	3/2 +	+2	0	+1	0	(4.42 ±0.44) ×10 ^{-23[h]}	$\sqrt{c} + \overline{u}^{\dagger}$
harmed igma ^[31]	Σ*+(2520)	udc	2517.5 ±2.3	1	3/2 +	+1	0	+1	0	>3.87 × 10 ^{-23[h]}	$\Lambda_{\varepsilon}^{+} + \underline{\pi}^{0}$
narmed igma ^[31]	Σ*°(2520)	ddc	2518.8±0.6	1	3/2+	0	0	+1	0	(4.54 ± 0.47) × 10 ^{-23 [h]}	<u>Λ</u> ⁺ + <u>π</u>

CHARMED BARYONS
$$(C = +1)$$

$$\begin{array}{lll} \varLambda_c^+ = udc, & \varSigma_c^{++} = uuc, & \varSigma_c^+ = udc, & \varSigma_c^0 = ddc, \\ & \Xi_c^+ = usc, & \Xi_c^0 = dsc, & \varOmega_c^0 = ssc \end{array}$$

$$\Sigma_c(2520)$$

$$I(J^P) = 1(\frac{3}{2}^+)$$

 J^P has not been measured; $\frac{3}{2}$ is the quark-model prediction.

$$\begin{split} & \Sigma_c(2520)^{++} \text{mass } m = 2518.4 \pm 0.6 \text{ MeV} \quad \text{(S} = 1.4) \\ & \Sigma_c(2520)^{+} \quad \text{mass } m = 2517.5 \pm 2.3 \text{ MeV} \\ & \Sigma_c(2520)^{0} \quad \text{mass } m = 2518.0 \pm 0.5 \text{ MeV} \\ & m_{\Sigma_c(2520)^{++}} - m_{\Lambda_c^+} = 231.9 \pm 0.6 \text{ MeV} \quad \text{(S} = 1.5) \\ & m_{\Sigma_c(2520)^{+}} - m_{\Lambda_c^+} = 231.0 \pm 2.3 \text{ MeV} \\ & m_{\Sigma_c(2520)^{0}} - m_{\Lambda_c^+} = 231.6 \pm 0.5 \text{ MeV} \quad \text{(S} = 1.1) \\ & m_{\Sigma_c(2520)^{++}} \quad m_{\Sigma_c(2520)^{0}} = 0.3 \pm 0.6 \text{ MeV} \quad \text{(S} = 1.2) \\ & \Sigma_c(2520)^{++} \quad \text{full width } \Gamma = 14.9 \pm 1.9 \text{ MeV} \\ & \Sigma_c(2520)^{+} \quad \text{full width } \Gamma < 17 \text{ MeV, CL} = 90\% \\ & \Sigma_c(2520)^{0} \quad \text{full width } \Gamma = 16.1 \pm 2.1 \text{ MeV} \end{split}$$

 $\Lambda_C^+\pi$ is the only strong decay allowed to a Σ_C having this mass.

Σ_c (2520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\Lambda_c^+ \pi$	pprox 100 %	180

The values of the mass of the charmed baryons, precisely the charmed Sigma $\Sigma_{\rm c}(2520)$, that are: 2517.9 ± 0.6 2517.5 ± 2.3 2518.8 ± 0.6 , are all very good approximations to the result obtained from the Ramanujan expression analyzed above: 2520.596

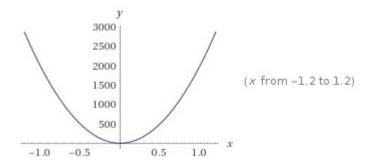
If we calculate the following simple integral:

$$1728/(2*10^56) \quad integrate \quad exp((((sqrt(1729)*3)+(((sqrt(1729)^2)-1))/24)))*((1/(sqrt(1729)*3)+(1/((2*(sqrt((1729))^2)3^2)))))) * (2.060144*10^-27)x$$

$$\frac{1728}{2 \times 10^{56}} \int \exp \left(\sqrt{1729} \times 3 + \frac{1}{24} \left(\sqrt{1729}^2 - 1 \right) \right) \\ \left(\frac{1}{\sqrt{1729} \times 3} + \frac{1}{2\sqrt{1729}^2 \times 3^2} \right) \times 2.060144 \times 10^{-27} \, x \, dx$$

Result:

1994.43 x^2



Alternate form assuming x is real:

$$1994.43 x^2 + 0$$

Indefinite integral assuming all variables are real:

the result 1994.43 is very near to the pseudoscalar meson strange D mass and to the vector meson D mass 1968.49 ± 0.34 2006.97 ± 0.19 with a difference of -26 and -12 (also 26 and 12 are significant numbers).

Now:

Entry 11(i). As x tends to ∞ ,

$$\sum_{k=0}^{\infty} \left(\frac{ex}{k} \right)^k \sim \sqrt{2\pi x} \exp \left(x - \frac{1}{24x} - \frac{1}{48x^2} - \left(\frac{1}{36} + \frac{1}{5760} \right) \frac{1}{x^3} + \cdots \right)$$

For x = 6, we obtain:

$$\sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \left(\frac{1}{36} + \frac{1}{5760}\right) \times \frac{1}{216}\right)$$

Exact result:

$$2 e^{7455 \, 439/1244 \, 160} \sqrt{3 \, \pi}$$

Decimal approximation:

2458.153445356729373894365991193740075188431305325431030927...

Continued fraction:

Series representations:

$$\sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \frac{1}{216} \left(\frac{1}{36} + \frac{1}{5760}\right)\right) = \exp\left(\frac{7455439}{1244160}\right) \sqrt{-1 + 12\pi} \sum_{k=0}^{\infty} (-1 + 12\pi)^{-k} \left(\frac{1}{2}\right)$$

$$\sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \frac{1}{216} \left(\frac{1}{36} + \frac{1}{5760}\right)\right) = \exp\left(\frac{7455439}{1244160}\right) \sqrt{-1 + 12\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 12\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \frac{1}{216} \left(\frac{1}{36} + \frac{1}{5760}\right)\right) = \exp\left(\frac{7455439}{1244160}\right) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (12\pi - z_0)^k z_0^{-k}}{k!}$$
for not $\left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)$

The result 2458,1534 is very near to the value of the mass of charmed Sigma: 2453.98 ± 0.16 2452.9 ± 0.4 2453.74 ± 0.16 of charmed Xi $2470.88(\pm0.34-0.80)$ and to decay width² of boson Z that is 2.4952 ± 0.0023 GeV/ $c^2 = 2495.2\pm2.3$ MeV/ c^2

$$f(E)\sim rac{1}{\left(E^2-M^2
ight)^2+M^2\Gamma^2}$$

(Questa equazione è scritta usando <u>unità naturali</u>, $\hbar = c = 1$.) Viene molto più spesso usata per modellare le <u>risonanze</u> (particelle instabili) nella <u>fisica</u> ad alta energia. In questo caso E è l'energia del centro di massa che produce la risonanza, M è la massa della risonanza, e Γ è la larghezza di risonanza (o larghezza di decadimento "decay width"), relativa alla sua vita media secondo la formula $\tau = \hbar/\Gamma$. La probabilità di produrre risonanza a una data energia E è proporzionale a f(E), in modo che il grafico del tasso di produzione della particella instabile in funzione dell'energia tracci la forma della distribuzione Breit-Wigner relativistica.

In generale, Γ può essere anche una funzione di E; questa dipendenza è in genere importante solo quando Γ non è piccola in confronto a M e la dipendenza spazio-fase della larghezza va presa in considerazione. (Per esempio, nel decadimento del mesone rho in una coppia di pioni.) Il fattore M^2 che moltiplica Γ^2 andrebbe sostituito con E^2 (o E^4/M^2 , ecc.) quando la risonanza è ampia. [2]

La forma della distribuzione Breit-Wigner relativistica sorge dal propagatore di una particella instabile, che ha un denominatore della forma $p^2 - M^2$ + iΓ. Qui p² è il quadrato del quadrimpulso portato dalla particella. Il propagatore appare nella ampiezza della meccanica quantistica per il processo che produce la risonanza; la distribuzione della probabilità risultante è proporzionale al quadrato assoluto dell'ampiezza, producendo la distribuzione Breit-Wigner relativistica per la funzione di densità della probabilità come descritta precedentemente.

La forma di questa distribuzione è simile alla soluzione dell'equazione classica del moto per una oscillatore armonico smorzato (dumped) condotto da una forza esterna sinusoidale"

² Note in Italian "La distribuzione Breit-Wigner relativistica (chiamata così dai nomi di <u>Gregory Breit</u> e <u>Eugene Wigner</u>) è una <u>distribuzione di</u> probabilità continua con la seguente funzione di densità di probabilità $f(E) \sim \frac{1}{(E^2-M^2)^2+M^2\Gamma^2}$.

$\Sigma_c(2455)$ MASSES

The masses are obtained from the mass-difference measurements that follow.

 $\Sigma_c(2455)^{++}$ MASS

VALUE (MeV)

2453.98 \pm 0.16 OUR FIT $\Sigma_c(2455)^{+}$ MASS

VALUE (MeV)

2452.9 \pm 0.4 OUR FIT $\Sigma_c(2455)^0$ MASS

VALUE (MeV)

2453.74 \pm 0.16 OUR FIT

From Wikipedia:

The W and Z bosons are together known as the weak or more generally as the intermediate vector bosons. These elementary particles mediate the weak interaction; the respective symbols are W, W, and Z. The W bosons have either a positive or negative electric charge of 1 elementary charge and are each other's antiparticles. The Z boson is electrically neutral and is its own antiparticle. The three particles have a spin of 1. The W bosons have a magnetic moment, but the Z has none.

Z bosons decay into a fermion and its antiparticle. As the Z boson is a mixture of the pre-symmetry-breaking W⁰ and B⁰ bosons (see weak mixing angle), each vertex factor includes a factor $T_3 - Q \sin^2 \theta_{W}$; where T_3 is the third component of the weak isospin of the fermion, Q is the electric charge of the fermion (in units of the elementary charge), and θ_{W} is the weak mixing angle. Because the weak isospin is different for fermions of different chirality, either left-handed or right-handed, the coupling is different as well.

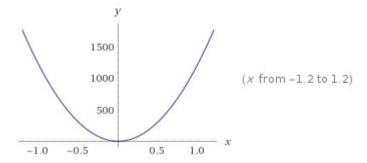
If we calculate the following simple integral:

integrate
$$(sqrt(12Pi)) * [(exp(6-1/144-1/(48*36)-(1/36+1/5760)*1/216))]x$$

Indefinite integral:

$$\int \sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \frac{1}{216} \left(\frac{1}{36} + \frac{1}{5760}\right)\right) x \, dx \approx \text{constant} + 1229.08 \, x^2$$

Plot of the integral:



we obtain the value very near to the delta baryons rest mass: 1232 ± 2

Δ(1232) BREIT-WIGNER MASSES

MIXED CHARGES				
VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
1230 to 1234 (≈ 1232) OUR E				
1228 ±2	ANISOVICH			Multichannel
1233.4 ± 0.4	ARNDT	06		$\pi N \rightarrow \pi N, \eta N$
1232 ±3	CUTKOSKY	80		$\pi N \rightarrow \pi N$
1233 ±2	HOEHLER	79	IPWA	$\pi N \rightarrow \pi N$
 • • We do not use the following 	ng data for average	s, fits,	limits,	etc. • • •
1231.1 <u>1</u> 0.2	SHRESTHA	12A	DPWA	Multichannel
1230 ±2	ANISOVICH	10	DPWA	Multichannel
1232.9 ± 1.2	ARNDT	04	DPWA	$\pi N \rightarrow \pi N, \eta N$
1228 ±1	PENNER	02C	DPWA	Multichannel
1234 ±5	VRANA	00	DPWA	Multichannel
1233	ARNDT	95	DPWA	$\pi N \rightarrow N \pi$
1231 ±1	MANLEY	92	IPWA	$\pi N \rightarrow \pi N \& N \pi \pi$
△(1232) ⁺⁺ MASS				
VALUE (MeV)	DOCUMENT ID	TE	CN CO	OMMENT
• • We do not use the following	19			
1230.55±0.20	GRIDNEV 06			$N \rightarrow \pi N$
1231.88±0.29	BERNICHA 96			t to PEDRONI 78
1230.5 ±0.2			15.5	$N \rightarrow \pi N$
1230.9 ±0.3				$N \rightarrow \pi N$
1231.1 ±0.2	PEDRONI 78			$N \rightarrow \pi N 70-370 \text{ MeV}$
Δ(1232) ⁺ MASS				
VALUE (McV)	DOCUMENT ID		СОММЕ	NT
• • • We do not use the following	-			0000
1234.9±1.4	MIROSHNIC			
A/1020\() NAACC				ARTICLE CO. A CONTROL OF THE CONTROL
△(1232) ⁰ MASS	DOCUMENT ID	-	ECN CO	DAMAGNE
VALUE (MeV) ■ ■ We do not use the followi		-		DMMENT
	9	20 5		
1231.3 ±0.6	BREITSCHOP06			sing new CHEX data
1233.40±0.22	GRIDNEV 06			$N \rightarrow \pi N$
1234.35 ± 0.75	BERNICHA 96			t to PEDRONI 78
1233.1 ±0.3	ABAEV 95			$N \rightarrow \pi N$
1233.6 ±0.5		B IP		$N \rightarrow \pi N$
1233.8 ±0.2	PEDRONI 78	3	π	$N \rightarrow \pi N 70-370 \text{ MeV}$

Entry 11(ii). As n tends to ∞ ,

$$I_n := \int_0^\infty \frac{x^{n-1} dx}{\sum_{k=0}^\infty (x/k)^k} \sim n^n \left(\frac{1}{n} + \frac{1}{2n^2} + \frac{1}{3n^3} + \frac{3}{8n^4} + \cdots \right).$$

For n = 6, we have:

$$6^{6} \left(\frac{1}{6} + \frac{1}{2 \times 6^{2}} + \frac{1}{3 \times 6^{3}} + \frac{3}{8 \times 6^{4}} \right)$$

8509.5

Possible closed forms:

$$\frac{27087\pi}{10} \approx 8509.63202$$

$$\csc^2\left(\frac{1}{40}(13-4\pi)\right) \approx 8509.41266$$

$$\frac{17019}{2} = 8509.5$$

For n = 8, we have

Decimal approximation:

2240682,666666

We calculate the following simple integrals 1):

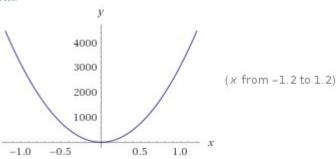
Input interpretation:

$$728 \times \frac{1}{10^3} \int 8509.5 \, x \, dx$$

Result:

 $3097.46 x^2$

Plot:



We observe that the result is practically equal to the mass of $J/\psi(1S)$ MASS value that is 3096.900±0.006 MeV.

The J/ψ (J/psi) meson or psion is a subatomic particle, a flavor-neutral meson consisting of a charm quark and a charm antiquark. Mesons formed by a bound state of a charm quark and a charm anti-quark are generally known as "charmonium". The

 J/ψ is the most common form of charmonium, due to its low rest mass. The J/ψ has a rest mass of 3.0969 GeV/ c^2 (or, equivalently, 3096.9 MeV/ c^2).

and 2):

Pi/1728 integral [2240682.666666]x

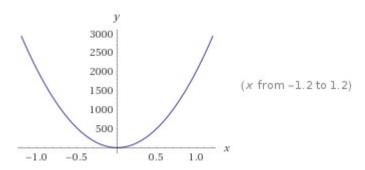
Input interpretation:

$$\frac{\pi}{1728} \int 2.240682666666 \times 10^6 \, x \, dx$$

Result:

2036.838022171 x2

Plot:



Indefinite integral assuming all variables are real:

678.9460073904 x3 + constant

The value 2036.838 is very near to the mass of charmed meson D (2010), i.e. $D^*(2010)^{\pm}$ mass, with a difference of a -26.

$D^*(2010)^{\pm}$ MASS

The fit includes D^{\pm} , D^{0} , D_{s}^{\pm} , $D^{*\pm}$, D^{*0} , $D_{s}^{*\pm}$, $D_{1}(2420)^{0}$, $D_{2}^{*}(2460)^{0}$, and $D_{s1}(2536)^{\pm}$ mass and mass difference measurements.

VALUE (MeV)	DOCUMENT ID	DOCUMENT ID			COMMENT
2010.26±0.05 OUR FIT			,	(4)	
• • • We do not use the	following data for averages	, fits	, limits,	etc. •	• •
2008 ±3	¹ GOLDHABER	77	MRK1	±	e+e-
2008.6 ±1.0	² PERUZZI	77	LGW	\pm	e+e-
	to $D^*(2010)^+$, $D^*(2007)$ difference below. It independent of FELDMA				

We have:

Example. For n, a > 0,

$$\int_0^\infty \frac{\cos(nx) dx}{a^2 + x^2} = \frac{\pi}{2a} e^{-na}.$$

For n = 1, a = 2, we obtain:

Input:

$$\frac{\pi}{4} \exp(-2)$$

Exact result:

$$\frac{\pi}{4 e^2}$$

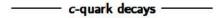
Decimal approximation:

0.106292082896909082109780590302250510262385101997650436041...

Value very near to the branching ratio of D^+ :

D+ BRANCHING RATIOS

Some now-obsolete measurements have been omitted from these Listings.



$\Gamma(c \rightarrow e^{+} \text{ anything})/\Gamma(c \rightarrow \text{ anything})$

For the Summary Table, we only use the average of e⁺ and μ^+ measurements from $Z^0 \to c \overline{c}$ decays; see the second data block below.

VALUE EVTS DOCUMENT ID TECN COMMENT

0.103
$$\pm$$
0.009 $^{+0.009}_{-0.008}$ 378

1 ABBIENDI 99K OPAL $Z^0 \rightarrow c\overline{c}$

$$\Gamma(K^+K^-\pi^+)/\Gamma(K^-2\pi^+)$$

VALUE

0.1059±0.0018 OUR FIT

0.1059±0.0018 OUR AVERAGE

0.106 ±0.002 ±0.003

0.117 ±0.013 ±0.007 181 ± 20

0.107 ±0.001 ±0.002 43k

AUBERT

0.108 BONVICINI 14 CLEO All CLEO-c runs

 $E^+e^-\approx \psi(3770)$
 $E^-e^-\approx \psi$

From the inverse of the expression, for n = 4, a = 3, we have:

$$\frac{1}{\frac{\pi}{6} \exp(-12)}$$

Exact result:

 $^{^1 \}text{ABBIENDI 99K}$ uses the excess of right-sign over wrong-sign leptons opposite reconstructed $D^*(2010)^+ \to D^0 \, \pi^+$ decays in $Z^0 \to c \, \overline{c}.$

$$\frac{6 e^{12}}{\pi}$$
Decimal approximation:

310838.7547946983720610230743772732246045552446944186835683...

 $310838.75479 = 3108.3875 * 10^2$ and 3108.38 is a value very near to the vector meson J/Psi = 3096.916 ± 0.011

If we calculate the following integral, we have:

 $(Pi^2)/1728$ integrate x/(Pi/6 * exp(-12))

$$\frac{\pi^2}{1728} \int \frac{x}{\frac{1}{6} \pi \exp(-12)} dx \approx \text{constant} + 887.69 \, x^2$$

Where 887,69 is a very good approximation to the masses of vector meson Kaon = 891.66 ± 0.026 896.00 ± 0.025

Now:

Entry 16. For a and n both real, and n integral in (iv),

(i)
$$\int_0^\infty \frac{\sinh(ax)}{\sinh(\pi x)} \cos(nx) \, dx = \frac{1}{2} \frac{\sin a}{\cosh n + \cos a}, \qquad |a| < \pi,$$

for a = 3 and n = 1/1728, we have:

$$\frac{1}{2} \times \frac{\sin(3)}{\cosh\left(\frac{1}{1728}\right) + \cos(3)}$$

Exact result:

$$\frac{\sin(3)}{2\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)}$$

Decimal approximation:

7.050592000652599429873182695638317820716138104149441462737...

If we calculate the following integral, we obtain:

 $Pi^6/3$ integrate 1/2 ((sin(3)/(((cosh(1/(1728))+cos(3)))x)

$$\frac{\pi^6}{3} \int \frac{\sin(3)}{2\left(\left(\cosh\left(\frac{1}{1728}\right) + \cos(3)\right)x\right)} dx \approx \text{constant} + 2259.45 \log(0.0200153 x)$$

(assuming a complex-valued logarithm)

Alternate forms:

$$\frac{1^{72}\sqrt[8]{e} \pi^{6} \sin(3) \log(2 x (\cos(3) + \cosh(\frac{1}{1728})))}{3 (1 + \sqrt[864]{e} + 2 \sqrt[1728]{e} \cos(3))}$$

$$\frac{\pi^{6} \sin(3) \log(2 x (\cos(3) + \cosh(\frac{1}{1728})))}{6 (\frac{1}{2^{1728}} + \frac{1^{72}\sqrt[8]{e}}{2} + \cos(3))}$$

$$\frac{^{172}\sqrt[8]{e} \ \pi^{6} \sin(3) \left(1728 \log \left(x \left(1+\sqrt[864]{e}+2\sqrt[1728]{e} \cos(3)\right)\right)-1\right)}{5184 \left(1+\sqrt[864]{e}+2\sqrt[1728]{e} \cos(3)\right)}$$

Alternate form assuming x>0:

$$\frac{\pi^6 \sin(3) \log(x)}{6 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + \frac{\pi^6 \sin(3) \log\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)}{6 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + \frac{\pi^6 \log(2) \sin(3)}{6 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)}$$

Series expansion of the integral at x=0:

$$\frac{\pi^6 \sin(3) \left(\log(x) + \log(2) + \log\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)\right)}{6 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + O\left(x^6\right)$$

(generalized Puiseux series)

Series expansion of the integral at x=∞:

$$\frac{\pi^6 \sin(3) \left(\log(x) + \log(2) + \log\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)\right)}{6 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(generalized Puiseux series)

where (2259.45 * 3.91) / 4 = 2209.32 or (2259.45 * 3.91) / 3.91 = 2259.45 that is very near to the mass of meson $f_2(2300)$:

f2(2300) MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
2297±28	¹ ETKIN 88	MPS	$22 \pi^- p \rightarrow \phi \phi n$
• • • We do not use	the following data for avera	iges, fits,	limits, etc. • • •
$2243 + 7 + 3 \\ -6 - 29$	UEHARA 13		5 5
2270 ± 12	VLADIMIRSK06	SPEC	$40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n$
$2327 \pm 9 \pm 6$	ABE 04	BELL	10.6 $e^+e^- \rightarrow e^+e^-K^+K^-$
2231 ± 10	BOOTH 86	OMEG	$85 \pi^- \text{Be} \rightarrow 2\phi \text{Be}$
2220^{+90}_{-20}	LINDENBAUM 84	RVUE	
2320 ± 40	ETKIN 82	MPS	$22 \pi^- p \rightarrow 2\phi n$

 $^{^1}$ Includes data of ETKIN 85. The percentage of the resonance going into $\phi\phi$ 2 $^+$ + S_2 , D_2 , and D_0 is 6 $^{+15}_{-}$, 25 $^{+18}_{-14}$, and 69 $^{+16}_{-27}$, respectively.

Indeed, if we take 2297, we have the minimum value 2297 - 28 = 2269. While 2209.32 is practically equal to the mass of meson $f_0(2200)$:

fo(2200) MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
2189±13 OUR AVE	RAGE	> ************************************			
$2170 \pm 20 + 10$		ABLIKIM	05Q	BES2	$\psi(2S) \rightarrow$
2210±50		1 BINON	05	GAMS	$ \gamma \pi^{+} \pi^{-} K^{+} K^{-} $ 33 $\pi^{-} p \rightarrow \eta \eta n$
2197 ± 17		² AUGUSTIN	88	DM2	$J/\psi \rightarrow \gamma K_S^0 K_S^0$
• • • We do not use t	he following	data for averages,	fits, lin	nits, etc.	• • •
2206±12± 8	381	3,4 DOBBS	15		$J/\psi \rightarrow \gamma K^+ K^-$
$2188 \pm 17 \pm 16$	203	3,4 DOBBS	15		$J/\psi \to \gamma K^+ K^-$ $\psi(2S) \to \gamma K^+ K^-$
~ 2122		HASAN	94	RVUE	
~ 2321		HASAN	94	RVUE	$\overline{p}p \rightarrow \pi\pi$

¹ First solution, PWA is ambiguous.

Indeed:
$$2189 + 13 = 2202$$
; $2197 + 17 = 2214$;

If we calculate the following integral:

125/2 integrate
$$1/2 ((\sin(3)/(((\cosh(1/(1728))+\cos(3)))x)$$

$$\frac{125}{2} \int \frac{\sin(3)}{2 \left(\left(\cosh\left(\frac{1}{1728}\right) + \cos(3) \right) x \right)} dx \approx \text{constant} + 440.662 \log(0.0200153 x)$$

(assuming a complex-valued logarithm)

² Cannot determine spin to be 0.

³ Using CLEO-c data but not authored by the CLEO Collaboration.

⁴ From a fit to a Breit-Wigner line shape with fixed $\Gamma = 238$ MeV.

Alternate forms:

$$\frac{125^{1728}\sqrt{e} \sin(3) \log(2 x (\cos(3) + \cosh(\frac{1}{1728})))}{2(1 + \sqrt[864]{e} + 2^{1728}\sqrt{e} \cos(3))}$$

$$\frac{125 \sin(3) \log \left(2 x \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)\right)}{4 \left(\frac{1}{2^{1728}\sqrt{e}} + \frac{1728\sqrt{e}}{2} + \cos(3)\right)}$$

$$\frac{125\sqrt[1728]{e}\sin(3)\left(1728\log\left(x\left(1+\sqrt[864]{e}+2\sqrt[1728]{e}\cos(3)\right)\right)-1\right)}{3456\left(1+\sqrt[864]{e}+2\sqrt[1728]{e}\cos(3)\right)}$$

Alternate form assuming x>0:

$$\frac{125 \sin(3) \log(x)}{4 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + \frac{125 \sin(3) \log\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)}{4 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + \frac{125 \log(2) \sin(3)}{4 \left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)}$$

Series expansion of the integral at x=0:

$$\frac{125\sin(3)\left(\log(x)+\log(2)+\log\left(\cos(3)+\cosh\left(\frac{1}{1728}\right)\right)\right)}{4\left(\cos(3)+\cosh\left(\frac{1}{1728}\right)\right)}+O\left(x^6\right)$$

(generalized Puiseux series)

Series expansion of the integral at $x=\infty$:

$$\frac{125\sin(3)\left(\log(x) + \log(2) + \log\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)\right)}{4\left(\cos(3) + \cosh\left(\frac{1}{1728}\right)\right)} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(generalized Puiseux series)

Now:

440.662 log(0.0200153)

-1723.54...

The result -1723,54 is exactly the mass with sign minus of $f_0(1710)$ that has been identified as possible particle named "glueball". Indeed:

fo(1710) MASS

)	EVTS	DOCUMENT ID		TECN	COMMENT
OUR	AVERAGE	Error includes so	ale fa	ctor of 1	1.6. See the ideogram below.
$^{+14}_{-25}$	5.5k	¹ ABLIKIM	13N	BES3	$e^+e^- \rightarrow J/\psi \rightarrow \gamma \eta \eta$
$^{+29}_{-18}$		UEHARA	13	BELL	$\gamma\gamma \to K_S^0 K_S^0$
+ 9 - 2	4k	² CHEKANOV	08	ZEUS	$\epsilon p \rightarrow K_S^0 K_S^0 X$
±13		ABLIKIM	06v	BES2	$e^+e^- \rightarrow J/\psi \rightarrow \gamma \pi^+\pi^-$
$+15 \\ -10$		³ ABLIKIM	05Q	BES2	$\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^+ K^-$
		ABLIKIM	04E	BES2	$J/\psi \rightarrow \omega K^{\dagger} K$
$^{+10}_{-25}$		⁴ BAI	03G	BES	$J/\psi \to \gamma K \overline{K}$
		⁴ BAI	00A	BES	$J/\psi \rightarrow \gamma (\pi^+\pi^-\pi^+\pi^-)$
		⁵ BARBERIS	00E		450 $pp \rightarrow p_f \eta \eta p_s$
± 11		⁶ BARBERIS	99D	OMEG	450 pp $\to K^+K^-, \pi^+\pi^-$
		7 FRENCH	99		$300 pp \rightarrow p_f (K^+ K^-) p_S$
		8 AUGUSTIN	88	DM2	$J/\psi \rightarrow \gamma K^+ K^-, K_S^0 K_S^0$
		8 AUGUSTIN	87	DM2	$J/\psi \rightarrow \gamma \pi^+ \pi^-$
± 10		9 BALTRUSAIT.	.87	MRK3	$J/\psi \rightarrow \gamma K^+ K^-$
		8 WILLIAMS	84	MPSF	$200 \pi^{-} N \rightarrow 2K_{5}^{0} X$
		BLOOM	83	CBAL	$J/\psi \rightarrow \gamma 2\eta$
	OUR +14 -25 +29 -18 + 9 - 2 ±13 +15 -10 +10 -25	OUR AVERAGE +14 -25 5.5k +29 -18 + 9 - 2 4k ±13 +15 -10 +10 -25	OUR AVERAGE Error includes so +14	OUR AVERAGE Error includes scale factor +14 -25 -25 -25 -18 -18 -18 -18 -18 -18 -18 -18 -18 -18	OUR AVERAGE Error includes scale factor of 1 +14 - 25 - 25 - 18 5.5k 1 ABLIKIM 13N BES3 +29 - 18 - 18 UEHARA 13 BELL + 9 - 2 - 2 - 4k 2 CHEKANOV 08 ZEUS ±13 - ABLIKIM 06V BES2 ±13 - ABLIKIM 05Q BES2 ABLIKIM 04E BES2 ABLIKIM 04E BES2 4 BAI 03G BES 4 BAI 00A BES 5 BARBERIS 00E 6 BARBERIS 99D OMEG 7 FRENCH 99 8 AUGUSTIN 87 DM2 4 BAITRUSAIT 87 MRK3 8 WILLIAMS 84 MPSF

Note that the value -1723,54 is practically equal to value 1723(+6, -5) in MeV with sign minus

From:

Strong Effective Coupling, Meson Ground States, and Glueball within Analytic Confinement

Gurjav Ganbold 1,2

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Our model has a minimal set of free parameters: $\{\hat{\alpha}, \Lambda, m_{ud}, m_s, m_c, m_b\}$. The glueball mass depends on $\{\hat{\alpha}, \Lambda\}$. We fix Λ by fitting the expected glueball mass. Particularly, for $\Lambda = 236$ MeV and $\hat{\alpha}(M_G)$ defined in Equation (28) we obtain new estimates:

$$M_{0++} = 1739 \text{ MeV}, \quad \hat{\alpha}(M_{0++}) = 0.451.$$
 (33)

The new value of $M_{0^{++}}$ in (33) agrees not only with our previous estimate [27], but also with other predictions expecting the lightest glueball located in the scalar channel in the mass range $\sim 1500 \div 1800 \,\text{MeV}$ [12,16,46,51]. The often referred quenched QCD calculations predict $1750 \pm 50 \pm 80 \,\text{MeV}$ for the mass of the lightest glueball [17]. The recent quenched lattice estimate with improved lattice spacing favors a scalar glueball mass $M_G = 1710 \pm 50 \pm 58 \,\text{MeV}$ [49].

Another important property of the scalar glueball is its size, the 'radius' which should depend somehow on the glueball mass. We estimate the glueball radius roughly as follows:

$$r_{0^{++}} \sim \frac{1}{2\Lambda} \sqrt{\frac{\int d^4x \ x^2 \ W_{\Lambda}(x) \ U^2(x)}{\int d^4x \ W_{\Lambda}(x) \ U^2(x)}} \approx \frac{1}{394.3 \ \text{MeV}} \approx 0.51 \ \text{fm} \,.$$
 (34)

This may indicate that the dominant forces binding gluons are provided by vacuum fluctuations of correlation length $\sim 0.5\, \mathrm{fm}$. On the other side, typical energy-momentum transfers inside a scalar glueball should occur in the confinement domain $\sim 236\, \mathrm{MeV} \sim 0.85\, \mathrm{fm}$, rather than at the chiral symmetry breaking scale $\Lambda_\chi \sim 1\, \mathrm{GeV} \sim 0.2\, \mathrm{fm}$.

The gluon condensate is a non-perturbative property of the QCD vacuum and may be partly responsible for giving masses to certain hadrons. The correlation function in QCD dictates the value of corresponding condensate. Particularly, with $\Lambda=236$ MeV and $\hat{\alpha}_s=0.451$ we calculate the lowest non-vanishing gluon condensate in the leading-order (ladder) approximation:

$$\frac{\hat{\alpha}_s}{\pi} \left\langle F_{\mu\nu}^A F_A^{\mu\nu} \right\rangle = \frac{16N_c}{\pi} \Lambda^4 \approx 0.0214 \text{ GeV}^4$$

which is in accordance with a refereed value [52]

$$\alpha_{S}\left\langle G^{2}\right\rangle = (7.0\pm1.3)\cdot10^{-2}\;GeV^{4}\,,\qquad {
m or},\qquad \frac{\alpha_{S}}{\pi}\left\langle G^{2}\right\rangle = (2.2\pm0.4)\cdot10^{-2}\;GeV^{4}\,.$$

7. Conclusions

In conclusion, we demonstrate that many properties of the low-energy phenomena such as strong running coupling, hadronization processes, mass generation for quark-antiquark and di-gluon bound states may be explained reasonably within a QCD-inspired model with infrared-confined propagators. We derived a meson mass equation and by exploiting it revealed a specific new behavior of the strong coupling $\alpha_S(M)$ in dependence of mass scale. An infrared freezing point $\alpha_S(0) = 1.03198$ at origin M = 0 has been found and it did not depend on the particular choice of the confinement scale $\Lambda > 0$. A new estimate of the lowest (scalar) glueball mass has been performed and it was found at ≈ 1739 MeV. The scalar glueball 'size' has also been calculated: $r_G \approx 0.51$ fm. A nontrivial value of the gluon condensate has also been obtained. We have estimated the spectrum of conventional mesons by introducing a minimal set of parameters: four masses of constituent quarks (u = d, s,

c, b) and Λ . The obtained values fit the latest experimental data with relative errors less than 1.8 percent. Accurate estimates of the leptonic decay constants of pseudoscalar and vector mesons have also been performed³.

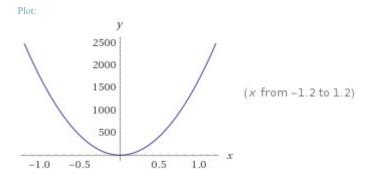
Now we take the integral of pg.76

integrate $(\text{sqrt}(12\text{Pi})) * [(\exp(6-1/144-1/(48*36)-(1/36+1/5760)*1/216))]x$ multiplied for $(1.08643)^4$. We obtain:

$$1.08643^4 \int \sqrt{12\pi} \exp\left(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \left(\frac{1}{36} + \frac{1}{5760}\right) \times \frac{1}{216}\right) x \, dx$$

Result:

 $1712.32 x^2$



Alternate form assuming x is real:

 $1712.32 x^2 + 0$

Indefinite integral assuming all variables are real:

 $570.775 x^3 + constant$

Also here the value 1712,32 is a good approximation to the mass of $f_0(1710)$.

We calculate this other integral (see pg.73):

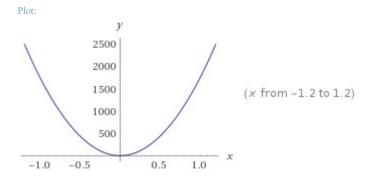
$$(729+9+16)*(1/10^56)$$
 integrate $exp((((sqrt(1729)*3)+(((sqrt(1729)^2)-1))/24)))*((1/(sqrt(1729)*3)+(1/((2*(sqrt((1729))^2)3^2)))))) * (2.060144*10^-27)x$

Note in Italian "In conclusione, dimostriamo che molte proprietà dei fenomeni di bassa energia, come il forte accoppiamento in movimento, i processi di adronizzazione, la generazione di massa per stati legati a quark-antiquark e di di-gluon possono essere spiegati ragionevolmente all'interno di un modello ispirato alla QCD con propagatori confinati con infrarossi. Abbiamo derivato un'equazione di massa del mesone e sfruttandola ha rivelato un nuovo comportamento specifico dell'accoppiamento forte αS (M) in dipendenza della scala di massa. È stato trovato un punto di congelamento a infrarossi αS (0) = 1.03198 all'origine M = 0 non dipendente dalla particolare scelta della scala di confinamento ΔS 0. È stata eseguita una nuova stima della massa di glueball più bassa (scalare) e è stato trovato a ≈ 1739 MeV. È stata calcolata anche la "dimensione" del glueball scalare: $rG \approx 0,51$ fm. È stato ottenuto anche un valore non banale del condensato di gluone. Abbiamo stimato lo spettro dei mesoni convenzionali introducendo un set minimo di parametri: quattro masse di quark costituenti (u = d, s, c, b) e ΔS . I valori ottenuti si adattano agli ultimi dati sperimentali con errori relativi inferiori all'1,8%. Sono state inoltre eseguite stime accurate delle costanti di decadimento dei mesoni pseudoscalari e vettoriali"

$$(729 + 9 + 16) \times \frac{1}{10^{56}} \int \exp\left(\sqrt{1729} \times 3 + \frac{1}{24} \left(\sqrt{1729}^2 - 1\right)\right) \\ \left(\frac{1}{\sqrt{1729} \times 3} + \frac{1}{2\sqrt{1729}^2 \times 3^2}\right) \times 2.060144 \times 10^{-27} \, x \, dx$$

Result:

 $1740.51 x^2$



Alternate form assuming x is real:

$$1740.51 x^2 + 0$$

Indefinite integral assuming all variables are real:

$$580.171 \, x^3 + constant$$

We note that this value i.e. 1740,51 correspond exactly to the new estimate of the lowest (scalar) glueball mass, that is ≈ 1739 MeV.

Furthermore $e^{1739} = 1,7302307644949 * 10^{754} = 1730,2307644949 * 10^{751}$ that is about a multiple of 1730.

Further, from pg.40, we can calculate the following integral:

$$-47.23265 - 58.8742714 - 382.257106 + 16507.8183 + 139489 - 139468 - 2209694 + 2085349$$

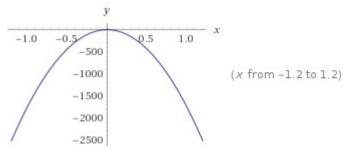
1/31 * integrate [-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349]x

$$\frac{1}{31} \int (-47.23265 - 58.8742714 - 382.257106 + 16507.8183 + 139489 - 139468 - 2209694 + 2085349) x dx$$

Result:

$$-1746.85 x^2$$

Plot:



Alternate form assuming x is real:

$$0 - 1746.85 x^2$$

Indefinite integral assuming all variables are real:

$$-582.283 x^3 + constant$$

The result -1746.85 is a good approximation of the new estimate of the lowest (scalar) glueball mass, that is ≈ 1739 MeV.

Note that:

$$(1/52) * (1/1728) * integrate [-2209694]x$$

Indefinite integral

$$\frac{1}{52 \times 1728} \int -2209694 \, x \, dx = -\frac{1104847 \, x^2}{89856} + \text{constant}$$

$$-12,29575097x^2$$

and

((1728+216)*1164.2696)/10^10 * integrate [-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349]x

Input interpretation:
$$\frac{(1728+216)\times 1164.2696}{10^{10}}$$

$$\int (-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349) x dx$$

Open code

Result:
$$-12.2565 x^2$$

results 12.29 and 12.25, that are very near to the value of the black hole entropy (12.19) with minus sign.

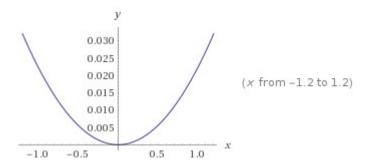
We have also:

$$(Pi^2)/(2e) * (1/(10^5)) * integrate (sqrt(12Pi)) * [(exp(6-1/144-1/(48*36)-(1/36+1/5760)*1/216))]x$$

Indefinite integral:

$$\frac{\pi^2}{(2\ e)\ 10^5} \int \sqrt{12\ \pi}\ \exp\Bigl(6 - \frac{1}{144} - \frac{1}{48\times36} - \frac{1}{216}\left(\frac{1}{36} + \frac{1}{5760}\right)\Bigr) x\ dx \approx \\ \text{constant} + 0.0223128\ x^2$$

Plot:



The value 0,0223128 is a good approximation to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \text{ GeV}^4$.

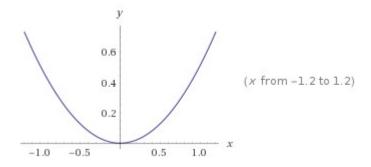
and:

$$(Pi^2)/(sqrt(9Pi/5)) * (1/(10^4)) * integrate (sqrt(12Pi)) * [(exp(6-1/144-1/(48*36)-(1/36+1/5760)*1/216))]x$$

Indefinite integral

$$\frac{\pi^2}{\sqrt{\frac{9\pi}{5}} \ 10^4} \int \sqrt{12\pi} \ \exp\Bigl(6 - \frac{1}{144} - \frac{1}{48 \times 36} - \frac{1}{216} \left(\frac{1}{36} + \frac{1}{5760}\right)\Bigr) x \, dx \approx \\ \text{constant} + 0.510114 \, x^2$$

Plot:



We observe that the value 0.510114 is exactly the value of the scalar glueball 'size' that is $r_G \approx 0.51\ fm$

From the integral of pg.80, we have:

$$0.57721 \quad 1/(10^3) \quad (Pi^2)/1728 \quad integrate \ x/(Pi/6 * exp(-12))$$

(where 0.57721 is the Eulero-Mascheroni constant)

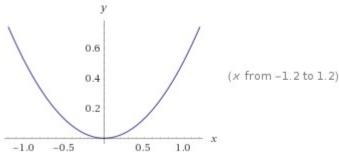
Input

$$0.57721 \times \frac{1}{10^3} \times \frac{\pi^2}{1728} \int \frac{x}{\frac{\pi}{6} \exp(-12)} dx$$

Result:

$$0.512383 x^2$$

Plot:



The value 0.512383 is very near to the value of the scalar glueball 'size' that is $r_G \approx 0.51 \ \text{fm}$

and

$$0.57721/(\text{sqrt}(2e))$$
 $1/(10^4)$ $(\text{Pi}^2)/1728$ integrate $x/(\text{Pi}/6 * \exp(-12))$

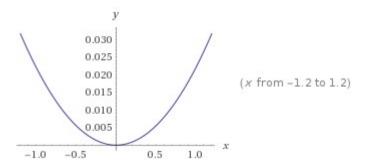
Input:

$$\frac{0.57721}{\sqrt{2e}} \times \frac{1}{10^4} \times \frac{\pi^2}{1728} \int \frac{x}{\frac{\pi}{6} \exp(-12)} dx$$

Result:

 $0.0219752 x^2$

Plot:



The value 0.0219752 is very near to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \text{ GeV}^4$.

Now:

Our version of Example (i) is different from that of Ramanujan, who writes that the maximum value of $a^x/\Gamma(x+1)$ is

$$\frac{a^{a-1/2}}{\Gamma(a+\frac{1}{2})} \exp\left(\frac{1}{1152a^3 + 323.2a}\right)$$

For a = 16

$$\frac{a^{a-1/2}}{\Gamma(a+\frac{1}{2})} \exp\left(\frac{1}{1152a^3 + 323.2a}\right)$$

4611686994701242309,804652561838 / (gamma 33/2)

$$\frac{4.611686994701242309804652561838\times10^{18}}{\Gamma\left(\frac{33}{2}\right)}$$

888571.9401322504183973732124617...

Gamma
$$3/2 = \frac{191898783962510625\sqrt{\pi}}{65536}$$

 $5.1899984530401250830724817743776669334491731891236353... \times 10^{12}$

$$16^3 = 4096$$
; $4096 * 1152 = 4718592$; $4718592 + 323.2*16 = 4723763,2$ and $e^{1/4723763,2} = 1,0000002116956467775140917669629$

 $(16^{15.5})*\ 1.0000002116956467775140917669629/(5.18999845304012508*10^{12})$

1/(32*8) integrate ((16^15.5)*

1.0000002116956467775140917669629)/ $(5.18999845304012508*10^12)$ x

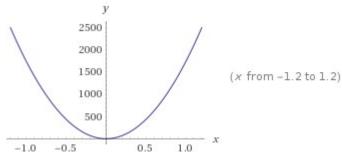
Input interpretation:

$$\frac{1}{32 \times 8} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} \, x \, dx$$

Result

 $1735.49 x^2$

Plot:



Alternate form assuming x is real:

$$1735.49 x^2 + 0$$

Indefinite integral assuming all variables are real:

$$578.497 x^3 + constant$$

Also this result 1735.49 is a good approximation of the new estimate of the lowest (scalar) glueball mass, that is ≈ 1739 MeV.

Note that:

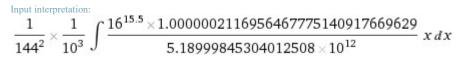
$$\frac{1}{144 \times 32 \times 8} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} \ x \ dx$$

Result:
$$12.052 x^2$$

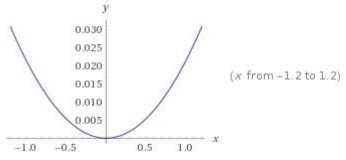
The result 12.052 is very near to the value of black hole entropy 12.19

Furthermore, we have:

1/(144^2) 1/(10^3) integrate ((16^15.5)* 1.0000002116956467775140917669629)/ $(5.18999845304012508*10^12)$ x



 $0.0214258 x^2$

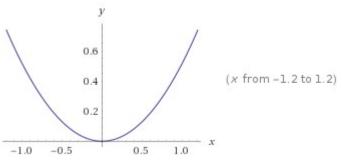


and

1/(Pi^2-3^2) 1/(10^6) integrate ((16^15.5)* $1.0000002116956467775140917669629))/(5.18999845304012508*10^12)x$

 $\frac{1}{\pi^2 - 3^2} \times \frac{1}{10^6} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} \ x \ dx$

 $0.510906 x^2$



The results 0.0214258 and 0.510906 are exactly the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214~\text{GeV}^4$ and the value of the scalar glueball 'size' that is $r_G \approx 0.51~\text{fm}$.

Now:

Entry 30(i). If n is a nonnegative integer, then

$$\int_0^\infty \frac{\sin^{2n+1} x}{x} \, dx = \int_0^\infty \frac{\sin^{2n+2} x}{x^2} \, dx = \frac{\sqrt{\pi} \, \Gamma(n + \frac{1}{2})}{2n!}.$$

We have, for n = 3:

$$\frac{\sqrt{\pi} \Gamma(\frac{7}{2})}{6!}$$
 $\frac{\pi}{384}$

0.008181230868723419891829800477290372094263461977539338075...

And

$$\frac{1}{\frac{\sqrt{\pi} \Gamma(\frac{7}{2})}{6!}}$$

122.2309962945756178705027302700910300424650079286705526382...

This result 122,23 is very near to the value of the mass of the Higgs boson $(125,09 \pm 0,24)$.

Multiplying the expression for the square of the golden ratio, we obtain:

$$\left(\frac{1}{2}\left(\sqrt{5} + 1\right)\right)^2 \times \frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}$$

Exact result:

$$\frac{(1+\sqrt{5})^2 \pi}{1536}$$

Decimal approximation:

0.021418740484127742328833730199275264911533876803636093597...

Property:

$$\frac{(1+\sqrt{5})^2 \pi}{1536}$$
 is a transcendental number

Alternate forms:

$$\frac{1}{768}\left(3+\sqrt{5}\right)\pi$$

$$\frac{\pi}{256} + \frac{\sqrt{5} \pi}{768}$$

Continued fraction:

Alternative representations:

$$\frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} = \frac{e^{-\log G(7/2) + \log G(9/2)}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2\sqrt{\pi}}{(1)_6}$$

$$\frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} = \frac{e^{-\log G(7/2) + \log G(9/2)}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2\sqrt{\pi}}{5!! \times 6!!}$$

$$\frac{\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\!\right)^{2}\left(\sqrt{\pi}\right.\left.\Gamma\!\left(\frac{7}{2}\right)\!\right)}{6\,!}=\frac{e^{-\log G\left(7/2\right)+\log G\left(9/2\right)}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2}\sqrt{\pi}}{e^{\log F\left(7\right)}}$$

Series representations:

$$\begin{split} \frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{2}\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \left(\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right. \\ &\left.\left(1+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^{k}\,(5-x)^{k}\,x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2} \\ &\left.\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{(-1)^{k_{1}}\,(\pi-x)^{k_{1}}\,x^{-k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}}\Gamma^{(k_{2})}(z_{0})}{k_{1}!\,k_{2}!}\right)\right/ \\ &\left.\left(4\sum_{k=0}^{\infty}\frac{(6-n_{0})^{k}\,\Gamma^{(k)}(1+n_{0})}{k!}\right) \end{split}$$

for $(x \in \mathbb{R} \text{ and } (n_0 \notin \mathbb{Z} \text{ or } n_0 \ge 0) \text{ and } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \text{ and } x < 0 \text{ and } n_0 \to 6)$

$$\begin{split} &\frac{\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\left(\sqrt{\pi}\right.\left.\Gamma\left(\frac{7}{2}\right)\right)}{6!} = \left(\left(\frac{1}{z_{0}}\right)^{1/2\left\lfloor \arg(\pi-z_{0})/(2\pi)\right\rfloor} z_{0}^{1/2+1/2\left\lfloor \arg(\pi-z_{0})/(2\pi)\right\rfloor} \\ & \left(1+2\left(\frac{1}{z_{0}}\right)^{1/2\left\lfloor \arg(5-z_{0})/(2\pi)\right\rfloor} z_{0}^{1/2+1/2\left\lfloor \arg(5-z_{0})/(2\pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!} + \\ & \left(\frac{1}{z_{0}}\right)^{\left\lfloor \arg(5-z_{0})/(2\pi)\right\rfloor} z_{0}^{1+\left\lfloor \arg(5-z_{0})/(2\pi)\right\rfloor} \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2} \right) \\ & \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}}\left(\pi-z_{0}\right)^{k_{1}} z_{0}^{-k_{1}} \Gamma^{(k_{2})}(z_{0})}{k_{1}! k_{2}!} \right) / \\ & \left(4\sum_{k=0}^{\infty} \frac{(6-n_{0})^{k} \Gamma^{(k)}(1+n_{0})}{k!}\right) \end{split}$$

for $((n_0 \notin \mathbb{Z} \text{ or } n_0 \ge 0) \text{ and } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \text{ and } n_0 \to 6)$

$$\begin{split} \frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{2}\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \frac{\Gamma\left(\frac{7}{2}\right)\left(1+\sqrt{5}\right)^{2}\sqrt{\pi}}{4\int_{0}^{\infty}e^{-t}\ t^{6}\ dt} \\ \frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{2}\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \\ &= i\pi\left(1+\sqrt{5}\right)^{2}\sqrt{\pi} \\ 2\left(720+e^{-\infty}\left(-(\infty+6\infty+30\infty+120\infty+360\infty+720)\infty+-720\right)\right)\oint_{L}\frac{e^{t}}{t^{7/2}}\ dt \\ \frac{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{2}\left(\sqrt{\pi}\ \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \frac{i\pi\left(1+\sqrt{5}\right)^{2}\sqrt{\pi}}{1440\oint_{L}\frac{e^{t}}{t^{7/2}}\ dt} \end{split}$$

And

$$24\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2 \times \frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}$$

$$\frac{1}{64} \left(1 + \sqrt{5}\right)^2 \pi$$

Decimal approximation:

0.514049771619065815892009524782606357876813043287266246341...

Property:

$$\frac{1}{64} \left(1 + \sqrt{5}\right)^2 \pi$$
 is a transcendental number

Alternate forms:

$$\frac{1}{32}\left(3+\sqrt{5}\right)\pi$$

$$\frac{3\pi}{32} + \frac{\sqrt{5}\pi}{32}$$

Continued fraction:

Alternative representations:

$$\frac{\left(24\left(\frac{1}{2}\left(\sqrt{5} + 1\right)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!} = \frac{24 e^{-\log G(7/2) + \log G(9/2)} \left(\frac{1}{2}\left(1 + \sqrt{5}\right)\right)^{2} \sqrt{\pi}}{(1)_{6}}$$

$$\frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right.\left.\Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{24\,e^{-\log G(7/2)+\log G(9/2)}\left(\frac{1}{2}\left(1+\sqrt{5}\right.\right)\right)^{2}\sqrt{\pi}}{5!!\times6!!}$$

$$\frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right.\left.\Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{24\,e^{-\log G(7/2)+\log G(9/2)}\left(\frac{1}{2}\left(1+\sqrt{5}\right.\right)\right)^{2}\sqrt{\pi}}{e^{\log \Gamma(7)}}$$

Series representations

$$\begin{split} \frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{2}\right)\left(\sqrt{\pi} \ \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \left(6\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right. \\ &\left.\left(1+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k} (5-x)^{k} x^{-k} \left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2} \\ &\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}} (\pi-x)^{k_{1}} x^{-k_{1}} \left(-\frac{1}{2}\right)_{k_{1}} \left(\frac{7}{2}-z_{0}\right)^{k_{2}} \Gamma^{(k_{2})}(z_{0})}{k_{1}! k_{2}!}\right)\right/ \\ &\left.\left(\sum_{k=0}^{\infty} \frac{(6-n_{0})^{k} \Gamma^{(k)}(1+n_{0})}{k!}\right)\right. \end{split}$$

for $(x \in \mathbb{R} \text{ and } (n_0 \notin \mathbb{Z} \text{ or } n_0 \ge 0) \text{ and } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \text{ and } x < 0 \text{ and } n_0 \to 6)$

$$\begin{split} &\frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right. \Gamma\left(\frac{7}{2}\right)\right)}{6!} = \left[6\left(\frac{1}{z_{0}}\right)^{1/2\left\lfloor \arg(\pi-z_{0})/(2\,\pi)\right\rfloor} z_{0}^{1/2+1/2\left\lfloor \arg(\pi-z_{0})/(2\,\pi)\right\rfloor} \\ & \left(1+2\left(\frac{1}{z_{0}}\right)^{1/2\left\lfloor \arg(5-z_{0})/(2\,\pi)\right\rfloor} z_{0}^{1/2+1/2\left\lfloor \arg(5-z_{0})/(2\,\pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!} + \\ & \left(\frac{1}{z_{0}}\right)^{\left\lfloor \arg(5-z_{0})/(2\,\pi)\right\rfloor} z_{0}^{1+\left\lfloor \arg(5-z_{0})/(2\,\pi)\right\rfloor} \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2}\right) \\ & \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}}\left(\pi-z_{0}\right)^{k_{1}} z_{0}^{-k_{1}} \Gamma^{(k_{2})}(z_{0})}{k_{1}! k_{2}!}\right) \\ & \left(\sum_{k=0}^{\infty} \frac{(6-n_{0})^{k} \Gamma^{(k)}(1+n_{0})}{k!}\right) \end{split}$$

for $((n_0 \notin \mathbb{Z} \text{ or } n_0 \ge 0) \text{ and } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \text{ and } n_0 \to 6)$

Integral representations:

$$\begin{split} \frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right. \, \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \frac{6 \, \Gamma\left(\frac{7}{2}\right)\left(1+\sqrt{5}\right)^{2} \, \sqrt{\pi}}{\int_{0}^{\infty} e^{-t} \, t^{6} \, dt} \\ \\ \frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right. \, \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \\ \frac{12 \, i \, \pi \left(1+\sqrt{5}\right)^{2} \, \sqrt{\pi}}{720 + e^{-\infty} \, \left(-(\infty+6\,\infty+30\,\infty+120\,\infty+360\,\infty+720)\,\infty+-720\right) \oint_{L} \frac{e^{t}}{t^{7/2}} \, dt} \\ \\ \frac{\left(24\left(\frac{1}{2}\left(\sqrt{5}\right.+1\right)\right)^{2}\right)\left(\sqrt{\pi}\right. \, \Gamma\left(\frac{7}{2}\right)\right)}{6!} &= \frac{12 \, i \, \pi \left(1+\sqrt{5}\right)^{2} \, \sqrt{\pi}}{720 \oint_{L} \frac{e^{t}}{t^{7/2}} \, dt} \end{split}$$

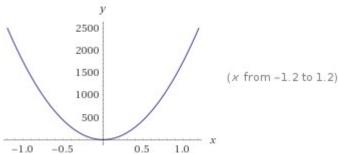
where the results 0.02141874 and 0.51404977 are exactly the value of the lowest non-vanishing gluon condensate, that is ≈0.0214 GeV⁴ and the value of the scalar glueball 'size' that is $r_G \approx 0.51$ fm.

Now, we calculate the following integral:

$$(9*Pi)$$
 integrate $x/[(((sqrt(Pi)*gamma(7/2)))/(6!)]$

$$(9 \pi) \int \frac{x}{\frac{\sqrt{\pi} \, \Gamma(\frac{7}{2})}{6!}} dx = 1728 \, x^2 + \text{constant}$$

Plot:



1728

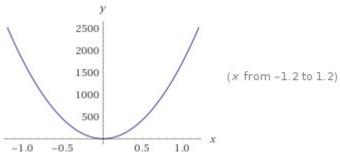
(sqrt(1.08643)+55/2) integrate x/[(((sqrt(Pi)*gamma(7/2)))/(6!)]

$$\sqrt{1.08643} + \frac{55}{2} \int \frac{x}{\frac{\sqrt{\pi} \, \Gamma(\frac{7}{2})}{6!}} dx$$

Result:

 $1744.38 x^2$

Plot:



The result 1744.38 is very near the new estimate of the lowest (scalar) glueball mass, that is ≈ 1739 MeV.

Note that:

1/144 * (sqrt(1.08643)+55/2) integrate x/[(((sqrt(Pi)*gamma(7/2)))/(6!)]

Input interpretation:

$$\frac{1}{144} \left(\sqrt{1.08643} + \frac{55}{2} \right) \int \frac{x}{\frac{\sqrt{\pi} \, \Gamma(\frac{7}{2})}{6!}} \, dx$$

Result:

 $12.1137 x^2$

The result 12.11 is very near to the value of black hole entropy 12.19 Now:

Entry 35. Let n denote a nonnegative integer, and let $\alpha, \beta > 0$ with $\alpha\beta = \pi$. Then

$$\sqrt{\alpha} \left\{ 1 + 2 \sum_{k=1}^{\infty} \frac{1}{(1 + \alpha^2 k^2)^{n+1}} \right\} = \sqrt{\beta} \frac{\Gamma(n + \frac{1}{2})}{\Gamma(n+1)} \left\{ 1 + 2 \sum_{k=1}^{\infty} e^{-2\beta k} \varphi(4\beta k) \right\},$$

where

$$\varphi(t) = \frac{n!}{(2n)!} \sum_{k=0}^{n} \frac{(n+k)!t^{n-k}}{(n-k)!k!}.$$

$$4 \int_0^\infty \frac{\cos(2\pi kx)}{(1+\alpha^2 x^2)^{n+1}} \, dx = \sqrt{\frac{\beta}{\alpha}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)} 2e^{-2\beta k} \varphi(4\beta k)$$

For n = 3, k = 2, $\alpha \beta = \pi$

[(3!)/(6!)] sum [5!t/(1!2!)]

$$\frac{\sum 60 t}{120}$$

0.5

sqrt(Pi)*(3.32335097)/(6)*(2(exp(-4Pi)*0.5(8Pi)

 $gamma(7/2) \ \ 3.323350970447842551184064031264647217745405230229475865400...$

gamma (4) = 6

 $((sqrt(Pi)*(3.32335097)/(6)))*(2*e^{(-4Pi)}*0.5(8Pi)))$

Input interpretation:

$$\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right) \left(2 e^{-4 \pi}\right) \times 0.5 (8 \pi)$$

Result:

0.0000860467...

$$\frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right) \left(2 e^{-4 \pi}\right) \times 0.5 (8 \pi)}$$

Result:

11621.6...

The result 11621,6 is very near to the value of the Ramanujan's class invariant 1164,2696 multiplied by 10.

Now:

$$(27*2)-(((sqrt(5)-1)/8)))*1/[((sqrt(Pi)*(3.32335097)/(6)))*(2*e^{(-4Pi)}*0.5(8Pi)))]$$

Input interpretation:
$$27 \times 2 - \left(\frac{1}{8} \left(\sqrt{5} - 1\right)\right) \times \frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right) \left(2 e^{-4 \pi}\right) \times 0.5 (8 \pi)}$$

Result:

-1741.63...

Series representations:

The result -1741,63 is very near the new estimate of the lowest (scalar) glueball mass, that is ≈ 1739 MeV.

$$27 \times 2 - \frac{\sqrt{5} - 1}{\frac{1}{6} \left(\left(2 e^{-4\pi} \right) \sqrt{\pi} \ 3.32335 \times 0.5 \ (8\pi) \right) 8} = \\ - \left(\left[0.0282095 \left(-e^{4\pi} + e^{4\pi} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \left(\frac{1}{2} \atop k \right) - 1914.25 \pi \sqrt{-1 + \pi} \right. \right. \\ \left. \sum_{k=0}^{\infty} \left(-1 + \pi \right)^{-k} \left(\frac{1}{2} \atop k \right) \right) / \left(\pi \sqrt{-1 + \pi} \sum_{k=0}^{\infty} \left(-1 + \pi \right)^{-k} \left(\frac{1}{2} \atop k \right) \right) \right)$$

$$\begin{aligned} 27 \times 2 - \frac{\sqrt{5} - 1}{\frac{1}{6} \left(\left(2 e^{-4\pi} \right) \sqrt{\pi} \ 3.32335 \times 0.5 \ (8\pi) \right) 8} = \\ - \left(\left[0.0282095 \left(-e^{4\pi} + e^{4\pi} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} \right)^k \left(-\frac{1}{2} \right)_k}{k!} - 1914.25 \pi \sqrt{-1 + \pi} \right. \right. \\ \left. \sum_{k=0}^{\infty} \frac{\left(-1 \right)^k \left(-1 + \pi \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right) \right] / \left[\pi \sqrt{-1 + \pi} \sum_{k=0}^{\infty} \frac{\left(-1 \right)^k \left(-1 + \pi \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right] \right] \end{aligned}$$

$$\begin{aligned} 27 \times 2 - \frac{\sqrt{5} - 1}{\frac{1}{6} \left(\left(2 \, e^{-4 \, \pi} \right) \sqrt{\pi} \, 3.32335 \times 0.5 \, (8 \, \pi) \right) 8} &= \\ - \left(\left(0.0282095 \left(-e^{4 \, \pi} + e^{4 \, \pi} \, \sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \, (5 - z_0)^k \, z_0^{-k}}{k!} - \right. \right. \\ &\left. 1914.25 \, \pi \, \sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \, (\pi - z_0)^k \, z_0^{-k}}{k!} \right) \right) / \\ \left(\pi \, \sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \, (\pi - z_0)^k \, z_0^{-k}}{k!} \right) \right) \, \text{for not} \left(\left(z_0 \in \mathbb{R} \, \text{and} \, -\infty < z_0 \le 0 \right) \right) \end{aligned}$$

$1/(192+9)^2$ integrate [-1741.63x]

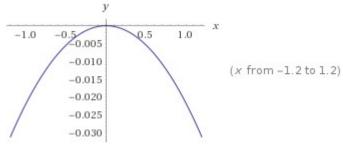
Input interpretation:

$$\frac{1}{(192+9)^2}\int -1741.63 \, x \, dx$$

Result:

 $-0.0215543 x^2$

Plot



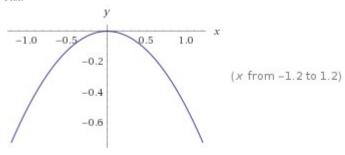
8* 1/(144-27)^2 integrate [-1741.63x]

$$8 \times \frac{1}{(144 - 27)^2} \int -1741.63 \, x \, dx$$

Result:

$$-0.508914 x^2$$

Plot:



Alternate form assuming x is real:

$$0 - 0.508914 x^2$$

Indefinite integral assuming all variables are real:

$$-0.169638 x^3 + constant$$

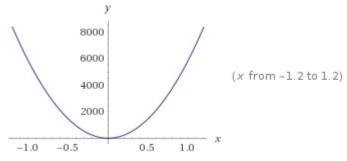
where the results -0.0215543 and -0.508914 are exactly the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214~\text{GeV}^4$ and the value of the scalar glueball 'size' that is $r_G \approx 0.51~\text{fm}$.

Now, we calculate the following integral already previously analyzed:

integrate $1/[((sqrt(Pi)*(3.32335097)/(6)))*(2*e^(-4Pi))*0.5(8Pi)))]x$

$$\int \frac{x}{\frac{1}{6} \left(\sqrt{\pi} \ 3.32335097 \right) \left(2 e^{-4\pi} \right) 0.5 \left(8 \pi \right)} dx = 5810.8 \, x^2 + \text{constant}$$

Plot of the integral:



Note that:

$$1729/(26Pi) * 1/10^4$$
 integrate $1/[((sqrt(Pi)*(3.32335097)/(6)))*(2*e^(4Pi))*0.5(8Pi)))]x$

$$\frac{1729}{26 \pi} \times \frac{1}{10^4} \int \frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right) \left(2 e^{-4 \pi}\right) \times 0.5 (8 \pi)} x \, dx$$

Result:

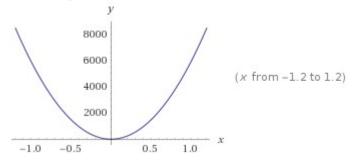
 $12.3001 x^2$

The result 12.30 is very near to the value of the black hole entropy 12.19-12.57 We have also (see pg. 19) this other integral and we can to obtain:

integrate [10361.2220016+1334.337561]x

$$\int (10361.2220016 + 1334.337561) x dx = 5847.78 x^2 + constant$$

Plot of the integral:



We note that:

1729/(26Pi) * 1/10^4 integrate [10361.2220016+1334.337561]x

Input interpretation:
$$\frac{1729}{26 \pi} \times \frac{1}{10^4} \int (10 \ 361.2220016 + 1334.337561) \ x \ dx$$
Open code

Result:

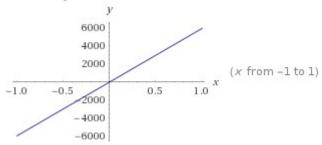
 $12.3784 x^2$

The result 12.378 is very near to the value of black hole entropy 12.19 - 12.57

integrate 1/2 [10361.2220016+1334.337561]+96

$$\int \left(\frac{10361.2220016 + 1334.337561}{2} + 96\right) dx = 5943.78 x + \text{constant}$$

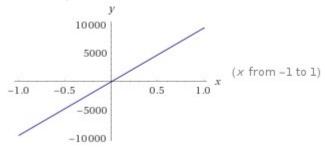
Plot of the integral:



integrate -(288+48)+2Pi/(ln1729)) 1/[((sqrt(Pi)*(3.32335097)/(6)))*(2*e^(-4Pi))*0.5(8Pi)))]

$$\int \left(-(288 + 48) + \frac{2\pi}{\frac{1}{6} \log(1729) \left(\left(\sqrt{\pi} \ 3.32335097 \right) \left(2 e^{-4\pi} \right) 0.5 \left(8 \pi \right) \right)} \right) dx = 9458.46 \ x + constant$$

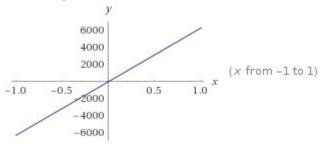
Plot of the integral:



integrate (96+48+9) + Pi/6 [11695.5595626]

$$\int \left((96 + 48 + 9) + \frac{\pi \, 11695.5595626}{6} \right) dx = 6276.78066691 \, x + \text{constant}$$

Plot of the integral:



We note that the results of the calculated integrals are very good approximations of the bottom Xi, bottom Sigma, charmed B meson and Upsilon meson.

Now, from:

A holographic description of heavy-flavoured baryonic matter decay involving glueball - Si-wen Li - arXiv:1812.03482v2 [hep-th]

$$y = r \cos \Theta, \quad z = r \sin \Theta,$$

$$U^{3} = U_{KK}^{3} + U_{KK}r^{2}, \quad \Theta = \frac{2\pi}{\beta}X^{4} = \frac{3}{2}\frac{U_{KK}^{1/2}}{R^{3/2}}.$$
 (B-8)

In the standard WSS model, the probe D8/ $\overline{D8}$ -branes are embedded at $\Theta = \pm \frac{1}{2}\pi$ respectively i.e. the position of y=0, which exactly corresponds to the antipodal D8/ $\overline{D8}$ -branes (blue) in Figure 1. In this case, the solution for the embedding function is $X^4(U) = \frac{1}{4}\beta$ and $U_0 = U_{KK}$. In addition, the (B-7) also allows the non-antipodal solution if we choose $\Theta = \pm \Theta_H \neq \pm \frac{1}{2}\pi$, $U_0 = U_{KK}$ which corresponds to the non-antipodal D8/ $\overline{D8}$ -branes (red) in Figure 1. On the other hand, while each endpoints of the IIL string could move along the flavoured branes, in our setup it is stretched between the heavy- (non-antipodal) and light-flavoured (antipodal) D8/ $\overline{D8}$ -branes. So it connects the positions respectively on the heavy- and light-flavoured D8/ $\overline{D8}$ -branes which are most close to each other and in the U X^4 plane, they are the positions of $(U_{KK}, 0)$ on the light-flavoured branes and $(U_H, 0)$ on the heavy-flavoured branes. And this is the configuration of the HL string with minimal length i.e. the VEV.

The eigenvalue equations for $H_{E,D,T}$ are given in (A-9) and (A-14). In the rescaled coordinate $Z \to \lambda^{-1/2} Z$, the equations are written as,

$$Z^2 = 1/\lambda$$

We have that:

Excitation of glueball (n)	n = 0
Glueball mass $M_E^{(n)}$	0.901
Glueball mass $M_{D,T}^{(n)}$	1.567
The coefficients	n = 0
\mathcal{C}_E	144.545
\mathcal{C}_D	29.772
\mathcal{C}_T	72.927

and:

$$\lambda = g_{\mathrm{YM}}^2 N_c, \ g_{\mathrm{YM}}^2 = 2\pi g_s l_s M_{KK},$$

$$= -2,31830159 * 10^{-34}$$

$$X^4 \sim X^4 + 2\pi \delta X^4$$
, $\delta X^4 = \frac{1}{M_{KK}}$.
For $\beta = 1$, $\frac{1}{4} + \frac{2\pi}{M_{KK}} = 0$; $\frac{1}{4} = -\frac{2\pi}{M_{KK}}$; $M_{KK} = -8\pi$

$$l = 0, N_Q = 1, N_c = 3, N_f = 2.$$

$$Z^2 = 1/\lambda = -4,313502611 * 10^{31}$$

Now, from the eq. (3.2):

$$\begin{split} H_{E}\left(z\right) &= \mathcal{C}_{E}\left(1 - \frac{3M_{E}^{2} + 16M_{KK}^{2}}{12M_{KK}^{2}\lambda}Z^{2}\right)\lambda^{-1/2}N_{c}^{-1}M_{KK}^{-1} + \mathcal{O}\left(\lambda^{-3/2}\right),\\ H_{D,T}\left(z\right) &= \mathcal{C}_{D,T}\left(1 - \frac{M_{D,T}^{2}}{4M_{KK}^{2}\lambda}Z^{2}\right)\lambda^{-1/2}N_{c}^{-1}M_{KK}^{-1} + \mathcal{O}\left(\lambda^{-3/2}\right). \end{split}$$

we obtain:

144.545((((1-(3*0.901^2+16*(64Pi^2)*-4.313502611*10^31)/(12*(64Pi^2)*-2.31830*10^-34))*((-2.0768980*10^16)*(1/3)*(-0.03978873))

$$\begin{aligned} 144.545 \left(\left(1 - \frac{3 \times 0.901^2 + 16 \left(64 \, \pi^2\right) \left(-4.313502611 \times 10^{31}\right)}{12 \left(64 \, \pi^2\right) \left(-2.31830 \times 10^{-34}\right)} \right) \\ \left(-2.0768980 \times 10^{16} \times \frac{1}{3} \times (-0.03978873) \right) \end{aligned}$$

Result:

$$-9.87771... \times 10^{81}$$

$$(-9.87771*10^81)+(8.958711419*10^47)$$

$$-9.87771 \times 10^{81} + 8.958711419 \times 10^{47}$$

Result:

$$-9.877709 * 10^{81}$$

Now:

 $(sqrt(sqrt([(-9.87771*10^81) + (8.958711419*10^47)]^2$

Result:

 $3.152565... \times 10^{20}$

And

 $(ln[-(-9.87771*10^81)+(8.958711419*10^47)]^1/11$

$$\sqrt[11]{\log(-(-9.87771\times10^{81})+8.958711419\times10^{47})}$$

Result:

1.61030894...

The result 1,61030894 is very near to the electric charge of the positron.

 $Pi^2/11 (ln[-(-9.87771*10^81)+(8.958711419*10^47)]^1/2$

$$\frac{\pi^2}{11} \sqrt{\log \left(-\left(-9.87771 \times 10^{81}\right) + 8.958711419 \times 10^{47}\right)}$$

Result:

12.3284273...

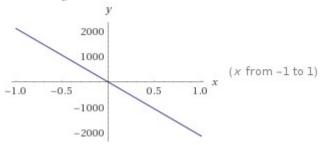
This result is very near to the value of Black Hole Entropy 12,57

Now, we calculate the following integral:

integrate $1/(27*1728) * 1/(10^37)^2 * [(-9.87771*10^81) + (8.958711419*10^47)]$

$$\int \frac{-9.87771 \times 10^{81} + 8.958711419 \times 10^{47}}{(27 \times 1728) \left(10^{37}\right)^2} \ dx = -2117.14 \ x + \text{constant}$$

Plot of the integral:



The result -2117,14 is very near to the rest mass of strange D meson 2112.3 \pm 0.5

Now, for the second expression of (3.2):

Input interpretation:

$$\left(1 - \frac{1.567^2 \left(-4.313502611 \times 10^{31}\right)}{4 \left(64 \, \pi^2\right)} \left(-2.0768980 \times 10^{16} \times \frac{1}{3} \times (-0.03978873)\right)\right)$$

Result:

 $8.9621493... \times 10^{47}$

 $8.9621493 * 10^{47}$

Percent increase:

$$8.958711419 \times 10^{47} + 29.772 \\ \left(1 - \frac{\left(1.567^2 \left(-4.313502611 \times 10^{31}\right)\right) \left(-2.0768980 \times 10^{16} \left(-0.03978873\right)\right)}{\left(4 \left(64 \, \pi^2\right)\right) 3}\right) = \\ 8.96215 \times 10^{47} \text{ is } 0.0383747\% \text{ larger than } 8.958711419 \times 10^{47} = 8.95871 \times 10^{47}.$$

Comparisons:

 $\approx 1.1 \times 10^{-6}$ × the size of the Monster group ($\approx 8.1 \times 10^{53}$) ≈ 0.017 × the number of chess positions ($\approx 5.2 \times 10^{49}$)

[ln(8.9621493*10^47)^1/10]

$$10 \log(8.9621493 \times 10^{47})$$

Result:

1.6006729829...

The result 1,6006729 is very near to the value of the electric charge of the positron.

[ln(8.9621493*10^47)Pi^2/89]

Input interpretation:

$$\log(8.9621493 \times 10^{47}) \times \frac{\pi^2}{89}$$

Result:

12.24435425...

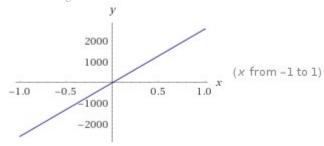
Note that the result 12.24 is very near to the value of black hole entropy 12.19

We calculate the following integral:

integrate $-16 + 1/((1728+1728)*10^41)$ [8.958711419*10^47 + 29.772((((1-(1.567^2 *(-4.313502611*10^31))/(4*(64Pi^2))*((-2.0768980*10^16)*(1/3)*(-0.03978873))]

$$\begin{split} \int & \left(-16 + \left(8.958711419 \times 10^{47} + \right. \right. \\ & \left. 29.772 \left(1 - \frac{1}{\left(4 \left(64 \, \pi^2 \right) \right) 3} \left(1.567^2 \left(-4.313502611 \times 10^{31} \right) \right) \right. \\ & \left. \left(-2.0768980 \times 10^{16} \left(-0.03978873 \right) \right) \right) \right) / \\ & \left. \left((1728 + 1728) \, 10^{41} \right) \right) dx = 2577.21 \, x + \text{constant} \end{split}$$

Plot of the integral:



The result 2577.21 is practically equal to 2575.6±3.1 and 2577.9±2.9 that are the values of the baryons charmed Xi prime.

We note that:

integrate $1/(144+64) * (-16 + 1/((1728+1728)*10^41) [8.958711419*10^47 + 29.772((((1-(1.567^2 * (-4.313502611*10^31))/(4*(64Pi^2))*((-2.0768980*10^16)*(1/3)*(-0.03978873))]$

$$\int \frac{1}{144 + 64} \left(-16 + \left(8.958711419 \times 10^{47} + 29.772 \left(1 - \frac{1}{\left(4 \left(64 \, \pi^2 \right) \right) 3} \left(1.567^2 \left(-4.313502611 \times 10^{31} \right) \right) \left(-2.0768980 \times 10^{16} \left(-0.03978873 \right) \right) \right) \right) \right)$$

$$\left((1728 + 1728) \, 10^{41} \right) dx = 12.3905 \, x + \text{constant}$$

The result 12.39 is very near to the value of black hole entropy 12.57

From the ratio between the two obtained results we have the following expression:

 $(((-9.877709 * 10^81) / (8.9621493 * 10^47))^2))^1/23)) - 16$

$$\sqrt[23]{\left(\frac{-9.877709\times10^{81}}{8.9621493\times10^{47}}\right)^2}-16$$

Result

896.42071...

Percent decrease:

$$\sqrt[23]{\left(-\frac{9.877709\times10^{81}}{8.9621493\times10^{47}}\right)^2} - 16 = 896.421 \text{ is } 1.75358$$
% smaller than $\sqrt[23]{\left(-\frac{9.877709\times10^{81}}{8.9621493\times10^{47}}\right)^2} = 912.421.$

The result 896,42 is equal to the value of rest mass of the Kaon 896.00±0.025

and:

$$(\ln(-(-9.877709 * 10^81) * (8.9621493 * 10^47)))^1/3$$

$$\sqrt[3]{\log(-(-9.877709\times10^{81})(8.9621493\times10^{47}))}$$

Result:

6.688479367...

(we observe that 6.68847 is very near to the value of $G_N = 6.70872$ that is the gravitational constant 4d of string theory)

and:

$$(\ln(-(-9.877709 * 10^81) * (8.9621493 * 10^47)))^1/12$$

$$\sqrt[12]{\log(-(-9.877709\times10^{81})(8.9621493\times10^{47}))}$$

Result:

1.6081695990...

The result 1,6081695 is very near to the value of the electric charge of the positron. Also:

 $2 * (-(-9.877709 * 10^81) + (8.9621493 * 10^47))^1/(139*3)$ where 139*3 = 417, is the difference between the value of "glueball" and the baryon Xi 1321.71 ± 0.07 . Indeed: $1739 - 1321.71 = 417.29 \approx 417$

Input interpretation:

$$2^{139\times3}\sqrt{-(-9.877709\times10^{81})+8.9621493\times10^{47}}$$

Result:

3.145284007...

This result is a good approximation to π

With the difference between the value of the strange D meson 2112.3 ± 0.5 and the value of "glueball" 1738 ± 30 , we have that: 2112.3-1738=374.3 that is about the value of the root: 375. Thence, we have:

$$(-(-9.877709 * 10^81) + (8.9621493 * 10^47))^1/(375)$$

Input interpretation

$$375 \sqrt{-(-9.877709 \times 10^{81}) + 8.9621493 \times 10^{47}}$$

Result:

1.654445314...

With the value of the Omega meson multiplied for 1/2, $(782.65\pm0.12)*1/2$, we obtain:

$$(-(-9.877709 * 10^81) + (8.9621493 * 10^47))^1/(782.65/2)$$

Input interpretation:

$$\frac{782.65}{2}\sqrt{-(-9.877709\times10^{81})+8.9621493\times10^{47}}$$

Result:

1.62006...

The results 1,65444 and 1,62006 are very near to fourteenth root of the Ramanujan's class invariant and to the mass of proton and the electric charge of the positron.

In conclusion, multiplying the two results and calculating the following integral, we obtain:

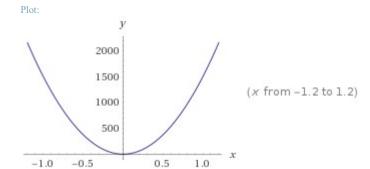
10 integrate ln(-(-9.877709 * 10^81)*(8.9621493 * 10^47))x

nput interpretation:

$$10 \int \log(-(-9.877709 \times 10^{81}) (8.9621493 \times 10^{47})) x \, dx$$

Result:

 $1496.07 x^2$



Alternate form assuming x is real:

$$1496.07 x^2 + 0$$

Indefinite integral assuming all variables are real:

$$498.69 x^3 + constant$$

where 1496 is a value very near to the $f_0(1500)$ mass:

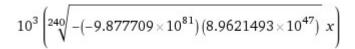
f₀(1500) MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1504± 6 OUR A	WERAGE	Error includes s	cale fac	ctor of 1	.3. See the ideogram below.
$1468 + 14 + 23 \\ -15 - 74$	5.5k	¹ ABLIKIM	13N	BES3	$e^+e^- o J/\psi o \gamma\eta\eta$
$1466 \pm 6 \pm 20$		ABLIKIM	06V	BES2	$e^+e^- \rightarrow J/\psi \rightarrow \gamma \pi^+\pi^-$
1515 ± 12		² BARBERIS	00A		450 $pp \rightarrow p_f \eta \eta p_s$
1511 ± 9		2,3 BARBERIS	00c		$450 pp \rightarrow p_f 4\pi p_s$
1510± 8		² BARBERIS	00E		450 $pp \rightarrow p_f \eta \eta p_s$
1522 ± 25		BERTIN	98	OBLX	$0.05-0.405 \ \overline{n}p \rightarrow \pi^{+}\pi^{+}\pi^{-}$
1449 ± 20		² BERTIN	97c	OBLX	$0.0 \overline{p}_P \rightarrow \pi^+ \pi^- \pi^0$
1515 ± 20		ABELE	96B	CBAR	$0.0 \overline{p}p \rightarrow \pi^0 K_I^0 K_I^0$
1500±15		4 AMSLER			$0.0 \overline{p}p \rightarrow 3\pi^0$
1505 ± 15		5 AMSLER			$0.0 \overline{p}p \rightarrow \eta \eta \pi^0$

We have that:

$$((10^3*(((-(-9.877709*10^81)*(8.9621493*10^47))^1/(240))x)))$$

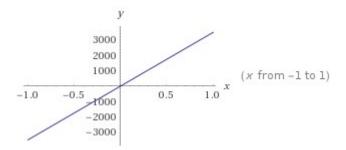
Input interpretation:



Result:

3478.93x

Plot:



Geometric figure:

line

Alternate form assuming x is real:

3478.93 x + 0

Property as a function:

Parity:

odd

Properties as a real function:

Domain:

R (all real numbers)

Range:

R (all real numbers)

Bijectivity:

bijective from its domain to R

Parity:

odd

Derivative:

$$\frac{d}{dx}(3478.93\,x) = 3478.93$$

Indefinite integral:

$$\int 10^{3} \left(\sqrt[240]{ - \left(-9.877709 \times 10^{81} \right) \left(8.9621493 \times 10^{47} \right)} \ x \right) dx = 1739.47 \ x^{2} + \text{constant}$$

1739.47

Definite integral after subtraction of diverging parts:

$$\int_0^\infty (3478.93 \, x - 3478.93 \, x) \, dx = 0$$

The result of indefinite integral is: 1739,47

And

$$((10^3 * (((-(-9.877709 * 10^81) + (8.9621493 * 10^47))^1/(152))x)))$$

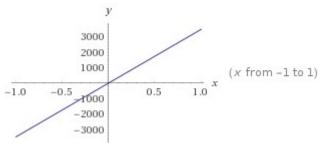
Input interpretation:

$$10^{3} \sqrt{\frac{152}{100} - (-9.877709 \times 10^{81}) + 8.9621493 \times 10^{47}} x$$

Result:

3462.89 x

Plot:



Geometric figure:

line

Alternate form assuming x is real:

3462.89 x + 0

Properties as a real function:

Domain

R (all real numbers)

Range:

R (all real numbers)

Bijectivity:

bijective from its domain to R

Parity:

odd

Derivative:

$$\frac{d}{dx}(3462.89 x) = 3462.89$$

Indefinite integral:

$$\int 10^{3} \left(\sqrt[152]{ - \left(-9.877709 \times 10^{81} \right) + 8.9621493 \times 10^{47}} \ x \right) dx = 1731.44 \ x^{2} + \text{constant}$$

1731.44

Definite integral after subtraction of diverging parts:

$$\int_0^\infty (3462.89 \, x - 3462.89 \, x) \, dx = 0$$

The result of indefinite integral is: 1731,44

We note that:

integrate 2Pi/1728 [-16+10^3 * (((-(-9.877709 * 10^81)+(8.9621493 * 10^47))^1/152)))]

$$\int \frac{2\pi \left(-16 + 10^{3} \sqrt[152]{ - \left(-9.877709 \times 10^{81}\right) + 8.9621493 \times 10^{47}\right)}}{1728} dx = 12.5332 x + \text{constant}$$

The result 12.53 is practically equal to the value of black hole entropy 12.57

We calculate also the following double integral:

((integrate integrate -16+ 10^3 * (((-(-9.877709 * 10^81)+(8.9621493 * 10^47))^1/152))))

Input interpretation:

$$\int \left(\int \left(-16 + 10^{3} \sqrt{-(-9.877709 \times 10^{81}) + 8.9621493 \times 10^{47}} \right) dx \right) dx$$

Result:

 $1723.44 x^2$

Indefinite integral assuming all variables are real:

Definite integral over a disk of radius R:

$$\iint_{2 \, x^2 < R^2} 3446.89 \, dx \, dx = 10828.7 \, R^2$$
Definite integral over a square of edge length 2 L:
$$\int_{-L}^{L} \int_{-L}^{L} 3446.89 \, dx \, dx = 13787.6 \, L^2$$

Result: 1723,44

Note that 1723.44 1739.47 and 1731.44 are results that are practically in the range of the mass of the candidate "glueball" $f_0(1710)$. Indeed, as we can see from the following Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the various integrations.

fo(1710) MASS

```
DOCUMENT ID TECN COMMENT
VALUE (MeV) EVTS
1723 + 6 OUR AVERAGE Error includes scale factor of 1.6. See the ideogram below.
1759 \pm 6 + \frac{14}{-25}
                                                         13N BES3 e^+e^- \rightarrow J/\psi \rightarrow \gamma \eta \eta
                        5.5k
                                    1 ABLIKIM
1750 + 6 + 29 \\ - 7 - 18
                                                          13 BELL \gamma \gamma \rightarrow \kappa_S^0 \kappa_S^0
                                      UEHARA
1701 ± 5 + 9
                                                          08 ZEUS ep \rightarrow K_S^0 K_S^0 X
                                    <sup>2</sup> CHEKANOV
                          4k
1765 + \frac{4}{3} \pm 13
                                                          06V BES2 e^+e^- \rightarrow J/\psi \rightarrow \gamma \pi^+\pi^-
                                      ABLIKIM
1760±15 +15
                                                          05Q BES2 \psi(2S) \rightarrow \gamma \pi^+ \pi^- K^+ K^-
                                    3 ABLIKIM
                                                          04E BES2 J/\psi \rightarrow \omega K^+ K^-
1738 \pm 30
                                       ABLIKIM
1740± 4 +10
                                    4 BAI
                                                          03<sub>G</sub> BES
                                                                          J/\psi \rightarrow \gamma K\overline{K}
1740 + 30
                                                                          J/\psi \rightarrow \gamma (\pi^+\pi^-\pi^+\pi^-)
                                    4 BAI
                                                         00A BES
1698 \pm 18
                                    5 BARBERIS
                                                         00E
                                                                          450 pp \rightarrow p_f \eta \eta p_S
                                    6 BARBERIS
                                                          99D OMEG 450 pp \rightarrow K^+K^-, \pi^+\pi^-
1710±12 ±11
                                                                         300 pp \rightarrow p_f(K^+K^-)p_s

J/\psi \rightarrow \gamma K^+K^-, K_s^0 K_s^0
                                    <sup>7</sup> FRENCH
                                                          99
1710 \pm 25
                                    8 AUGUSTIN
                                                          88
1707 \pm 10
                                                               DM<sub>2</sub>
                                    8 AUGUSTIN
                                                          87 DM2
                                                                          J/\psi \rightarrow \gamma \pi^+ \pi^-
1698 \pm 15
                                    9 BALTRUSAIT...87
1720 \pm 10 \pm 10
                                                                MRK3 J/\psi \rightarrow \gamma K^+ K^-
                                    8 WILLIAMS
1742 \pm 15
                                                          84 MPSF 200 \pi^- N \rightarrow 2K_c^0 X
                                                          83 CBAL J/\psi \rightarrow \gamma 2\eta
1670 \pm 50
                                      BLOOM

    • • • We do not use the following data for averages, fits, limits, etc. • • •
                         381 10,11 DOBBS
1744 \pm 7 \pm 5
                                                          15
                                                                           J/\psi \rightarrow \gamma \pi^{+}\pi^{-}
                         237 10,11 DOBBS
                                                                          \psi(2S) \rightarrow \gamma \pi^+ \pi^-
1705 \pm 11 \pm 5
                                                         15
                        1.0k 10,11 DOBBS
1706 \pm 4 \pm 5
                                                         15
                                                                          J/\psi \rightarrow \gamma K^+ K^-
                         349 10,11 DOBBS
                                                                          \psi(2S) \rightarrow \gamma K^+ K^-
1690 \pm 8 \pm 3
                                                          15
                                                               CBAR 1.64 pp \rightarrow K^+K^-\pi^0
1750 \pm 13
                                      AMSLER
                                                          06
                         80k 12,13 UMAN
                                                                E835 5.2 \overline{p}p \rightarrow \eta \eta \pi^0
1747 \pm 5
                                                          06
                                                                SPEC 40 \pi^- p \rightarrow K_S^0 K_S^0 n
1776 \pm 15
                                      VLADIMIRSK...06
1790 + 40
                                    3 ABLIKIM
                                                                 BES2 J/\psi \rightarrow \phi \pi^+ \pi^-
                                  12 BINON
                                                          05 GAMS 33 \pi^- p \rightarrow \eta \eta n
1670 \pm 20
                                  <sup>13</sup> CHEKANOV
                                                         04 ZEUS ep \rightarrow K_S^0 K_S^0 X
1726 \pm 7
                          74
                                  <sup>14</sup> ANISOVICH
                                                          03
1732 + 15
                                      TIKHOMIROV 03 SPEC 40.0 \pi^-C \rightarrow \kappa_S^0 \kappa_S^0 \kappa_I^0 X
1682 \pm 16
                        3.6k 4,15 NICHITIU
                                                         02 OBLX
1670 \pm 26
                               16,17 ANISOVICH
                                                         99B SPEC 0.6–1.2 p\overline{p} \rightarrow \eta \eta \pi^0
1770 \pm 12
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```
<sup>4</sup> BARBERIS
                                                                                    OMEG 450 pp \rightarrow p_s p_f K^+ K^-
1730 \pm 15
                                                                                      OMEG 450 pp \rightarrow p_s p_f \pi^+ \pi^-
                                                 <sup>4</sup> BARBERIS
1750±20
                                               <sup>18</sup> ANISOVICH
                                                                              98B RVUE Compilation
1750 \pm 30
                                                                                                     J/\psi \rightarrow \gamma \pi^0 \pi^0
1720±39
                                                    BAI
                                                                              98H BES
                                               19 BARKOV
                                                                              98 \pi^{-}p \to K_{S}^{0}K_{S}^{0}n
96c DLPH Z^{0} \to K^{+}K^{-} + X
1775 \pm 1.5
                                               <sup>20</sup> ABREU
1690 \pm 11
                                                 9 BAI
                                                                              96c BES
                                                                                                    J/\psi \rightarrow \gamma K^+ K^-
1696 \pm 5
                                                                                                    J/\psi \rightarrow \gamma K^+ K^-
                                                 4 BAI
                                                                              96c BES
1781 ± 8
                                                 BALOSHIN 95 SPEC 40 \pi^- \text{C} \rightarrow K_S^0 K_S^0 \text{X}
^{21} BUGG 95 MRK3 J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^+ \pi^-
^{9} BUGG 95 MRK3 J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^+ \pi^-
^{8} ARMSTRONG 93C E760 \overline{p}p \rightarrow \pi^0 \eta \eta \rightarrow 6\gamma
1768±14
                                               <sup>21</sup> BUGG
1750 \pm 15
1620 \pm 16
1748 \pm 10
                                                                                                     pp \rightarrow pp\pi^{+}\pi^{-}\pi^{+}\pi^{-}
                                                    BREAKSTONE 93
                                                                                       SFM
\sim 1750
1744 \pm 15
                                                                              92D GAM2 38 \pi^- p \rightarrow \eta \eta n
                                               <sup>23</sup> ARMSTRONG 89D OMEG 300 pp \rightarrow ppK^+K
1713 \pm 10
                                               <sup>23</sup> ARMSTRONG 89D OMEG 300 pp \rightarrow ppK_S^0K_S^0
1706 \pm 10
                                                                             88 SPEC 40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n

88 SPEC 40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n

88 SPEC 40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n

88 DM2 J/\psi \rightarrow \phi K^{+} K^{-}, K_{S}^{0} K_{S}^{0}

88 DM2 J/\psi \rightarrow \phi K^{+} K^{-}, K_{S}^{0} K_{S}^{0}
                                                 9 BOLONKIN
1700 \pm 15
                                                 <sup>4</sup> BOLONKIN
1720 \pm 60
                                               <sup>24</sup> FALVARD
1638 \pm 10
                                               <sup>25</sup> FALVARD
1690 \pm 4
                                                                              86c GAM2 38 \pi^- p \to n2\eta
                                               <sup>26</sup> ALDE
1755 \pm 8
1730 + 2 \\ -10
                                                                                       RVUE 22 \pi^- p \rightarrow n2K_S^0
                                               <sup>27</sup> LONGACRE
                                                                                       MRK2 J/\psi \rightarrow \gamma 2\rho
1650 \pm 50
                                                    BURKE
                                                                              82
                                          28,29 EDWARDS
                                                                              82D CBAL J/\psi \rightarrow \gamma 2\eta
1640 \pm 50
                                                                              82C MPS 23 \pi^- p \to n2K_S^0
                                               30 ETKIN
1730±10 ±20
```

Now:

Excitation of glueball (n)	n = 0	n = 1	n = 2	n = 3	n = 4
Glueball mass $M_E^{(n)}$	0.901	2.285	3.240	4.149	5.041
Glueball mass $M_{D,T}^{(n)}$	1.567	2.485	3.373	4.252	5.124
The coefficients	n = 0	n = 1	n = 2	n = 3	n = 4
\mathcal{C}_E	144.545	114.871	131.283	146.259	157.832
\mathcal{C}_D	29.772	36.583	42.237	47.220	51.724
\mathcal{C}_T	72.927	89.609	103.46	115.664	126.696

We note that dividing the two numbers, 3.240 for M_E n = 2 and 2.485 for M_D n = 1, that are the mass of the dilatonic and exotic scalar glueball, we obtain 3.240 / 2.485 =

1,303822937 which is very near to the mass ratio of the glueball candidates. Indeed, we have: 1.723 / 1.504 = 1,1456117 that with the recent value of $f_0(1710)$ is:

1.739 / 1.504 = 1,15625. If we take 5.041 for M_E n = 4 and 4.252 for M_D n = 3, we obtain: 5.041 / 4.252 = 1,18555973 very near to 1,15625. We observe, also that 1,18555 is practically equal to $(1,08643)^2 = 1,18033$ where 1,08643 is the "new Ramanujan's constant".

Now:

	I	II	III	IV
Γ	$0.0392\lambda^{-1}$	$0.0628\lambda^{-1}$	$0.0785\lambda^{-1}$	$0.1046\lambda^{-1}$
	V	VI	VII	VIII
Γ	$0.1674\lambda^{-1}$	$0.2093\lambda^{-1}$	$0.6316\lambda^{-1}$	$1.0527\lambda^{-1}$

Table 3: The corresponding decay rates in the units of m_H to the transitions in (3.7) by setting $l = 0, N_Q = 1, N_c = 3, N_f = 2$.

and this result would be in agreement with the previous discussion in [19]. Therefore we could conclude that only the decay process VIII in (3.7) might be realistic. This transition describes the decay of the baryonic meson consisted of one heavy- and one light- flavoured quark. So while the identification of the other transitions might be less clear, the transition VIII could be interpreted as the decay of the baryonic B-meson involving the glueball candidate f_0 (1710) as discussed e.g. in [8, 9, 10] since the corresponding quantum numbers of the states could be identified.

From Table 3 we have the decay process VIII that is:

$$1.0527\lambda^{-1} = 1.0527 * -4,313502611 * 10^{31} = -4,5408241985997 * 10^{31}$$

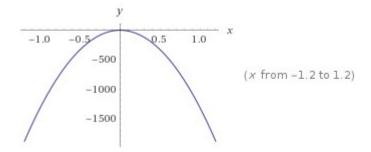
Now, we have that:

The rest mass of baryon Xi is 1314.86 ± 0.20 . We have that:

Indefinite integral:

$$\int -\frac{\left(4.5408241985997 \times 10^{31}\right)x}{1729 \times 10^{25}} dx = -1313.1359741468 x^2 + \text{constant}$$

Plot of the integral:



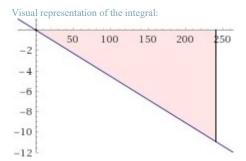
Or:

integrate $1/(10^33)$ [(-4.5408241985997 * 10^31)] x,[1729/(1.63721868)^4, 0]

Definite integral:

$$\int_{\frac{1729}{1627318684}}^{0} -\frac{\left(4.5408241985997 \times 10^{31}\right)x}{10^{33}} dx = 1314.74$$

The result 1314.74 is equal to the rest mass of baryon Xi that is 1314.86±0.20



Riemann sums

left sum
$$1314.74 + \frac{1314.74}{n} = 1314.74 + \frac{1314.74}{n} + O((\frac{1}{n})^2)$$

(assuming subintervals of equal length)

Indefinite integral:

$$\int -\frac{\left(4.5408241985997 \times 10^{31}\right)x}{10^{33}} dx = -0.022704120992999 x^2 + \text{constant}$$

The value 0,02270412 is a good approximation to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \text{ GeV}^4$

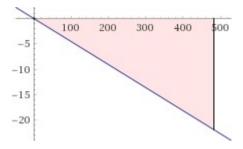
And:

integrate $1/(10^33)$ [(-4.5408241985997 * 10^31)] x,[1728/(ln36), 0]

$$\int_{\frac{1728}{\log(36)}}^{0} - \frac{\left(4.5408241985997 \times 10^{31}\right)x}{10^{33}} dx = 5279.2564694966$$

The result 5279.256 is practically equal to the rest mass of B meson, that is 5279.15 ± 0.31 5279.53 ± 33

Visual representation of the integral:



Riemann sums:

left sum
$$\frac{5279.256469497}{n} + 5279.2564694966 = 5279.2564694966 + \frac{5279.256469497}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$$

(assuming subintervals of equal length)

Indefinite integral:

$$\int -\frac{\left(4.5408241985997 \times 10^{31}\right)x}{10^{33}} dx = -0.022704120992999 x^2 + \text{constant}$$

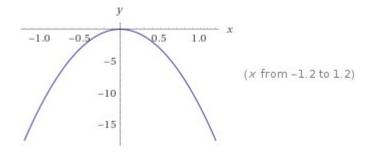
Also here, the value 0,02270412 is a good approximation to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \text{ GeV}^4$

We have that:

integrate 1/((1729*3*(sqrt(5)+5)/2 *10^26)) [(-4.5408241985997 * 10^31)]x

Indefinite integral:
$$\int -\frac{\left(4.5408241985997\times10^{31}\right)x}{\frac{1729}{2}\times3\left(\sqrt{5}\right.}\,dx = -12.098061896138\,x^2 + \text{constant}$$
 Open code

Plot of the integral:



The result -12.098 is very near to the value of black hole entropy 12.19, but with the sign minus

Now:

The value of Q corresponds to the situation of a baryonic bound state consisting of N_Q heavy flavoured quarks. The eigenfunctions and mass spectrum of (C-7) can be evaluated by solving its Schrodinger equation, respectively they are obtained as⁸,

$$\psi(y_I) = R(\rho)T^{(l)}(a_I), \ R(\rho) = e^{-\frac{m_y \omega_\rho}{2}\rho^2} \rho^{\tilde{l}} Hypergeometric_1 F_1\left(-n_\rho, \tilde{l} + 2; m_y \omega_\rho \rho^2\right),$$

$$E(l, n_\rho, n_z) = \omega_\rho \left(\tilde{l} + 2n_\rho + 2\right) = \sqrt{\frac{(l+1)^2}{6} + \frac{640}{3}a^2\pi^4Q^2} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}}.$$
(C-9)

Notice that $T^{(l)}(a_I)$ satisfies $\nabla^2_{S^3}T^{(l)} = -l(l+2)T^{(l)}$ which is the function of the spherical part because H_y can be written with the radial coordinate ρ as,

$$H_{y} = -\frac{1}{2m_{y}} \left[\frac{1}{\rho^{3}} \partial_{\rho} (\rho^{3} \partial_{\rho}) + \frac{1}{\rho^{2}} \left(\nabla_{S^{3}}^{2} - 2m_{y} \mathcal{Q} \right) \right] + \frac{1}{2} m_{y} \omega_{\rho}^{2} \rho^{2}. \tag{C-10}$$

For 1 = 0 and Q = -3,29867 we obtain:

$$((sqrt(640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2)) + ((2*(3+2)+2))/(sqrt(6))$$

Input interpretation:

$$\sqrt{\frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} \, + \frac{2 \, (3+2) + 2}{\sqrt{6}}$$

Result

4.9699805...

Series representations:

$$\begin{split} \sqrt{\frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \frac{12 + \sqrt{z_0}^2 \, \sum_{k_1 = 0}^\infty \sum_{k_2 = 0}^\infty \frac{(-1)^{k_1 + k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \, (6 - z_0)^{k_1} \left(\frac{0.0497541}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1 - k_2}}{k_1! \, k_2!} \\ \sqrt{z_0} \, \sum_{k=0}^\infty \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6 - z_0)^k \, z_0^{-k}}{k!} \\ & \text{for not} \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right) \end{split}$$

$$\begin{split} \sqrt{\frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(12 + \exp\!\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \exp\!\left(i \, \pi \left\lfloor \frac{\arg\!\left(\frac{0.0497541}{\pi^2} - x\right)}{2 \, \pi} \right\rfloor\right) \sqrt{x}^2 \right. \\ \left. \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, (6-x)^{k_1} \left(\frac{0.0497541}{\pi^2} - x\right)^{k_2} \, x^{-k_1 - k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! \, k_2!} \right) / \\ \left. \left(\exp\!\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \sqrt{x} \, \sum_{k=0}^{\infty} \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \, \text{for} \, (x \in \mathbb{R} \, \text{and} \, x < 0) \right. \end{split}$$

$$\begin{split} \sqrt{\frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, + \, \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(\left(\frac{1}{z_0}\right)^{-1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \, z_0^{-1/2 - 1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \\ \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/(2 \, \pi) \right\rfloor} \right) \\ & \qquad \qquad \frac{1 + 1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/(2 \, \pi) \right\rfloor}{z_0} \\ & \qquad \qquad \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, \left(-\frac{1}{2}\right)_{k_1} \, \left(-\frac{1}{2}\right)_{k_2} \, (6 - z_0)^{k_1} \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1 - k_2}}{k_1! \, k_2!} \\ & \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6 - z_0)^k \, z_0^{-k}}{k!} \right) \end{split}$$

For $l = 1,616 * 10^{-35}$ and Q = -3,29867 we obtain:

 $((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2)) + ((2*(3+2)+2))/(sqrt(6))$

Input interpretation:

$$\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \,\pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2 \,(3+2) + 2}{\sqrt{6}}$$

Result

5.31335590...

Series representations

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \frac{12 + \sqrt{z_0}^2 \, \sum_{k_1 = 0}^\infty \sum_{k_2 = 0}^\infty \frac{\left(-1\right)^{k_1 + k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \, (6 - z_0)^{k_1} \left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1 - k_2}}{k_1! \, k_2!} \\ \sqrt{z_0} \, \sum_{k = 0}^\infty \frac{\left(-1\right)^{k} \left(-\frac{1}{2}\right)_{k} \, (6 - z_0)^{k} \, z_0^{-k}}{k!} \\ \text{for not} \, \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right) \end{split}$$

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(12 + \exp\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \exp\left(i \, \pi \left\lfloor \frac{\arg\left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - x\right)}{2 \, \pi} \right\rfloor\right) \sqrt{x}^2 \\ \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, (6-x)^{k_1} \left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - x\right)^{k_2} \, x^{-k_1 - k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! \, k_2!} \right) \\ \left(\exp\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \sqrt{x} \, \sum_{k=0}^{\infty} \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \, \text{for } (x \in \mathbb{R} \text{ and } x < 0) \end{split}$$

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-3.29867)^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(\left(\frac{1}{z_0}\right)^{-1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \, z_0^{-1/2 - 1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \\ &- \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - z_0\right) \middle/(2 \, \pi) \right\rfloor} \right. \\ &- \left. \frac{1 + 1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - z_0\right) \middle/(2 \, \pi) \right\rfloor}{z_0} \right. \\ &- \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, \left(-\frac{1}{2}\right)_{k_1} \, \left(-\frac{1}{2}\right)_{k_2} \, (6 - z_0)^{k_1} \, \left(\frac{1}{6} + \frac{0.0497541}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1 - k_2}}{k_1 \, ! \, k_2 \, !} \\ &- \left. \frac{1}{2} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \, \left(-\frac{1}{2}\right)_k \, (6 - z_0)^k \, z_0^{-k}}{k \, !} \right) \right. \end{split}$$

For $l = 1,616 * 10^{-35}$ and Q = -54,192473 we obtain:

$$((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-54.192473)^2)) + ((2*(3+2)+2))/(sqrt(6))$$

Input interpretation:

$$\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \left(-54.192473\right)^2 + \frac{2 \, (3+2) + 2}{\sqrt{6}}}$$

Result

6.13480446...

Series representations

$$\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, (-54.1925)^2} + \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \frac{12 + \sqrt{z_0}^2 \, \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-1\right)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \, (6-z_0)^{k_1} \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1-k_2}}{k_1! k_2!} \\ \sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{\left(-1\right)^{k} \left(-\frac{1}{2}\right)_{k} \, (6-z_0)^{k} \, z_0^{-k}}{k!} \\ \text{for not } \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)$$

$$\begin{split} \sqrt{\frac{1}{6}} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3} \right)^2 640 \, \pi^4 \, (-54.1925)^2 \, + \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(12 + \exp \left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor \right) \exp \left(i \, \pi \left\lfloor \frac{\arg\left(\frac{1}{6} + \frac{13.4286}{\pi^2} - x \right)}{2 \, \pi} \right\rfloor \right) \sqrt{x}^2 \\ \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, (6-x)^{k_1} \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - x \right)^{k_2} \, x^{-k_1 - k_2} \left(-\frac{1}{2} \right)_{k_1} \left(-\frac{1}{2} \right)_{k_2}}{k_1! \, k_2!} \right) / \\ \left(\exp \left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor \right) \sqrt{x} \, \sum_{k=0}^{\infty} \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right) \, \text{for} \, (x \in \mathbb{R} \, \text{and} \, x < 0) \end{split}$$

For $l = 1,616 * 10^{-35}$ and Q = 50,893800 we obtain:

$$((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(50.893800)^2)) + ((2*(3+2)+2))/(sqrt(6))$$

Input interpretation

$$\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \times 50.893800^2} + \frac{2 \, (3+2) + 2}{\sqrt{6}}$$

Result:

6.06802466...

Series representations

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, 50.8938^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \frac{12 + \sqrt{z_0}^2 \, \sum_{k_1=0}^\infty \sum_{k_2=0}^\infty \frac{\left(-1\right)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \, (6-z_0)^{k_1} \left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right)^{k_2} \, z_0^{-k_1-k_2}}{k_1! \, k_2!} \\ \sqrt{z_0} \, \sum_{k=0}^\infty \frac{\left(-1\right)^{k} \left(-\frac{1}{2}\right)_{k} \, (6-z_0)^{k} \, z_0^{-k}}{k!} \\ & \text{for not} \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right) \end{split}$$

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, 50.8938^2} \, &+ \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(12 + \exp\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \exp\left(i \, \pi \left\lfloor \frac{\arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - x\right)}{2 \, \pi} \right\rfloor\right) \sqrt{x}^2 \\ & \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \frac{(-1)^{k_1 + k_2} \, (6-x)^{k_1} \left(\frac{1}{6} + \frac{11.8435}{\pi^2} - x\right)^{k_2} \, x^{-k_1 - k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! \, k_2!} \right) / \\ \left(\exp\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \sqrt{x} \, \sum_{k=0}^{\infty} \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \, \text{for } (x \in \mathbb{R} \text{ and } x < 0) \end{split}$$

$$\begin{split} \sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \, 50.8938^2} \, + \, \frac{2 \, (3+2) + 2}{\sqrt{6}} = \\ \left(\left(\frac{1}{z_0}\right)^{-1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \, z_0^{-1/2 - 1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor} \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right) \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right) \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right) \right. \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right. \right) \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg(6-z_0)/(2 \, \pi) \rfloor + 1/2 \, \left\lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right) \right. \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor \arg\left(\frac{1}{6} + \frac{11.8435}{\pi^2} - z_0\right) \middle/ (2 \, \pi) \right\rfloor} \right. \right. \right) \right. \\ \left. \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \\ \left. \left(12 + \left(\frac{1}{z_0}\right)^{1/2 \, \lfloor 3/2 \, \rfloor} \right) \right. \\ \left. \left(12 + \left(\frac{1}{z_0$$

Now, we calculate the exp of the above expression for $l = 1,616 * 10^{-35}$ and Q = -54,192473 and multiplied it for π . We obtain:

Pi * exp((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-54.192473)^2)) + ((2*(3+2)+2))/(sqrt(6))

$$\pi \exp\left(\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \,\pi^3}\right)^2 \pi^4 \left(-54.192473\right)^2} + \frac{2 \,(3+2) + 2}{\sqrt{6}}\right)$$

Result:

1450.3125...

Series representations:

$$\pi \exp\left[\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \left(-54.1925\right)^2} \right. \\ + \frac{2 \, (3+2) + 2}{\sqrt{6}}\right] = \\ \pi \exp\left[\frac{12}{\sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(6 - z_0\right)^k z_0^{-k}}{k!}} + \sqrt{z_0} \, \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - z_0\right)^k z_0^{-k}}{k!}\right] \\ \text{for not } \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)$$

$$\pi \exp\left[\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 640 \, \pi^4 \left(-54.1925\right)^2} \right. + \frac{2 \, (3+2) + 2}{\sqrt{6}}\right] = \\ \pi \exp\left[\frac{12}{\exp\left(i \, \pi \left\lfloor \frac{\arg(6-x)}{2 \, \pi} \right\rfloor\right) \sqrt{x} \, \sum_{k=0}^{\infty} \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right. \\ \left. \exp\left[i \, \pi \left\lfloor \frac{\arg\left(\frac{1}{6} + \frac{13.4286}{\pi^2} - x\right)}{2 \, \pi} \right\rfloor\right] \sqrt{x} \right. \\ \left. \sum_{k=0}^{\infty} \frac{(-1)^k \, \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - x\right)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. \right.$$
 for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\pi \exp\left[\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \pi^3}\right)^2 640 \pi^4 \left(-54.1925\right)^2} + \frac{2 (3+2) + 2}{\sqrt{6}}\right] = \\ \pi \exp\left[\frac{12 \left(\frac{1}{z_0}\right)^{-1/2 \left[\arg (6-z_0)/(2\pi)\right]} z_0^{-1/2 - 1/2 \left[\arg (6-z_0)/(2\pi)\right]}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!}} + \left(\frac{1}{z_0}\right)^{1/2 \left[\arg \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - z_0\right) / (2\pi)\right]} \\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!} + \left(\frac{1}{z_0}\right)^{1/2 \left[\arg \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - z_0\right) / (2\pi)\right]} \\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{6} + \frac{13.4286}{\pi^2} - z_0\right)^k z_0^{-k}}{k!} \right] \\ \pi \exp\left[\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \pi^3}\right)^2 640 \pi^4 \left(-54.1925\right)^2} + \frac{2 (3+2) + 2}{\sqrt{6}}\right] = \\ \left(12 \left(\frac{1}{2}\right)^{-1/2 \left[\arg (6-z_0)/(2\pi)\right]} z_0^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} \right] + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} \right] + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]\right)} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}}\right)^{1/2 \left(-1 - \left[\arg (6-z_0)/(2\pi)\right]} + \left(\frac{12 (3+2) + 2}{\sqrt{6}$$

$$\pi \exp\left[\sqrt{\frac{1}{6} + \frac{1}{3} \left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4} \left(-54.1925\right)^{2} + \frac{2 (3+2)+2}{\sqrt{6}}}\right] = \\
\pi \exp\left[\frac{12 \left(\frac{1}{z_{0}}\right)^{-1/2 \left[\arg(6-z_{0})/(2\pi)\right]} z_{0}^{1/2 \left(-1-\left[\arg(6-z_{0})/(2\pi)\right]\right)}}{\sum_{k=0}^{\infty} \frac{\left(-1\right)^{k} \left(-\frac{1}{2}\right)_{k} (6-z_{0})^{k} z_{0}^{-k}}{k!}} + \left(\frac{1}{z_{0}}\right)^{1/2} \left[\frac{\arg\left(\frac{1}{6} + \frac{13.4286}{\pi^{2}} - z_{0}\right)}{k!}\right] z_{0}^{\infty}} - \frac{1/2 \left(1 + \left[\arg\left(\frac{1}{6} + \frac{13.4286}{\pi^{2}} - z_{0}\right)\right]/(2\pi)\right]}{\sum_{k=0}^{\infty} \frac{\left(-1\right)^{k} \left(-\frac{1}{2}\right)_{k} \left(\frac{1}{6} + \frac{13.4286}{\pi^{2}} - z_{0}\right)^{k} z_{0}^{-k}}{k!}}{k!}\right]$$

From:

72. $\rho(1450)$ and $\rho(1700)$

Updated November 2015 by S. Eidelman (Novosibirsk), C. Hanhart (Juelich) and G. Venanzoni (Frascati).

This scenario with two overlapping resonances is supported by other data. Bisello [9] measured the pion form factor in the interval 1.35–2.4 GeV, and observed a deep minimum around 1.6 GeV. The best fit was obtained with the hypothesis of ρ -like resonances at 1420 and 1770 MeV, with widths of about 250 MeV. Antonelli [10] found that the e+e- $\rightarrow \eta \pi$ + π - cross section is better fitted with two fully interfering Breit-Wigners, with parameters in fair agreement with those of [2] and [9]. These results can be considered as a confirmation of the ρ (1450).

The result of the above expression is 1450,3125 that is practically equal to the mass of meson $\rho(1450)$. Indeed, as we can see from the next Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the analyzed expression.

ρ(1450) MASS

VALUE (MeV) DOCUMENT ID

1465±25 OUR ESTIMATE This is only an educated guess; the error given is larger than the error on the average of the published values.

$\eta \rho^0$ MODE

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
• • • We do n	ot use the f	ollowing data for av	erage	es, fits, li	mits, etc. • • •
1500 ± 10	7.4k	¹ ACHASOV	18	SND	1.22–2.00 $e^+e^- \to \eta \pi^+\pi^-$
1497 + 14		² AKHMETSHIN	01B	CMD2	$e^+e^- \rightarrow \eta \gamma$
1421 ± 15		3 AKHMETSHIN	00D	CMD2	$e^+e^- \rightarrow \eta \pi^+\pi^-$
1470±20		ANTONELLI	88	DM2	$e^+e^- \rightarrow \eta \pi^+\pi^-$
1446 ± 10		FUKUI	88	SPEC	8.95 $\pi^- p \rightarrow \eta \pi^+ \pi^- n$

¹ From the combined fit of AULCHENKO 15 and ACHASOV 18 in the model with the interfering $\rho(1450)$, $\rho(1700)$ and $\rho(2150)$ with the parameters of the $\rho(1450)$ and $\rho(1700)$ floating and the mass and width of the $\rho(2150)$ fixed at 2155 MeV and 320 MeV, respectively. The phases of the resonances are π , 0 and π , respectively.

$\omega \pi$ MODE

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
• • • We do no	t use the fo	ollowing data for ave	rages	, fits, lin	nits, etc. • • •
1510± 7	10.2k	¹ ACHASOV	16 D	SND	$1.05-2.00 e^+e^- \rightarrow \pi^0\pi^0\gamma$
$1544 \pm 22 + 11 \\ -46$	821	² MATVIENKO	15	BELL	$\overline{B}^0 \rightarrow D^{*+} \omega \pi^-$
1491±19 1582±17±25	7815 2382	³ ACHASOV ⁴ AKHMETSHIN	13 103B	SND CMD2	$1.05-2.00 e^{+}e^{-} \rightarrow \pi^{0}\pi^{0}\gamma$ $e^{+}e \rightarrow \pi^{0}\pi^{0}\gamma$
$1349 \pm 25 + 10$	341	5 ALEXANDER	01 B	CLE2	$B \rightarrow D(*)\omega\pi^-$
1523±10 1463±25		⁶ EDWARDS ⁷ CLEGG	00A 94	CLE2 RVUE	$ au^- \rightarrow \omega \pi^- \nu_T$
1250		8 ASTON			20–70 $\gamma p \rightarrow \omega \pi^0 p$
1290 ± 40		⁸ BARBER	80c	SPEC	$3-5 \gamma p \rightarrow \omega \pi^0 p$

¹ From a phenomenological model based on vector meson dominance with interfering $\rho(770)$, $\rho(1450)$, and $\rho(1700)$. The $\rho(1700)$ mass and width are fixed at 1720 MeV and 250 MeV, respectively. Systematic uncertainties not estimated. Supersedes ACHASOV 13.

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 $^{^2}$ Using the data of AKHMETSHIN 01B on $e^+e^-\to\eta\gamma$, AKHMETSHIN 00D and ANTONELLI 88 on $e^+e^-\to\eta\pi^+\pi^-$.

³ Using the data of ANTONELLI 88, DOLINSKY 91, and AKHMETSHIN 00D. The energy-independent width of the $\rho(1450)$ and $\rho(1700)$ mesons assumed.

ACHASOV 13. Using Breit-Wigner parameterization of the $\rho(1450)$ and assuming equal probabilities of the $\rho(1450) \to \pi\pi$ and $\rho(1450) \to \omega\pi$ decays.

³ From a phenomenological model based on vector meson dominance with the interfering $\rho(1450)$ and $\rho(1700)$ and their widths fixed at 400 and 250 MeV, respectively. Systematic uncertainty not estimated.

4π MODE

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
• • • We do not use	the following data for avera	ges,	fits, limi	its, etc. • • •
1435 ± 40				$0.0 \overline{p} n \rightarrow 2\pi^- 2\pi^0 \pi^+$
1350 ± 50				$e^{+}e^{-} \rightarrow 2(\pi^{+}\pi^{-})$
1449 ± 4	1 ARMSTRONG	89E	OMEG	300 $pp \to pp2(\pi^{+}\pi^{-})$

¹ Not clear whether this observation has l=1 or 0.

$\pi\pi$ MODE

VALUE (MeV)		EVTS	DOCUMENT ID		TECN	COMMENT
\	We do r	ot use	the fol	lowing data for avera	ages, fi	ts, limit	s, etc. • • •
1326.35	5± 3.40	6		¹ BARTOS	17	RVUE	$e^+e^- \rightarrow \pi^+\pi^-$
1342.33	1 ± 46.62	2		² BARTOS	17A	RVUE	$e^+e^- \rightarrow \pi^+\pi^-$
1373.83	3 ± 11.3	7		3 BARTOS	17A	RVUE	$ au^- ightarrow ~\pi^- \pi^0 u_ au$
1429	± 41		20K	4 LEES	17 C	BABR	$J/\psi \rightarrow \pi^{+}\pi^{-}\pi^{0}$
1350	± 20	$^{+20}_{-30}$	63.5k	5 ABRAMOWIC	Z12	ZEUS	$ep \rightarrow e\pi^{+}\pi^{-}p$
1493	±15			6 LEES	12G	BABR	$e^+e^- ightarrow \pi^+\pi^-\gamma$
1446	± 7	± 28	5.4M	7,8 FUJIKAWA	08	BELL	$ au^- ightarrow ~\pi^- \pi^0 u_ au$
1328	± 15			9 SCHAEL	05C	ALEP	$ au^- ightarrow ~\pi^- \pi^0 u_ au$
1406	± 15		87k	7,10 ANDERSON	00A	CLE2	$ au^- ightarrow ~\pi^- \pi^0 u_ au$
~ 1368	i i			11 ABELE	99C	CBAR	7 1
1348	± 33			BERTIN	98	OBLX	$0.05-0.405 \overline{n}p \rightarrow$
1411	±14			12 ABELE	97	CBAR	$\frac{2\pi^{+}\pi^{-}}{pn \to \pi^{-}\pi^{0}\pi^{0}}$
1370	$^{+90}_{-70}$			ACHASOV	97	RVUE	$e^+e^- \rightarrow \pi^+\pi^-$
1359	±40			10 BERTIN	97c	OBLX	$0.0 \overline{p}p \rightarrow \pi^+\pi^-\pi^0$
1282	± 37			BERTIN	97D	OBLX	$0.05 \overline{p}p \rightarrow 2\pi^{+}2\pi^{-}$
1424	± 25			BISELLO	89	DM2	$e^+e^- \rightarrow \pi^+\pi^-$
1265.5	±75.3			DUBNICKA	89	RVUE	$e^+e^- \rightarrow \pi^+\pi^-$
1292	±17			¹³ KURDADZE	83	OLYA	$0.64-1.4 e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}$

¹ Applies the Unitary & Analytic Model of the pion electromagnetic form factor of DUB-NICKA 10 to analyze the data of LEES 12G and ABLIKIM 16C.

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⁴ Using the data of AKHMETSHIN 03B and BISELLO 91B assuming the $\omega\pi^0$ and $\pi^+\pi^-$ mass dependence of the total width. $\rho(1700)$ mass and width fixed at 1700 MeV and 240 MeV, respectively.

⁵ Using Breit-Wigner parameterization of the $\rho(1450)$ and assuming the $\omega\pi^-$ mass dependence for the total width.

 $^{^6}$ Mass-independent width parameterization. $\rho(1700)$ mass and width fixed at 1700 MeV and 235 MeV respectively.

⁷ Using data from BISELLO 91B, DOLINSKY 86 and ALBRECHT 87L.

⁸ Not separated from $b_1(1235)$, not pure $J^P = 1^-$ effect.

²Applies the Unitary & Analytic Model of the pion electromagnetic form factor of DUB-NICKA 10 to analyze the data of ACHASOV 06, AKHMETSHIN 07, AUBERT 09AS, and AMBROSINO 11A.

and AMBROSINO 11A.

Applies the Unitary & Analytic Model of the pion electromagnetic form factor of DUB-NICKA 10 to analyze the data of FUJIKAWA 08.

⁴ From a Dalitz plot analysis in an isobar model with $\rho(1450)$ and $\rho(1700)$ masses and widths floating.

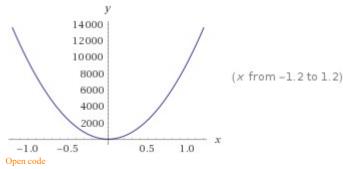
Now, we calculate the following integral:

integrate (((
$$1728*9+(728+1164.2696) + Pi * exp((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-54.192473)^2)) + ((2*(3+2)+2))/(sqrt(6)))))x$$

$$\int \left(1728 \times 9 + (728 + 1164.2696) + \frac{1}{6} \left(\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \, \pi^3} \right)^2 \pi^4 \left(-54.192473 \right)^2 + \frac{2 \, (3+2) + 2}{\sqrt{6}} \right) \right)$$

$$x \, dx = 9447.29 \, x^2 + \text{constant}$$

Plot of the integral:



The result 9447,29 is a good approximation to the rest mass of Upsilon meson, that is 9460.30±0.26

Note that 1164,2696 is the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$

$$\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3 = 1164,269601267364$$

Note that, we have also:

integrate 1/(744+6.582119)) (((1728*9+(728+1164.2696) + Pi * exp((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-54.192473)^2)) + ((2*(3+2)+2))/(sqrt(6)))))x

$$\int \frac{1}{744 + 6.582119} \left[1728 \times 9 + (728 + 1164.2696) + \frac{1}{744 + 6.582119} \left(\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \, \pi^3} \right)^2 \pi^4 \left(-54.192473 \right)^2} + \frac{2 \, (3 + 2) + 2}{\sqrt{6}} \right) \right]$$

$$x \, dx = 12.5866 \, x^2 + \text{constant}$$

The result 12,5866 is practically equal to the value of black hole entropy 12,57

Now we calculate the exp for l = 0 and Q = -3,29867

$$((sqrt(640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2)) + ((2*(3+2)+2))/(sqrt(6))$$

 $5- \ exp((((sqrt(640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2)) + ((2*(3+2)+2))/(sqrt(6))) + ((2*(3+2)+2))/(sqrt(6)) + ((2*(3+2)+2)/(sqrt(6)) + ((2*(3+2)+2)/$

$$5 - \exp\left(\sqrt{\frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2 \, (3+2) + 2}{\sqrt{6}}\right)$$

Result:

-139.0241...

Series representations

$$\begin{aligned} & 5 - \exp\!\left(\!\sqrt{\frac{1}{3} \left(\frac{1}{216\,\pi^3}\right)^{\!2} 640\,\pi^4 \left(-3.29867\right)^2} \right. \\ & + \frac{2\,(3+2)+2}{\sqrt{6}} \right) = \\ & 5 - \exp\!\left(\!\frac{12}{\sqrt{z_0}\,\sum_{k=0}^\infty \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(6-z_0\right)^k z_0^{-k}}{k!}} + \sqrt{z_0}\,\sum_{k=0}^\infty \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.0497541}{\pi^2} - z_0\right)^k z_0^{-k}}{k!} \right) \\ & \text{for not } \left(\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right) \end{aligned}$$

$$\begin{aligned} & 5 - \exp\!\left(\!\sqrt{\frac{1}{3} \left(\frac{1}{216\,\pi^3}\right)^2 640\,\pi^4 \, (-3.29867)^2} \, + \frac{2\,(3+2)+2}{\sqrt{6}}\right) = 5 \, - \\ & \exp\!\left(\!\frac{12}{\exp\!\left(\!i\,\pi \left\lfloor \frac{\operatorname{arg}(6-x)}{2\,\pi}\right\rfloor\!\right) \sqrt{x} \, \sum_{k=0}^\infty \frac{(-1)^k \, (6-x)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} + \exp\!\left(\!i\,\pi \left\lfloor \frac{\operatorname{arg}\!\left(\frac{0.0497541}{\pi^2} - x\right)}{2\,\pi}\right\rfloor\!\right) \right) \\ & \sqrt{x} \, \sum_{k=0}^\infty \frac{(-1)^k \, \left(\frac{0.0497541}{\pi^2} - x\right)^k \, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \, \right] \text{ for } (x \in \mathbb{R} \text{ and } x < 0) \end{aligned}$$

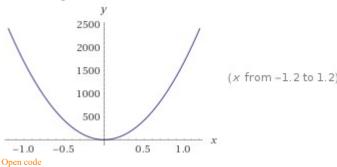
$$\begin{split} & 5 - \exp\left(\sqrt{\frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \, 640 \, \pi^4 \, (-3.29867)^2 \, + \frac{2 \, (3+2) + 2}{\sqrt{6}}}\right) = 5 \, - \\ & \exp\left(\frac{12 \left(\frac{1}{z_0}\right)^{-1/2 \, \lfloor \arg(6-z_0)/(2\,\pi)\rfloor} \, z_0^{-1/2 - 1/2 \, \lfloor \arg(6-z_0)/(2\,\pi)\rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6-z_0)^k \, z_0^{-k}}{k!}} + \left(\frac{1}{z_0}\right)^{1/2 \, \left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor} \right) \\ & \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6-z_0)^k \, z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \left(\frac{-1}{2}\right)^k \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!} \right) \\ & 5 - \exp\left(\sqrt{\frac{1}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \, 640 \, \pi^4 \, (-3.29867)^2 \, + \frac{2 \, (3+2) + 2}{\sqrt{6}}}{\sum_{k=0}^{\infty} \left(\frac{-1}{2}\right)^k \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right) \\ & 5 - \exp\left(\frac{12 \left(\frac{1}{z_0}\right)^{-1/2 \, \lfloor \arg(6-z_0)/(2\,\pi)\rfloor} \, z_0^{1/2 \, (-1-\lfloor \arg(6-z_0)/(2\,\pi)\rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6-z_0)^k \, z_0^{-k}}{k!}} + \left(\frac{1}{z_0}\right)^{1/2 \, \left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor} \right) \\ & \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, (6-z_0)^k \, z_0^{-k}}{k!} \\ & \frac{1/2 \left(1+\left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right) \\ & \frac{1/2 \left(1+\left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right) \\ & \frac{1/2 \left(1+\left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right)} \\ & \frac{1/2 \left(1+\left\lfloor \arg\left(\frac{0.0497541}{\pi^2} - z_0\right) \middle/ (2\,\pi)\right\rfloor}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right)}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}} \right)}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \, \left(\frac{0.0497541}{\pi^2} - z_0\right)^k \, z_0^{-k}}{k!}}$$

The result -139,0241 is practically equal, but with sign minus, to the rest mass of Pion meson, that is 139.57018 ± 0.00035

integrate (((1728+1164.2696+Pi*exp((sqrt(640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2))+((2*(3+2)+2))/(sqrt(6)))))x

$$\int \left[1728 + 1164.2696 + \pi \exp\left(\sqrt{\frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2 \, (3+2) + 2}{\sqrt{6}} \right) \right] x \, dx = 1672.37 \, x^2 + \text{constant}$$

Plot of the integral:



This value 1672,37 is practically equal to the rest mass of Omega baryon 1672.45±0.29

For $l = 1,616 * 10^{-35}$ and Q = -3,29867 and the previously expression

$$((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2)) + ((2*(3+2)+2))/(sqrt(6))$$

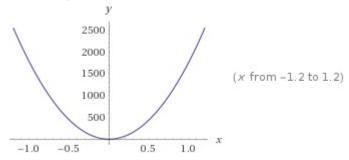
we calculate the exp together to the following integral:

integrate (((1728+1164.2696+Pi*exp((sqrt(1/6+640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2))+((2*(3+2)+2))/(sqrt(6)))))x

$$\int \left(1728 + 1164.2696 + \pi \exp\left(\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \,\pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2 \,(3+2) + 2}{\sqrt{6}} \right) \right) x$$

$$dx = 1765.05 \,x^2 + \text{constant}$$

Plot of the integral:



We note that the result 1765,05 is a good approximation to the mass of strange meson $K_2(1770)$. Indeed, as we can see from the next Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the analyzed expression.

K2(1770) MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
1773 ± 8 OUR /	AVERAGE					
$1777 \pm 35 + \frac{122}{77}$	4289	¹ AAIJ	17 c	LHCB		$B^+ \rightarrow J/\psi \phi K^+$
1773± 8		² ASTON	93	LASS		$11K^-p \rightarrow K^-\omega p$
• • • We do not u	se the follo	wing data for aver	ages,	fits, lim	its, etc	. • • •
1743±15		TIKHOMIROV	03	SPEC		$K_{S}^{0}K_{S}^{0}K_{I}^{0}X$
1810 ± 20		FRAME	86	OMEG	+	13 $K^+ p \rightarrow \phi K^+ p$
~ 1730		ARMSTRONG	83	OMEG	_	$18.5 K^- p \rightarrow 3Kp$
~ 1780		³ DAUM	81 C	CNTR	-	$63 K^- p \rightarrow K^- 2\pi p$
1710 ± 15	60	CHUNG	74	HBC	1_2	$7.3 K^- p \rightarrow K^- \omega p$
1767 ± 6		BLIEDEN	72	MMS		11-16 K ⁻ p
1730 ± 20	306	⁴ FIRESTONE	72B	DBC	+	12 K ⁺ d
1765 ± 40		⁵ COLLEY	71	HBC	+	$10 K^+ p \rightarrow K 2\pi N$
1740		DENEGRI	71	DBC	_	$12.6 K^- d \rightarrow \overline{K} 2\pi d$
1745 ± 20		AGUILAR	70 C	HBC	_	$4.6 K^{-}p$
1780 ± 15		BARTSCH	70 c	HBC	_	10.1 K ⁻ p
1760 ± 15		LUDLAM	70	HBC	_	12.6 K ⁻ p

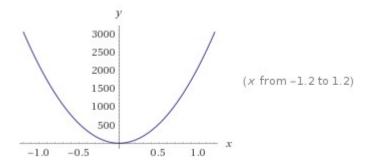
For $l = 1,616 * 10^{-35}$ and Q = 50,893800 and the previously expression ((sqrt $(1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(50.893800)^2)) + ((2*(3+2)+2))/(sqrt(6))$ we calculate the exp together to the following integral:

integrate (((1728+1164.2696-24+Pi*exp((sqrt(1/6+640/3*(1/(216Pi^3))^2*Pi^4*(50.893800)^2))+((2*(3+2)+2))/(sqrt(6)))))x

$$\int \left(1728 + 1164.2696 - 24 + \frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \pi^3} \right)^2 \pi^4 50.893800^2 + \frac{2(3+2)+2}{\sqrt{6}} \right) dx$$

$$x \, dx = 2112.45 \, x^2 + \text{constant}$$

Plot of the integral:



This value 2112,45 is practically equal to the rest mass of strange D meson 2112.3±0.5

Indeed, as we can see from the next Table, the value of mass is equal to the result of the analyzed expression.

D* MASS

The fit includes D^{\pm} , D^0 , D_s^{\pm} , $D^{*\pm}$, D^{*0} , and $D_s^{*\pm}$ mass and mass difference measurements.

Now, from the precedent integrals, we can to obtain also:

$$\int \left(1728 \times 9 + (728 + 1164.2696) + \pi \exp\left(\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \, \pi^3}\right)^2 \pi^4 \left(-54.192473\right)^2} + \frac{2 \, (3+2) + 2}{\sqrt{6}}\right)\right)^{0.048} dx = 1.60422 \, x + \text{constant}$$

integrate [(((1728*9+(728+1164.2696) + Pi * exp((sqrt (1/6 + 640/3*(1/(216Pi^3))^2*Pi^4*(50.893800)^2)) + ((2*(3+2)+2))/(sqrt(6))))]^0.048

$$\int \left(1728 \times 9 + (728 + 1164.2696) + \frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \pi^3}\right)^2 \pi^4 50.893800^2 + \frac{2(3+2)+2}{\sqrt{6}}\right)^{0.048}$$

$$dx = 1.60384 x + \text{constant}$$

Results that practically are equals to the value of the electric charge of the positron.

Also:

integrate $(((1728+1164.2696+Pi*exp((sqrt(640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2))+((2*(3+2)+2))/(sqrt(6)))))^(1/17)$

$$\int_{17}^{17} 1728 + 1164.2696 + \pi \exp\left(\sqrt{\frac{640}{3} \left(\frac{1}{216 \pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2(3+2)+2}{\sqrt{6}}\right)$$

$$dx = 1.61182 x + \text{constant}$$

and

integrate (((1728+1164.2696+Pi*exp((sqrt(1/6+640/3*(1/(216Pi^3))^2*Pi^4*(-3.29867)^2))+((2*(3+2)+2))/(sqrt(6)))))^(1/17)

$$\int_{17}^{17} 1728 + 1164.2696 + \pi \exp\left(\sqrt{\frac{1}{6} + \frac{640}{3} \left(\frac{1}{216 \pi^3}\right)^2 \pi^4 \left(-3.29867\right)^2} + \frac{2(3+2)+2}{\sqrt{6}}\right)$$

$$dx = 1.61694 x + \text{constant}$$

are results that practically are very near to the value of the electric charge of the positron.

Now, we have:

$$D_{M}D_{M}\Phi_{N} - D_{N}D_{M}\Phi_{M} + 2\mathcal{F}_{NM}\Phi_{M} + \mathcal{O}(\lambda^{-1}) = 0.$$

$$D_{M}(D_{0}\Phi_{M} - D_{M}\Phi_{0}) - \mathcal{F}^{0M}\Phi_{M} - \frac{1}{64\pi^{2}a}\epsilon_{MNPQ}K_{MNPQ} + \mathcal{O}(\lambda^{-1}) = 0, \qquad (2.8)$$

where $x^{M} = \{x^{i}, Z\}$, i = 1, 2, 3 and the 4-form K_{MNPQ} is given as,

$$K_{MNPQ} = \partial_{M} \mathcal{A}_{N} \partial_{P} \Phi_{Q} + \mathcal{A}_{M} \mathcal{A}_{N} \partial_{P} \Phi_{Q} + \partial_{M} \mathcal{A}_{N} \mathcal{A}_{P} \Phi_{Q} + \frac{5}{6} \mathcal{A}_{M} \mathcal{A}_{N} \mathcal{A}_{P} \Phi_{Q}. \tag{2.9}$$

Since the holographic approach is valid in the strongly coupling limit $\lambda \to \infty$, the contributions from $\mathcal{O}\left(\lambda^{-1}\right)$ have been dropped off. Note that the light flavoured gauge field \mathcal{A}_a satisfies the equations of motion obtained by varying the action (C-1), so their solution remains to be (C-2) in the large λ limit. And we could further define $\Phi_a = \phi_a e^{\pm i m_H x^0}$ in the heavy quark limit i.e. $m_H \to \infty$ as in [25, 26] so that $D_0 \Phi_M = (D_0 + i m_H) \phi_M$ where "+" corresponds to quark and anti-quark respectively. By keeping these in mind, altogether we find the full solution for (2.8) as,

$$\phi_0 - \frac{1}{1024a\pi^2} \left[\frac{25\rho}{2(x^2 + \rho^2)^{5/2}} + \frac{7}{\rho(x^2 + \rho^2)^{3/2}} \right] \chi,$$

$$\phi_M = \frac{\rho}{(x^2 + \rho^2)^{3/2}} \sigma_M \chi,$$
(2.10)

where χ is a spinor independent on x^M . Then in the double limit i.e. $\lambda \to \infty$ followed by $m_H \to \infty$, the Hamiltonian for the collective modes involving the heavy flavour could be calculated as in (C-7) by following the procedures in Appendix C.

We have that:

Input interpretation:

$$-\frac{216\,\pi^3}{1024\,\pi^2}\times\frac{25\left(-6.5677261\times10^{16}\right)}{2\left(1+\left(-6.5677261\times10^{16}\right)^2\right)^{2.5}}$$

Result

$$4.45198... \times 10^{-67}$$

Input interpretation:

$$-\frac{216 \pi^3}{1024 \pi^2} \left(-\frac{7}{6.5677261 \times 10^{16} \left(1 + \left(-6.5677261 \times 10^{16}\right)^2\right)^{1.5}}\right)$$

Result:

$$2.49311... \times 10^{-67}$$

$$(4.45198*10^{\circ}-67) + (2.49311*10^{\circ}-67)$$

Input interpretation:

$$4.45198 \times 10^{-67} + 2.49311 \times 10^{-67}$$

$$6.94509 \times 10^{-67}$$

And

$$(((-6.5677261*10^16)*585))/(1+(6.5677261*10^16)^2))^1.5))))\\$$

$$\frac{-6.5677261 \times 10^{16} \times 585}{\left(1 + \left(6.5677261 \times 10^{16}\right)^2\right)^{1.5}}$$

Result:

$$-1.35621... \times 10^{-31}$$

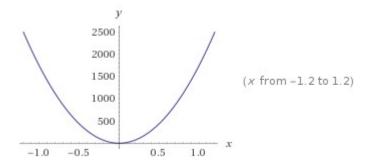
We calculate the following integrals:

integrate
$$1/(((1.65578)^1.08643^(2Pi))) * (1164.2696*10^67) * [(4.45198*10^-67)+(2.49311*10^-67)]x$$

Indefinite integral:

$$\int \frac{\left(1164.2696 \times 10^{67}\right) \left(4.45198 \times 10^{-67} + 2.49311 \times 10^{-67}\right) x}{1.65578^{1.08643^{2}\pi}} \ dx = 1729.88 \ x^{2} + \text{constant}$$

Plot of the integral:



We have that:

1/(142) integrate 1/(((1.65578)^1.08643^(2Pi))) * (1164.2696* 10^67) * [(4.45198*10^-67)+(2.49311*10^-67)]x

$$\frac{1}{142} \int \frac{1}{1.65578^{1.08643^{2}\pi}} \left(1164.2696 \times 10^{67}\right) \left(4.45198 \times 10^{-67} + 2.49311 \times 10^{-67}\right) x \, dx$$

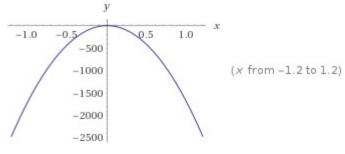
Result: 12.1823 x²

The result 12,182 is practically equal to the value of black hole entropy 12,19

integrate $(\sqrt{1.65578})*1.08643^{(2Pi)})*1164.2696*10^{31}((-6.5677261*10^{16})*585))/(1+(6.5677261*10^{16})^{2}))^{1.5})))x$

$$\int \frac{\left(\sqrt{1.65578} \ 1.08643^{2\,\pi}\right)1164.2696\times10^{31}\left(\left(-6.5677261\times10^{16}\,585\right)x\right)}{\left(1+\left(6.5677261\times10^{16}\right)^{2}\right)^{1.5}}\,dx=\\ -1710.24\,x^{2}+\text{constant}$$

Plot of the integral:



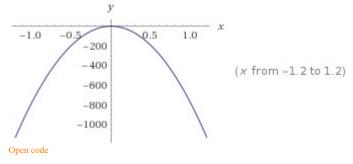
Now:

$$\frac{6.94509}{10^{67}} - \frac{1.35621}{10^{31}}$$

We calculate the following integral of the algebraic sum of the two results: integrate $(1164.2696 - 9)*10^31 [(6.94509/10^67) - (1.35621/10^31)]x$

$$\int (1164.2696 - 9) \, 10^{31} \left(\left(\frac{6.94509}{10^{67}} - \frac{1.35621}{10^{31}} \right) x \right) dx = -783.394 \, x^2 + \text{constant}$$

Plot of the integral:



This result -783.394 is practically equal, with sign minus, to the rest mass of Omega meson 782.65 ± 0.12 . Indeed

ω (782) MASS

EVTS	DOCUMENT ID		TECN	COMMENT
VERAGE	Error includes scale	factor	of 1.9.	See the ideogram below.
18680	AKHMETSHIN	05	CMD2	0.60 -1.38 $e^+e^- \rightarrow \pi^0 \gamma$
11200	¹ AKHMETSHIN	04	CMD2	$e^{+}e^{-'} \rightarrow \pi^{+}\pi^{-}\pi^{0}$
1.2M	² ACHASOV	03D		$0.44-2.00 e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\pi^{0}$
19500	WURZINGER	95	SPEC	1.33 $pd \rightarrow {}^{3}\text{He}\omega$
11k	³ AMSLER	94C	CBAR	$0.0 \ \overline{p}p \rightarrow \omega \eta \pi^0$
3463	⁴ AMSLER	94C	CBAR	$0.0 \ \overline{p}p \rightarrow \omega \eta \pi^0$
15k	AMSLER	93B	CBAR	$0.0 \ \overline{p}p \rightarrow \omega \pi^0 \pi^0$
270k	WEIDENAUER	93	ASTE	$\overline{p}p \rightarrow 2\pi^{+}2\pi^{-}\pi^{0}$
1488	KURDADZE	83B	OLYA	$e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\pi^{0}$
7000	⁵ KEYNE	76	CNTR	$\pi^- p \rightarrow \omega n$
the follow	ing data for averages	fits,	limits, e	tc. • • •
	⁶ BARKOV	87	CMD	$e^+e^- \rightarrow \pi^+\pi^-\pi^0$
433	CORDIER	80	DM1	$e^+e^- \rightarrow \pi^+\pi^-\pi^0$
33260	ROOS	80	RVUE	0.0-3.6 pp
3000	BENKHEIRI	79	OMEG	9-12 $\pi^{\pm} p$
1430	COOPER	78B	HBC	$0.7-0.8 \ \overline{p}p \rightarrow 5\pi$
535	VANAPEL	78	HBC	$7.2 \overline{p}p \rightarrow \overline{p}p\omega$
2100	GESSAROLI	77	HBC	$11 \pi^- p \rightarrow \omega n$
418	AGUILAR	72B	HBC	3.9,4.6 K ⁻ p
248	BIZZARRI	71	HBC	$0.0 p\overline{p} \rightarrow K^+K^-\omega$
510	BIZZARRI	71	HBC	$0.0 \ p\overline{p} \rightarrow K_1 K_1 \omega$
3583	7 COYNE	71	HBC	$3.7 \pi^+ p \rightarrow$
				$p_{\pi}^{+} + \pi^{+} + \pi^{-} \pi^{0}$
750	ABRAMOVI	70	HBC	3.9 $\pi^{-}p$
	⁸ BIGGS	70B	CNTR	$<$ 4.1 γ C \rightarrow $\pi^+\pi^-$ C
2400	BIZZARRI	69	HBC	0.0 p p
	VERAGE 18680 11200 1.2M 19500 11k 3463 15k 270k 1488 7000 the follow 433 33260 3000 1430 535 2100 418 248 510 3583 750	VERAGE Error includes scale 18680 AKHMETSHIN 11200 1 AKHMETSHIN 1.2M 2 ACHASOV 19500 WURZINGER 11k 3 AMSLER 3463 4 AMSLER 15k AMSLER 270k WEIDENAUER 1488 KURDADZE 7000 5 KEYNE the following data for averages 6 BARKOV 433 CORDIER 33260 ROOS 3000 BENKHEIRI 1430 COOPER 535 VANAPEL 2100 GESSAROLI 418 AGUILAR 248 BIZZARRI 510 BIZZARRI 510 BIZZARRI 3583 7 COYNE 750 ABRAMOVI 8 BIGGS	## Page 18	VERAGE Error includes scale factor of 1.9. 18680 AKHMETSHIN 05 CMD2 11200 1 AKHMETSHIN 04 CMD2 1.2M 2 ACHASOV 03D RVUE 19500 WURZINGER 95 SPEC 11k 3 AMSLER 94C CBAR 3463 4 AMSLER 94C CBAR 15k AMSLER 93B CBAR 270k WEIDENAUER 93 ASTE 1488 KURDADZE 83B OLYA 7000 5 KEYNE 76 CNTR the following data for averages, fits, limits, etc. 6 BARKOV 87 CMD 433 CORDIER 80 DM1 33260 ROUS 80 RVUE 3000 BENKHEIRI 79 OMEG 1430 COOPER 78B HBC 535 VANAPEL 78 HBC 248 BIZZARRI 71 HBC 248 BIZZARRI 71 HBC 1 HBC 1

Furthermore, from the above integral we have also:

(26+1)/(1728) integrate $(1164.2696 - 9)* 10^31 [(6.94509/10^67) - (1.35621/10^31)]x$

$$\frac{26+1}{1728} \int (1164.2696-9) \times 10^{31} \left(\left(\frac{6.94509}{10^{67}} - \frac{1.35621}{10^{31}} \right) x \right) dx$$

Result:
$$-12.2405 x^2$$

The result -12,24 is very near to the value of black hole entropy 12,19 with sign minus

Now, we have:

For the dilatonic scalar glueball, the following formula:

$$\begin{split} \frac{\mathcal{L}_{\Psi}^{D}}{a\mathcal{C}_{D}} = & v^{2} \frac{(N_{f}+1)^{2}}{N_{f}^{2}} \bigg[-\frac{\partial^{i}\partial^{j}G_{D}}{3M_{D}^{2}M_{KK}} \Phi_{i}^{\dagger}\Phi_{j} + \frac{2G_{D}}{3M_{KK}} \eta^{ij}\Phi_{i}^{\dagger}\Phi_{j} \\ & + \frac{\partial^{2}G_{D}}{6M_{D}^{2}M_{KK}} \eta^{ij}\Phi_{i}^{\dagger}\Phi_{j} + \frac{G_{D}}{3M_{KK}} \Phi_{Z}^{\dagger}\Phi_{Z} + \frac{\partial^{2}G_{D}}{6M_{D}^{2}M_{KK}} \Phi_{Z}^{\dagger}\Phi_{Z} \bigg]. \end{split}$$

(1.2371318784*10^63)[-(0.249996/185.1395)-(0.999992i/(24Pi))+(0.249996/(48Pi*2.455489))-(0.499996i/(24Pi))+(0.249996/(48Pi*2.455489)]

$$\begin{aligned} 1.2371318784 \times 10^{63} \left(-\frac{0.249996}{185.1395} - 0.999992 \times \frac{i}{24\,\pi} + \right. \\ \left. \frac{0.249996}{48\,\pi \times 2.455489} - 0.499996 \times \frac{i}{24\,\pi} + \frac{0.249996}{48\,\pi \times 2.455489} \right) \end{aligned}$$

Result:

Polar coordinates:

$$r = 2.46118 \times 10^{61}$$
 (radius), $\theta = -90.^{\circ}$ (angle)

And:

 $[2.46118*10^{6}1]*((1/(216*Pi^{3}))*29.772$

$$2.46118 \times 10^{61} \left(\frac{1}{216 \, \pi^3} \times 29.772 \right)$$

Result:

$$1.09408... \times 10^{59}$$

Comparison

$$\approx 1.4 \times 10^5$$
 ×the size of the Monster group ($\approx 8.1 \times 10^{53}$)

Now, we have the following integral:

 $(1164.2696+1729-144)*(1/(10^59))$ integrate $[(2.46118*10^61)*((1/(216*Pi^3))*29.772]x$

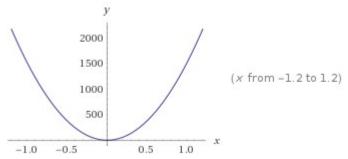
Input interpretation:

$$(1164.2696 + 1729 - 144) \times \frac{1}{10^{59}} \int 2.46118 \times 10^{61} \left(\frac{1}{216 \pi^3} \times 29.772 \, x \right) dx$$

Result:

 $1503.96 x^2$

Plot:



Alternate form assuming x is real:

$$1503.96 x^2 + 0$$

Indefinite integral assuming all variables are real:

$$501.319 x^3 + constant$$

The result 1503.96 is practically equal to the following value of meson $f_0(1500)$ mass:

f₀(1500) MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1504 ± 6 OUR A	VERAGE	Error includes se	cale fac	ctor of 1	.3. See the ideogram below.
$1468 + 14 + 23 \\ -15 - 74$	5.5k	¹ ABLIKIM	1 3N	BES3	$e^+e^- \rightarrow J/\psi \rightarrow \gamma \eta \eta$
1466± 6± 20		ABLIKIM	06V	BES2	$e^+e^- \rightarrow J/\psi \rightarrow \gamma \pi^+\pi^-$
1515±12		² BARBERIS	00A		450 $pp \rightarrow p_f \eta \eta p_s$
1511± 9		2,3 BARBERIS	00c		$450 pp \rightarrow p_f 4\pi p_S$
1510± 8		² BARBERIS	00E		450 $pp \rightarrow p_f \eta \eta p_s$
1522±25		BERTIN	98	OBLX	$0.05-0.405 \ \overline{n}p \rightarrow \pi^{+}\pi^{+}\pi^{-}$
1449 ± 20		² BERTIN	97c	OBLX	$0.0 \overline{p}p \rightarrow \pi^{+}\pi^{-}\pi^{0}$
1515 ± 20		ABELE	96B	CBAR	$0.0 \overline{p}p \rightarrow \pi^0 K_I^0 K_I^0$
1500 ± 15		4 AMSLER			$0.0 \overline{p}p \rightarrow 3\pi^0$
1505 ± 15		⁵ AMSLER	95c	CBAR	$0.0 \overline{p}p \rightarrow \eta \eta \pi^0$

Indeed, we have also that, from:

Production of f0(1710), f0(1500), and f0(1370) in J/ψ hadronic decays

Frank E. Close and Qiang Zhao

The importance of glueball- $Q\bar{Q}$ mixing is also highlighted by the indispensible contributions from the doubly disconnected processes, which turn out to be nonperturbative and violate the OZI rule. Since the coupling $gg \to QQ$ in the doubly disconnected processes is essentially the same as the glueball- $Q\bar{Q}$ mixing, the nonperturbative feature of the doubly disconnected processes is self-consistent with the proposed configuration mixing scheme for these three f_0 states. In this sense, our results not only provide an understanding of the recent "puzzling" experimental data from BES [13, 14, 15], but also highlight the strong possibility of the existence of glueball contents in the $f_0(1500)$, and its sizeable interferences in $f_0(1710)$. Furthermore, due to the configuration mixing, the $|nn\rangle$ dominant $f_0(1370)$ tends to have a lower mass lower than 1370 MeV, which also agrees with a recent more refined analysis [13, 22].

With Eqs. (9) and (10), and applying the method of Ref. [6], the the relative decay widths (excluding phase space) for $f_0^i \rightarrow \gamma \gamma$ are found to be $f_0(1370): f_0(1500): f_0(1710) \sim 12: 2: 1$. The results are consistent with those of Ref. [6], which should not be surprizing since the mixing matrices are similar to each other.

In this factorization scheme, a quantitative normalization of the scalar glueball production rate in the $J/\psi \to VG$ is also accessible. With the pure glueball mass in a range of 1.46 - 1.52 GeV, we obtain the branching ratios $br_{J/\psi \to \phi G} \simeq \frac{1}{2}br_{J/\psi \to \omega G} \simeq (1 \sim 2) \times 10^{-4}$. Although a direct measurement of the glueball production seems

In this sense, our results not only provide an understanding of the recent "puzzling" experimental data from BES [13, 14, 15], but also highlight the strong possibility of the existence of glueball contents in the f0(1500), and its sizeable interferences in f0(1710). Furthermore, due to the configuration mixing, the |n ni dominant f0(1370) tends to have a lower mass lower than 1370 MeV, which also agrees with a recent more refined analysis.

From the above integral, we have also that:

$$1/(192-64-e^*1.65578)$$
 (1164.2696+1729-144) * (1/(10^59)) integrate [(2.46118*10^61)*((1/(216*Pi^3))*29.772]x

Input interpretation:

$$\frac{1}{192-64+e\times(-1.65578)}\left((1164.2696+1729-144)\times\frac{1}{10^{59}}\right)\\ \int 2.46118\times10^{61}\left(\frac{1}{216\,\pi^3}\times29.772\,x\right)dx$$

Result:

$$12.1779 x^2$$

Furthermore:

Input interpretation:

$$\sqrt{\log(1729 \times 2.46118 \times 10^{61})}$$

Result:

12.198919...

And

sqrt(ln[64Pi*1729*(1.09408*10^59))

Input interpretation:
$$\sqrt{log(64 \pi \times 1729 \times 1.09408 \times 10^{59})}$$

Result:

12.194316...

All the results 12,1779 12,1989 and 12,1943 are very near to the value of black hole entropy 12,19

From the following formula of the the exotic scalar glueball:

$$\begin{split} \mathcal{L}_{\Psi}^{E} &= -\,v^2 \frac{(N_f+1)^2}{N_f^2} \bigg[-\frac{5}{12 M_E^2 M_{KK}} \partial^i \partial^j G_E \Phi_i^\dagger \Phi_j + \frac{5}{24 M_E^2 M_{KK}} \partial^2 G_E \delta^{ij} \Phi_i^\dagger \Phi_j \\ &- \frac{5}{12 M_{KK}} G_E \Phi_Z^\dagger \Phi_Z + \frac{5}{24 M_E^2 M_{KK}} \partial^2 G_E \Phi_Z^\dagger \Phi_Z \bigg]. \end{split}$$

(1.2371318784*10^63)[-5*-0.153644/(12*0.901^2*(-8Pi))+(5*(-0.153644)/(24*0.901^2*(-8Pi))-(5*0.391974i)/(12*(-8Pi))+(5*-0.153644/(24*0.901^2*(-8Pi)]

$$1.2371318784 \times 10^{63} \left(-5 \left(-\frac{0.153644}{12 \times 0.901^2 (-8 \pi)}\right) + \left(5 \left(-\frac{0.153644}{24 \times 0.901^2 (-8 \pi)}\right) - \frac{5 \times 0.391974 i}{12 (-8 \pi)} + 5 \left(-\frac{0.153644}{24 \times 0.901^2 (-8 \pi)}\right)\right)\right)$$

Result:

$$8.03937... \times 10^{60} i$$

Polar coordinates:

$$r = 8.03937 \times 10^{60}$$
 (radius), $\theta = 90^{\circ}$ (angle)

We calculate the following integral:

 $sqrt[(((sqrt(5)+1)/2)))^2 + sqrt(5)/2]*(1729*1164.2696+729*4)/1.65578)*(1/(10^63)) integrate [8.03937*10^60]x$

Input interpretation

$$\sqrt{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\frac{\sqrt{5}}{2}}\times\frac{1729\times1164.2696+729\times4}{1.65578}\times\frac{1}{10^{63}}\int8.03937\times10^{60}\,x\,dx$$

Result:

 $9459.63 x^2$

or:

 $(1/(10^63))$

sqrt[(((sqrt(5)+1)/2)))^2+sqrt(5)/2]*(1729*1164.2696+(1729+729))*1/1.65578) integrate [8.03937*10^60]x

$$\frac{1}{10^{63}} \left(\sqrt{\left(\frac{1}{2} \left(\sqrt{5} + 1\right)\right)^2 + \frac{\sqrt{5}}{2}} \right. (1729 \times 1164.2696 + (1729 + 729)) \times \frac{1}{1.65578} \right) \\ \int 8.03937 \times 10^{60} \ x \ dx$$

Result:

 $9457.48 x^2$

The two results 9459.63 and 9457.48 are practically equals to the rest mass of Upsilon meson 9460.30±0.26

Now:

Pi+ln [[[sqrt[(((sqrt(5)+1)/2)))^2+sqrt(5)/2]*(1729*1164.2696+729*4)/1.65578) * (1/(10^63)) integrate [8.03937*10^60]x]]]

Input interpretation:
$$\pi + \log \left(\sqrt{\left(\frac{1}{2} \left(\sqrt{5} + 1\right)\right)^2 + \frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696 + 729 \times 4}{1.65578} \times \frac{1}{10^{63}} \right)$$

$$\int 8.03937 \times 10^{60} \ x \ dx$$

Result:

$$\log(9459.63 \, x^2) + \pi$$

Input interpretation:

 $\pi + \log(9459.63)$

Result:

12.29638...

$$\pi + \log \left(\frac{1}{10^{63}} \left(\sqrt{\left(\frac{1}{2} \left(\sqrt{5} + 1\right)\right)^2 + \frac{\sqrt{5}}{2}} \right) (1729 \times 1164.2696 + (1729 + 729)) \times \frac{1}{1.65578} \right)$$

$$\int 8.03937 \times 10^{60} \ x \ dx$$

Result:

$$\log(9457.48 \, x^2) + \pi$$

Input interpretation:

 $\pi + \log(9457.48)$

Result:

12.29615...

The results 12.29638 and 12.29615 are very near to the value of black hole entropy 12.19

We have also that:

Input interpretation:

$$\sqrt{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^2+\frac{\sqrt{5}}{2}}\times\frac{1729\times1164.2696+729}{288+\pi}\times\frac{1}{10^{61}}\int8.03937\times10^{60}~x~dx$$

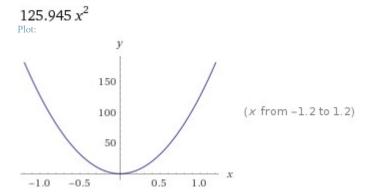
This result 5374.04 is very near to the rest mass of Strange B meson 5366.3±0.6 We have also that:

 $1/75 * (sqrt(2.618+sqrt(5)/2)*1729*1164.2696)/1.65578) * (1/(10^63)) integrate [8.03937*10^60]x$

Input interpretation:

$$\frac{1}{75} \times \frac{\sqrt{2.618 + \frac{\sqrt{5}}{2}} \times 1729 \times 1164.2696}{1.65578} \times \frac{1}{10^{63}} \int 8.03937 \times 10^{60} \ x \, dx$$

Result:



The result 125.945 is very near to the value of the mass of Higgs boson that is 125.09 ± 0.24

And

 $sqrt(13) + ln[sqrt[(((sqrt(5)+1)/2)))^2 + sqrt(5)/2]*(((1729*1164.2696)+729))/(288+Pi) *(1/(10^61)) integrate [8.03937*10^60]x]$

Input interpretation:
$$\sqrt{13} + \log \left(\sqrt{\left(\frac{1}{2} \left(\sqrt{5} + 1\right)\right)^2 + \frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696 + 729}{288 + \pi} \times \frac{1}{10^{61}} \int 8.03937 \times 10^{60} \ x \ dx \right)$$

Result:

$$\log(5374.04 \, x^2) + \sqrt{13}$$

Input interpretation:

$$\sqrt{13} + \log(5374.04)$$

Result:

12.19489...

The result 12.19489 is practically equal to the value of the black hole entropy 12.19 From:

Monstrous Moonshine and the Entropy of the Smallest Black Hole

Last Update: 14th September 2008

The reason for the j-function being of interest is too lengthy to discuss, so suffice it to say that it has played a role in mathematics since Gauss and features strongly in number theory. It also features in the theory of elliptic curves, which provides an alternative, purely algebraic, definition. However, what we are interested in is the Laurent expansion of j in powers of q,

$$j(\tau) = \frac{1}{q} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots,$$

Finally we see the moonshine. The coefficient of 'q' is none other than the dimension of the first non-trivial representation of the Monster group (plus 1). Coincidence? It could have been, but for the fact that the subsequent coefficients of the j-function also have a simple numerical relationship with the dimensions of the Monster group's representations, as follows,

$$196,884 = 1 + 196,883$$

$$21,493,760 = 1 + 196,883 + 21,296,876$$

$$864,299,970 = 1 + 1 + 196,883 + 196,883 + 21,296,876 + 842,609,326$$

Relationships along these lines have been proved to continue for all the expansion coefficients and dimensions. Moreover, this is not all. It turns out that there are further numerical coincidences connecting the j-function and the Monster group.

The conformal theory considered has the interesting property that the cosmological constant is quantised by an integer k = 1, 2, 3... The total vacuum energy of the spacetime is also quantised by this integer. The magnitude of the cosmological constant in our universe is notoriously tiny when expressed in Planck units, of order 10^{-123} . In Witten's 3d spacetime it is $-1/(16k)^2$, and hence, as well as being of different sign, is comparatively enormous in magnitude for modest vales of k (i.e. $\sim 10^{-3}$).

However, for any given k, there is a minimum size of black hole which can exist in this spacetime. What Witten does is to find the number of quantum states of a black hole of minimum size, and how it depends upon k. He does this by arguing that the partition function of the theory should differ from that of the corresponding classical theory only by linear terms, and that it should also be expressible as a power series in the j-function. This leads to a set of functions,

$$Z_1(q) = j(q) = q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots$$

$$Z_2(q) = j(q)^2 - 393767 = q^{-2} + 1 + 42987520q + 40491909396q2 + \dots$$

$$Z_3(q) = j(q)^3 - 590651j(q) - 64481279$$

= $q^{-3} + q^{-1} + 1 + 2593096794q + 12756091394048q^2 + \dots$

$$Z_4(q) = j(q)^4 - 787535j(q)^2 - 8597555039j(q) - 644481279$$

= $q^{-4} + q^{-2} + q^{-1} + 2 + 81026609428q + 1604671292452452276q^2 + \dots$

(where j has been redefined for convenience by omitting the constant term, 744).

The coefficient of q in each of the functions Z_k is the number of quantum states of the minimal black hole for that value of k (to an accuracy of within one or two states, at least). Thus, for k = 1, the entropy of the minimal black hole is $\ln(196884) = 12.190$, whereas for k = 4 the entropy is $\ln(81026609428) = 25.118$.

Now the point here is that the entropy of a black hole is also known from semi-classical arguments (i.e. neglecting the quantisation of gravity) by a formula known as the Bekenstein-Hawking formula, which in this case becomes $S = 4\pi\sqrt{k}$. So for k = 1 we expect the result $4\pi = 12.566$, which compares with Witten's 12.190, and for k = 4 we expect $8\pi = 25.133$ which compares with Wittens' 25.118. The comparison for the first 4 values of k is,

k	Bekenstein- Hawking	Witten	Difference (%)
1	12.566	12.190	3.0%
2	17.772	17.576	1.1%
3	21.766	21.676	0.4%
4	25.133	25.118	0.06%

We note that the results 12.194316 and 12.198919 are very near to the value obtained from Witten for k = 1, for the entropy of a black hole, considering the ln (196884) that is 12.190

We note also that:

$$728 / 7 = 104$$
; $ln(1729*104) = 12.09968$

$$ln(729/6*1729) = 12.255$$

all results that are very near to the value of black hole entropy 12,19

Note that from the sum of the Ramanujan's numbers

 $(14258^3 + 1 + 1010^3 - 1 + 172^3 - 1 + 12^3 + 1 + 9^3 - 1)$ we calculate the following expressions:

$$ln(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - ((sqrt(5)+5)/2))$$

$$\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2}(\sqrt{5} + 5)$$

Exact result:

$$\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2\,897\,481\,469\,606)$$

Decimal approximation:

25.07682902808574525405759734333809832819946081084836122198...

Property:

$$\frac{1}{2}(-5-\sqrt{5}) + \log(2897481469606)$$
 is a transcendental number

Alternate forms:

$$-\frac{5}{2} - \frac{\sqrt{5}}{2} + \log(2897481469606)$$

$$\frac{1}{2} \left(-5 - \sqrt{5} + 2 \log(2897481469606) \right)$$

$$\frac{1}{2} \left(-5 - \sqrt{5} + 2 \log(2) + 2 \log(1448740734803) \right)$$

Continued fraction:

[25; 13, 62, 1, 5, 4, 1, 4, 1, 2, 1, 4, 4, 4, 1, 1, 2, 13, 5, 6, 2, 2, 1, 14, 2, 12, 1, 3, 3, ...]

Alternative representations

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) = \log_{e}(-1 - 9^{3} - 12^{3} - 172^{3} - 1010^{3} + 14258^{3}) + \frac{1}{2}(-5 - \sqrt{5})$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) =$$

$$-\text{Li}_{1}(2 + 9^{3} + 12^{3} + 172^{3} + 1010^{3} - 14258^{3}) + \frac{1}{2}(-5 - \sqrt{5})$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) = \log(a)\log_{a}(-1 - 9^{3} - 12^{3} - 172^{3} - 1010^{3} + 14258^{3}) + \frac{1}{2}(-5 - \sqrt{5})$$

Series representations:

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) = -\frac{5}{2} - \frac{\sqrt{5}}{2} + \log(2897481469605) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605}\right)^{k}}{k}$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) =$$

$$-\frac{5}{2} - \frac{\sqrt{5}}{2} + 2i\pi \left[\frac{\arg(2897481469606 - x)}{2\pi} \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^{k} (2897481469606 - x)^{k} x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2} \left(\sqrt{5} + 5\right) =$$

$$-\frac{5}{2} - \frac{\sqrt{5}}{2} + \left[\frac{\arg(2897481469606 - z_{0})}{2\pi}\right] \log\left(\frac{1}{z_{0}}\right) + \log(z_{0}) +$$

$$\left[\frac{\arg(2897481469606 - z_{0})}{2\pi}\right] \log(z_{0}) - \sum_{k=1}^{\infty} \frac{(-1)^{k} (2897481469606 - z_{0})^{k} z_{0}^{-k}}{k}$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) = -\frac{5}{2} - \frac{\sqrt{5}}{2} + \int_{1}^{2897481469606} \frac{1}{t} dt$$

$$\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2}(\sqrt{5} + 5) = -\frac{5}{2} - \frac{\sqrt{5}}{2} - \frac{i}{2\pi} \int_{-i + \gamma}^{i + \gamma} \frac{2897481469605^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$1/2 ((\ln(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - ((\operatorname{sqrt}(5) + 5)/2)))$$

$$\frac{1}{2} \left(log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right)$$

Exact result:
$$\frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$

12.53841451404287262702879867166904916409973040542418061099...

Property:
$$\frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$
is a transcendental number

$$\frac{1}{4} \left(-5 - \sqrt{5} + 2 \log(2897481469606) \right)$$

$$-\frac{5}{4}-\frac{\sqrt{5}}{4}+\frac{\log(2\,897\,481\,469\,606)}{2}$$

$$\frac{1}{4} \left(-5 - \sqrt{5}\right) + \frac{\log(2\,897\,481\,469\,606)}{2}$$

Continued fraction:

$$[12; 1, 1, 6, 125, 1, 2, 9, 1, 1, 1, 6, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 26, 2, 1, 1, 2, 1, 2, \dots]$$

Alternative representations:

$$\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{1}{2} \left(\log_e \left(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

$$\begin{split} &\frac{1}{2} \left(log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} \right. + 5 \right) \right) = \\ &\frac{1}{2} \left(- \text{Li}_1 \left(2 + 9^3 + 12^3 + 172^3 + 1010^3 - 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right) \end{split}$$

$$\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{1}{2} \left(\log \left(a \right) \log_a \left(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

Series representations

$$\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = -\frac{5}{4} - \frac{\sqrt{5}}{4} + \frac{\log \left(2897481469605 \right)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605} \right)^k}{k}$$

$$\begin{split} &\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \\ &- \frac{5}{4} - \frac{\sqrt{5}}{4} + i \pi \left[\frac{\arg (2897481469606 - x)}{2 \pi} \right] + \frac{\log (x)}{2} - \\ &- \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left(2897481469606 - x \right)^k x^{-k}}{k} \quad \text{for } x < 0 \end{split}$$

$$\begin{split} &\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \\ &- \frac{5}{4} - \frac{\sqrt{5}}{4} + i \pi \left[\frac{\pi - \arg \left(\frac{1}{z_0} \right) - \arg (z_0)}{2 \pi} \right] + \\ &- \frac{\log (z_0)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k \left(2897481469606 - z_0 \right)^k z_0^{-k}}{k} \end{split}$$

$$\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = -\frac{5}{4} - \frac{\sqrt{5}}{4} + \frac{1}{2} \int_{1}^{2897481469606} \frac{1}{t} dt$$

$$\frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = -\frac{5}{4} - \frac{i}{4} - \frac{i}{4\pi} \int_{-i + \gamma}^{i + \gamma} \frac{2897481469605^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$5 + \frac{1}{2} \left(log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right)$$

Exact result:

$$5 + \frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$

Decimal approximation:

 $17.53841451404287262702879867166904916409973040542418061099\dots$

Property

$$5 + \frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$
 is a transcendental number

Alternate forms:

$$\frac{1}{4} \left(15 - \sqrt{5} + 2 \log(2897481469606) \right)$$

$$\frac{15}{4} - \frac{\sqrt{5}}{4} + \frac{\log(2\,897\,481\,469\,606)}{2}$$

$$\frac{1}{4} \left(15 - \sqrt{5}\right) + \frac{\log(2\,897\,481\,469\,606)}{2}$$

Continued fraction:

$$[17;\,1,\,1,\,6,\,125,\,1,\,2,\,9,\,1,\,1,\,1,\,6,\,1,\,1,\,1,\,1,\,1,\,1,\,1,\,1,\,1,\,3,\,1,\,26,\,2,\,1,\,1,\,2,\,1,\,2,\,\ldots]$$

Alternative representations:

$$5 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 5 + \frac{1}{2} \left(\log_e(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

$$5 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 5 + \frac{1}{2} \left(-\text{Li}_1 \left(2 + 9^3 + 12^3 + 172^3 + 1010^3 - 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

$$5 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 5 + \frac{1}{2} \left(\log(a) \log_a \left(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

Series representations

$$5 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{15}{4} - \frac{\sqrt{5}}{4} + \frac{\log \left(2897481469605 \right)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605} \right)^k}{k}$$

$$\begin{aligned} 5 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) &= \\ \frac{15}{4} - \frac{\sqrt{5}}{4} + i \pi \left[\frac{\arg(2897481469606 - x)}{2\pi} \right] + \frac{\log(x)}{2} - \\ \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2897481469606 - x)^k x^{-k}}{k} & \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} 5 + \frac{1}{2} \left(\log \left(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) &= \\ \frac{15}{4} - \frac{\sqrt{5}}{4} + i \pi \left| \frac{\pi - \arg \left(\frac{1}{z_{0}} \right) - \arg (z_{0})}{2 \pi} \right| + \\ \frac{\log(z_{0})}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k} (2897481469606 - z_{0})^{k} z_{0}^{-k}}{k} \end{aligned}$$

Integral representations:

$$5 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{15}{4} - \frac{\sqrt{5}}{4} + \frac{1}{2} \int_{1}^{2897481469606} \frac{1}{t} dt$$

$$5 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{15}{4} - \frac{\sqrt{5}}{4} - \frac{i}{4\pi} \int_{-i + \gamma}^{i + \gamma} \frac{2897481469605^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

 $9 + 1/2 ((\ln(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12^3 + 1 - 12$ ((sqrt(5)+5)/2))

$$9 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right)$$

$$9 + \frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$

21.53841451404287262702879867166904916409973040542418061099...

9 +
$$\frac{1}{2} \left(\frac{1}{2} \left(-5 - \sqrt{5} \right) + \log(2897481469606) \right)$$
 is a transcendental number

$$\frac{1}{4} \left(31 - \sqrt{5} + 2 \log(2897481469606) \right)$$

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$$\frac{31}{4} - \frac{\sqrt{5}}{4} + \frac{\log(2897481469606)}{2}$$

$$\frac{1}{4} \left(31 - \sqrt{5}\right) + \frac{\log(2\,897\,481\,469\,606)}{2}$$

Continued fraction:

$$9 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 9 + \frac{1}{2} \left(\log_e(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

$$9 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 9 + \frac{1}{2} \left(-\text{Li}_1 \left(2 + 9^3 + 12^3 + 172^3 + 1010^3 - 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

$$9 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = 9 + \frac{1}{2} \left(\log(a) \log_a \left(-1 - 9^3 - 12^3 - 172^3 - 1010^3 + 14258^3 \right) + \frac{1}{2} \left(-5 - \sqrt{5} \right) \right)$$

Series representations:

$$9 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{31}{4} - \frac{\sqrt{5}}{4} + \frac{\log(2897481469605)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605} \right)^k}{k}$$

$$\begin{aligned} 9 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) &= \\ \frac{31}{4} - \frac{\sqrt{5}}{4} + i \pi \left[\frac{\arg(2897481469606 - x)}{2\pi} \right] + \frac{\log(x)}{2} - \\ \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2897481469606 - x)^k x^{-k}}{k} & \text{for } x < 0 \end{aligned}$$

$$9 + \frac{1}{2} \left(\log(14258^{3} + 1 - 1010^{3} - 1 - 172^{3} - 1 - 12^{3} + 1 - 9^{3} - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) =$$

$$\frac{31}{4} - \frac{\sqrt{5}}{4} + i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_{0}}\right) - \arg(z_{0})}{2\pi} \right| +$$

$$\frac{\log(z_{0})}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k} (2897481469606 - z_{0})^{k} z_{0}^{-k}}{k}$$

Integral representations

$$9 + \frac{1}{2} \left(\log(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{31}{4} - \frac{\sqrt{5}}{4} + \frac{1}{2} \int_{1}^{2897481469606} \frac{1}{t} dt$$

$$9 + \frac{1}{2} \left(\log \left(14258^3 + 1 - 1010^3 - 1 - 172^3 - 1 - 12^3 + 1 - 9^3 - 1 \right) - \frac{1}{2} \left(\sqrt{5} + 5 \right) \right) = \frac{31}{4} - \frac{\sqrt{5}}{4} - \frac{i}{4\pi} \int_{-i + \gamma}^{i + \gamma} \frac{2897481469605^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

The results obtained 25,076 12,538 17,538 and 21,538 are very near to the various values of the black hole entropy as showed in the following table:

k	Bekenstein- Hawking	Witten	Difference (%)
1	12.566	12.190	3.0%
2	17.772	17.576	1.1%
3	21.766	21.676	0.4%
4	25.133	25.118	0.06%

If

(i)
$$\frac{1+53x+9x^{2}}{1-82x-82x^{2}+x^{3}} = a_{0}+a_{1}x+a_{2}x^{2}+a_{3}x^{3}+\cdots$$

or $\frac{a_{0}}{x}+\frac{a_{1}}{x_{1}}+\frac{a_{1}}{x_{3}}+\cdots$

(i) $\frac{2-26x-12x^{2}}{1-82x-82x^{2}+x^{3}} = b_{0}+b_{1}x+b_{1}x^{2}+b_{1}x^{2}+\cdots$

or $\frac{b_{0}}{x}+\frac{b_{1}}{x_{1}}+\frac{b_{1}}{x_{2}}+\cdots$

or $\frac{b_{0}}{x}+\frac{b_{1}}{x_{1}}+\frac{b_{1}}{x_{2}}+\cdots$

or $\frac{b_{0}}{x}+\frac{b_{1}}{x_{1}}+\frac{b_{1}}{x_{2}}+\cdots$

Ithin

$$a_{m}^{3}+b_{m}^{3}=c_{m}^{3}+(-1)^{m}$$

and $a_{m}^{3}+b_{m}^{3}=x_{1}^{3}+(-1)^{m}$

Snumples

$$135^{3}+138^{3}=172^{3}-1$$

$$11161^{3}+11468^{3}=14258^{3}+1$$

$$79/^{3}+8/8^{3}=1010^{3}-1$$

Conclusion

From the different results highlighted during this research, it is possible to propose the number 1729 (and also the 728), defined by the mathematical genius S. Ramanujan as "very interesting", which plays a fundamental role in number theory, as a new physical constant from which emerge various properties of the Standard Model particles, including the masses, and also the mass values of the "glueballs" and also in many cases, the value of the entropy of black holes. Since the entropy of black holes also takes negative values, we tend to propose that they be white holes. For supersymmetry, as for each particle there is a superpartner, with each black hole there is a white hole. As from a black hole nothing can come out, from a white hole the reverse happens. It is therefore easy to think that all white holes are big bang singularities. In reality, not all the black holes that evaporate pass the information to the corresponding white holes from which possible bubble universes will emerge, but only a well-defined number that will form the universes subsets of the multiverse. For all others, information will pass directly into the infinite-dimensional Hilbert space. This further strengthens the proposal of a multiverse composed of a very high but finite set of bubbles (perhaps 8.08 * 10 ^ 53). Once the expansion-acceleration phase is complete, every bubble-universe of the multiverse becomes the final phase, when each galaxy, star, etc. ends its cycle, an immense black hole. The final giant n-black holes, connected to each other in a sort of entanglement, as happens for the particles, will evaporate simultaneously in an incalculable but finite time, passing once they become infinitely small, more than an atomic nucleus, (symmetry with the initial singularity) the n-information in the infinite-dimensional space. The evident similar behavior of the physics of black holes and particles, even in the entanglement effect, could explain the evident connection that is obtained from the equations of the physics of subatomic particles inherent to the Standard Model, whose solutions are very close and often even equal to the entropy value of a black hole. This with the appropriate use of Ramanujan's mathematics which can then be applied to both black holes and particle physics. This could also be a further indication that elementary particles, such as electrons, mediators of fundamental forces, massive and scalar bosons and glueballs, are in fact a sort of quantum black holes. All these connections obtained by integrating and / or using different equations from various expressions of Ramanujan in different ways, reinforce our belief that this mathematics can be the way to go to reach a sort of "mathematical TOE"

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