Roots of the equation $x = 1 + \cos(3 - 3x)$

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abstract

In this note we give , the three real roots of the equation

$$x = 1 + \cos(3 - 3x) \quad , x \in \mathbb{R}$$

1. Introduction: the roots of the equation: $x = 1 + \cos(3 - 3x)$.

Entry 1. If $x = 1 + \cos(3 - 3x)$, $x \in \mathbb{R}$ then

$$x = \begin{cases} \alpha = 0.0206332020... \\ \beta = 0.1122737055... \\ \gamma = 1.3900403166... \end{cases} \tag{1}$$

Entry 2.

$$\alpha = 1 + \cos\left(3\cos\left(3\cos\left(3...\right)\right)\right) \tag{2}$$

$$\beta = 1 - \left\{ \frac{\pi}{3} - \frac{1}{3}\cos^{-1}\left(\frac{\pi}{3} - \frac{1}{3}\cos^{-1}\left(\frac{\pi}{3} - ... \right) \right) \right\} \tag{3}$$

$$\gamma = 1 + \left\{ \frac{\pi}{6} - \frac{1}{3}\sin^{-1}\left(\frac{\pi}{6} - \frac{1}{3}\sin^{-1}\left(\frac{\pi}{6} - ... \right) \right) \right\} \tag{4}$$

$$\gamma = 1 + \frac{1}{3}\cos^{-1}\left(\frac{1}{3}\cos^{-1}\left(\frac{1}{3}\cos^{-1}\left(\frac{1}{3} ... \right) \right) \right) \tag{5}$$

Entry 3.

$$\alpha_{n+1} = \cos\left(3\alpha_n\right) \quad , \alpha_0 = 0 \Rightarrow \alpha_n \to \alpha - 1 \tag{6}$$

$$\beta_{n+1} = \frac{\pi}{3} - \frac{1}{3}\cos^{-1}\beta_n \quad , \beta_0 = 0 \Rightarrow \beta_n \to 1 - \beta \tag{7}$$

$$\gamma_{n+1} = \frac{\pi}{6} - \frac{1}{3}\sin^{-1}\gamma_n \quad , \gamma_0 = 0 \Rightarrow \gamma_n \to \gamma - 1 \tag{8}$$

$$\delta_{n+1} = \frac{1}{3}\cos^{-1}\delta_n \quad , \delta_0 = 0 \Rightarrow \delta_n \to \gamma - 1 \tag{9}$$

2. Integrals

Entry 4.

$$\int_0^{2\pi} \frac{e^{xi}}{-1 + \frac{1}{10}e^{xi} - \cos\left(-3 + \frac{3}{10}e^{xi}\right)}\,dx = \frac{20\pi}{1 - 3\sin\left(3 - 3\alpha\right)} \tag{10}$$

$$\int_0^{2\pi} \frac{e^{xi}}{-\frac{9}{10} + \frac{1}{20}e^{xi} - \cos\left(-\frac{27}{10} + \frac{3}{20}e^{xi}\right)}\,dx = \frac{40\pi}{1 - 3\sin\left(3 - 3\beta\right)} \tag{11}$$

$$\int_0^{2\pi} \frac{e^{xi}}{e^{xi} - 2\cos\left(\frac{3}{2}e^{xi}\right)}\,dx = \frac{2\pi}{1 - 3\sin\left(3 - 3\gamma\right)} \tag{12}$$

3. Formulas

Entry 5.

$$\pi = 3 - 3\alpha + \tan^{-1}\left(\frac{\sqrt{\alpha\left(2-\alpha\right)}}{1-\alpha}\right) = 3 - 3\alpha + 2\tan^{-1}\sqrt{\frac{\alpha}{2-\alpha}} \tag{13}$$

$$\pi = 3 - 3\alpha + \sqrt{2\alpha}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-3n}\alpha^n}{2n+1} \tag{14}$$

$$\pi = 3 - 3\beta + \tan^{-1}\left(\frac{\sqrt{\beta(2-\beta)}}{1-\beta}\right) = 3 - 3\beta + 2\tan^{-1}\sqrt{\frac{\beta}{2-\beta}} \tag{15}$$

$$\pi = 3 - 3\beta + \sqrt{2\beta}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-3n}\beta^{n}}{2n+1} \tag{16}$$

$$\pi = 6\gamma - 6 + 2\sin^{-1}(\gamma - 1) \tag{17}$$

$$\pi = 6\gamma - 6 + 2\tan^{-1}\left(\frac{\gamma-1}{\sqrt{\gamma(2-\gamma)}}\right) \tag{18}$$

Remark: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592...$

4. Graphics

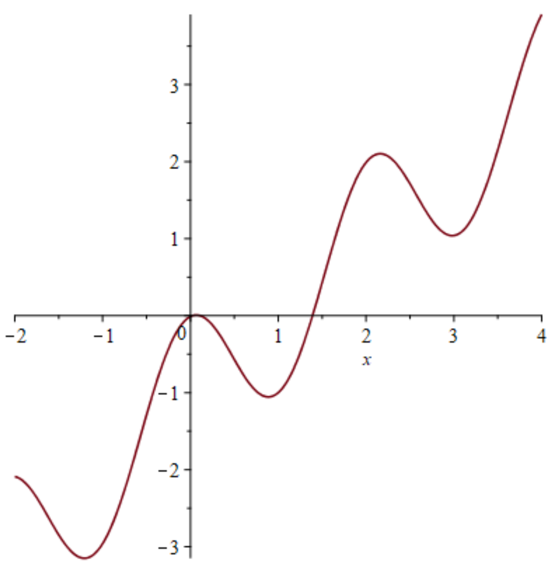

Figure 1. $x - 1 - \cos(3 - 3x)$, $-2 < x < 4$.

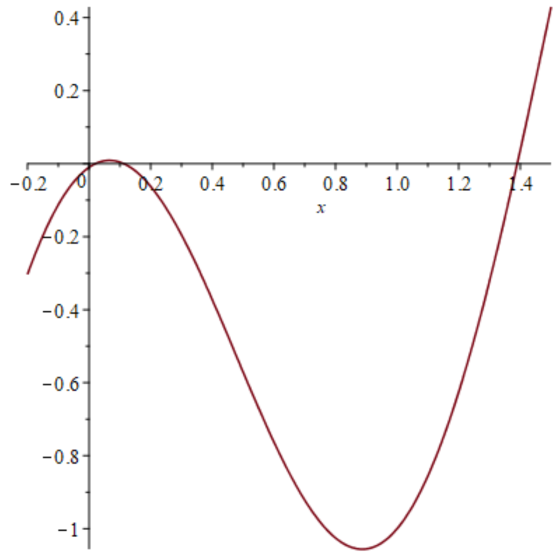

Figure 2. $x-1-\cos(3-3x)$, $-0.2<x<1.5$.

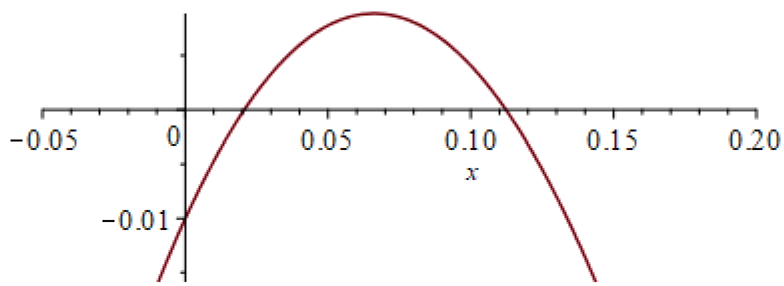

Figure 3. $x-1-\cos(3-3x)$, $-0.01<x<0.15$.

References

1. Olver, F.W.J., Lozier, D.W., Boisvert, R.F., Clark, C.W. : NIST Handbook of Mathematical Functions. Cambridge University Press , 2010.