# Determination of the Fundamental Impedance of Free Space due to Radiation Energy from the Cosmic Microwave Background and Information Horizons 

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#### Abstract

Here, the fundamental constants namely vacuum permeability and permittivity, which comprise the numerical definition of the speed of light in vacuum, are determined. They are found to be composites correlated to Planck's constant, Wien's constant and the mass energy of the cosmic microwave background. Derivations for both a new fundamental composite speed of light in vacuum and vacuum impedance are performed. Furthermore, this newly suggested definition is correlated to a confined quantized radiation spectrum of the cosmic particle horizon.


## 1 Introduction

Recently, the cosmic microwave background (CMB) radiation was linked to a neutrino mass and furthermore to Newton's Gravity Law [2] [3]. This method suggests replacing the Planck mass as a fundamental particle using the mass energy potential. Newton's Gravity Law was modified during the unification of all four fundamental forces incorporating a discrete spectrum within a confinement using conditions of wavelength boundaries from the cosmic horizon down to high frequencies of Planck length, $l_{p}$ [1]. Planck's gravitational constant is equated to the modified CMB mass energy gravitational constant. From solving this equation, the speed of light $c$ (in vacuum), vacuum permittivity $\epsilon_{0}$, vacuum permeability $\mu_{0}$, and vacuum impedance $Z_{0}$ can all be determined using Wien's constant, the corresponding mass of the CMB energy, Planck's constant and the cosmic particle horizon. Additionally, a new definition of the speed of light is introduced which is a composite of the CMB equivalent mass, the reduced Planck's constant, Wien's constant, Plank length and the dimension of the cosmic particle horizon. It is suggested that the fundamental magnetic and electric constant can be isolated from the elements of the new speed of light definition. Henceforth, it appears that the magnetic and electric constants are composites constants themselves.

## 2 Method

### 2.1 Fundamental constants using force

The fundamental energy between the two Planck masses, $m_{p}$, at a distance $x$ was found previously [1]. The two logarithmic ratios represent a spectrum range that is associated to the energy difference of the virtual particles in the regions of the forces. This is due to virtual particle momenta around the objects. It should be noted that the displayed logarithmic division represents the $1 / k$ multiplication (elongation) of the wavelength in a finite summation and represents an integral denoting a physics action (momentum times length) over the range of allowed modes between the logarithmic numerator and denominator. Physically, the two logarithmic ratios represent the push and pull around the objects. The numerator represents the unshared energy waves while the denominator represents the total amount of energy waves. The cosmic horizon used is $\Theta=8.8 \cdot 10^{26} \mathrm{~m}$. Additionally, $\gamma$ is Euler-Mascheroni's constant.

$$
\begin{equation*}
E_{G 0}=\frac{4 h c}{2 \pi x} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{1}
\end{equation*}
$$

Now assume the two objects are composed of a certain number of Planck masses namely $N_{1}$ and $N_{2}$. Each Planck mass number is associated with a virtual particle and their combinations of interactions of pairs of masses will result in a double summation [5].

$$
\begin{equation*}
E_{G}=\sum_{i=1}^{N_{1}} \sum_{j=1}^{N_{2}} E_{G 0} \tag{2}
\end{equation*}
$$

Compute both summations to obtain and simplify to obtain the following.

$$
\begin{equation*}
E_{G}=\frac{4 \hbar c N_{1} N_{2}}{x} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{3}
\end{equation*}
$$

Now, rewrite the number of Planck masses using $N_{1}=m_{1} / m_{p}$ and $N_{2}=m_{2} / m_{p}$ [1].

$$
\begin{equation*}
E_{G}=\frac{4 \hbar c m_{1} m_{2}}{m_{p}^{2} x} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{4}
\end{equation*}
$$

Next substitute in for $m_{p}^{2}=\hbar c / G$ to obtain the following.

$$
\begin{equation*}
E_{G}=\frac{4 G m_{1} m_{2}}{x} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{5}
\end{equation*}
$$

Using (5) one can also write the energy equation using the fundamental neutrino mass where $G_{c m b}=\frac{\hbar c}{m_{c m b}^{2}} \cdot \frac{\beta \pi^{2} l_{p}}{4 \Theta}$ [2] [3]. Here, Wien's constant is $\beta=\frac{1}{4.965114}$ and the CMB mass is $m_{\text {cmb }}=2.0792027 \cdot 10^{-39} \mathrm{~kg}[3][2]$.

$$
\begin{equation*}
E_{G}=\frac{4 m_{1} m_{2}}{x} \frac{\hbar c}{m_{c m b}^{2}} \cdot \frac{\beta \pi^{2} l_{p}}{4 \Theta} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{6}
\end{equation*}
$$

Take the derivative of the energy equation to obtain the force equation. Take the absolute value as only the magnitude matters.

$$
\begin{equation*}
F_{g}=A\left(\frac{\ln (B / x)(\ln (C x)-1)+\ln (C x)}{x^{2}}\right) m_{1} m_{2} \tag{7}
\end{equation*}
$$

Where,

$$
A=\frac{4 \hbar c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta \ln ^{2}\left(\frac{\theta c \gamma}{2 p_{p}}\right)}, \quad B=\Theta e^{\gamma}, \quad C=\frac{e^{\gamma}}{2 l_{p}}
$$

Next isolate the new gravitational constant namely $G_{\text {new }}$ from the force equation of $F=\frac{G_{\text {new }} m_{1} m_{2}}{x^{2}}$. Also take $x=1 \mathrm{~m}$ to show the fundamental constants at this key distance.

$$
\begin{equation*}
G_{\text {new }}=A(\ln (B)(\ln (C)-1)+\ln (C)) \tag{8}
\end{equation*}
$$

Where,

$$
A=\frac{4 \hbar c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta \ln ^{2}\left(\frac{\left.\theta e^{\gamma}\right)}{2 l_{p}}\right)}, \quad B=\Theta e^{\gamma}, \quad C=\frac{e^{\gamma}}{2 l_{p}}
$$

Next, set $G_{\text {new }}$ equal to $G=\frac{l_{p}^{2} c^{3}}{\hbar}$ [8] [4].

$$
\begin{equation*}
\frac{l_{p^{2}}^{2}}{\hbar}=A(\ln (B)(\ln (C)-1)+\ln (C)) \tag{9}
\end{equation*}
$$

Plug in for $A, B$, and $C$ into (9).

$$
\begin{equation*}
\frac{l_{p}^{2} c^{3}}{\hbar}=\frac{4 \hbar c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta \ln ^{2}\left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)}\left(\ln \left(\Theta e^{\gamma}\right)\left(\ln \left(\frac{e^{\gamma}}{2 l_{p}}\right)-1\right)+\ln \left(\frac{e^{\gamma}}{2 l_{p}}\right)\right) \tag{10}
\end{equation*}
$$

For simplicity denote all the logarithmic terms as $\zeta$.

$$
\begin{equation*}
\frac{l_{p}^{2} c^{3}}{\hbar}=\frac{4 \hbar c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta} \zeta \tag{11}
\end{equation*}
$$

Next solve for $c^{2}$. Notice nothing has been cancelled in order to identify units.

$$
\begin{equation*}
c^{2}=\frac{4 \hbar^{2} c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta l_{p}^{2} c} \zeta \tag{12}
\end{equation*}
$$

Take the square root of both sides. Note: all calculations assume the speed of light is in a vacuum.

$$
\begin{equation*}
c=\sqrt{\frac{4 \hbar^{2} c \beta \pi^{2} l_{p}}{4 m_{c m b}^{2} \Theta l_{p}^{2} c} \zeta} \tag{13}
\end{equation*}
$$

The speed of light can be reduced to the following in terms of the CMB parameters, the cosmic horizon and Planck length. Note that $c$ has the correct units of $\left[\mathrm{m} \mathrm{s}^{-1}\right]$.

$$
\begin{equation*}
c=\sqrt{\frac{\hbar^{2} \beta \pi^{2}}{m_{c m b}^{2} \Theta l_{p}} \zeta} \tag{14}
\end{equation*}
$$

Using (14) rewrite in the form $c=\sqrt{\frac{1}{\epsilon_{0} \mu_{0}}}$.

$$
\begin{equation*}
c=\sqrt{\frac{1}{\frac{4 m_{c m b}^{2} \Theta \rho_{p}^{2} c}{4 \hbar^{2} c \beta \pi^{2} l_{p} \zeta}}} \tag{15}
\end{equation*}
$$

Now identify both $\mu_{0}$ and $\epsilon_{0}$.

$$
\begin{gather*}
\epsilon_{0}=\frac{8 m_{c m b} \Theta l_{p}}{\hbar \zeta}  \tag{16}\\
\mu_{0}=\frac{m_{c m b}}{8 \pi^{2} \beta \hbar} \tag{17}
\end{gather*}
$$

Next find the vacuum impedance using the formula $Z_{0}=\sqrt{\mu_{0} / \epsilon_{0}}$.

$$
\begin{equation*}
Z_{0}=\frac{1}{8 \pi} \sqrt{\frac{\zeta}{\beta \Theta l_{p}}} \tag{18}
\end{equation*}
$$

Numerically the three vacuum constants converge but some units are lost since $\Theta$ has embedded units that are cancelled. This will be seen and identified when $\Theta$ is substituted below. Substitute in for $\Theta=\frac{\beta \pi^{2} \hbar^{2}}{4 m_{c m b}^{2} c^{2} l_{p}}$ into (12) [2].

$$
\begin{equation*}
\frac{l_{p}^{2} c^{3}}{\hbar}=\frac{4 \hbar c 4 m_{c m b}^{2} c^{2} \beta \pi^{2} l_{p}^{2}}{4 m_{c m b}^{2} \beta \pi^{2} \hbar^{2}} \zeta \tag{19}
\end{equation*}
$$

Next solve for $c^{2}$ on both sides.

$$
\begin{equation*}
c^{2}=\frac{16 \hbar^{2} m_{c m b}^{2} c^{2} \beta \pi^{2} l_{p}^{2}}{4 \hbar^{2} m_{c m b}^{2} \beta \pi^{2} l_{p}^{2}} \zeta \tag{20}
\end{equation*}
$$

Here is the subtle portion. Only certain items should be canceled in order to identify $\epsilon_{0}$ and $\mu_{0}$. First cancel one order of $m_{c m b}$ and $l_{p}$ and solve for $c^{2}$ on both sides.

$$
\begin{equation*}
c^{2}=\frac{16 \hbar^{2} m_{c m b} c^{2} \beta \pi^{2} l_{p}}{4 \hbar^{2} m_{c m b} \beta \pi^{2} l_{p}} \zeta \tag{21}
\end{equation*}
$$

Next, isolate $\hbar l_{p}$ in the numerator and denominator.

$$
\begin{equation*}
c^{2}=\frac{16 \hbar m_{c m b} c^{2} \beta \pi^{2} l_{p}}{4 \hbar m_{c m b} \beta \pi^{2} l_{p}} \frac{\hbar l_{p}}{\hbar l_{p}} \zeta \tag{22}
\end{equation*}
$$

The numerical value of the ratio of $\hbar l_{p}$ will cancel but the units will remain for both the numerator and denominator. The units of $\hbar l_{p}$ are $\left[\mathrm{kg} \mathrm{m}^{3} \mathrm{~s}^{-1}\right]$. Therefore, define $s=1\left[\mathrm{~kg} \mathrm{~m}^{3} \mathrm{~s}^{-1}\right]$ and replace the ratio which will be simply $s / s$.

$$
\begin{equation*}
c^{2}=\frac{16 \hbar m_{c m b} c^{2} \beta \pi^{2} l_{p}}{4 \hbar m_{c m b} \beta \pi^{2} l_{p}} \frac{s}{s} \zeta \tag{23}
\end{equation*}
$$

Now use the equation $c^{2}=\frac{1}{\epsilon_{0} \mu_{0}}$ to rewrite the equation.

$$
\begin{equation*}
c^{2}=\frac{1}{\frac{4 \hbar m_{c m b} \beta \pi^{2} l_{p} s}{16 \hbar m_{c m b} c^{2} \beta \pi^{2} l_{p} s \zeta}} \tag{24}
\end{equation*}
$$

Therefore the speed of light can be written as the following.

$$
\begin{equation*}
c=\sqrt{\frac{1}{\frac{4 \hbar m_{c m b} \beta \pi^{2} l_{p} s}{16 \hbar m_{c m b} c^{2} \beta \pi^{2} l_{p} s \zeta}}} \tag{25}
\end{equation*}
$$

Finally rewrite to identify $\epsilon_{0}$ and $\mu_{0}$ by using unit identification and dimensional analysis. Notice $\epsilon_{0}$ will have the $\zeta$ term since it will most likely be variable.

$$
\begin{gather*}
c=\sqrt{\frac{1}{\frac{4 \pi^{2} \beta \hbar}{2 \zeta m_{c m b} c^{2} s} \frac{m_{c m b} s}{2 \hbar 4 \pi^{2} \beta}}}  \tag{26}\\
\epsilon_{0}=\frac{\left(4 \pi^{2}\right) \beta \hbar}{2 \zeta m_{c m b} c^{2} s}  \tag{27}\\
\mu_{0}=\frac{m_{c m b} s}{2 \hbar\left(4 \pi^{2} \beta\right)} \tag{28}
\end{gather*}
$$

Simplify to obtain the following.

$$
\begin{gather*}
\epsilon_{0}=\frac{2 \pi^{2} \beta \hbar}{\zeta m_{c m b} c^{2} s}  \tag{29}\\
\mu_{0}=\frac{m_{c m b} s}{8 \hbar \pi^{2} \beta} \tag{30}
\end{gather*}
$$

The only dimensional units missing are the Coulombs [ $\mathrm{C}^{2}$ ] in numerator of $\mu_{0}$ and the denominator of $\epsilon_{0}$. Since the gravity equation does not have charge associated to it, simply add these units to match the traditional units of both constants.

One can also write $\mu_{0}$ in terms of the neutrino (CMB) momentum divided by the angular wavelength energy corresponding to the dimension of one meter.

$$
\begin{equation*}
\mu_{0}=\frac{m_{c m b} c s}{8 \hbar c \pi^{2} \beta} \tag{31}
\end{equation*}
$$

Next compute the vacuum impedance $Z_{0}=\sqrt{\frac{\mu_{0}}{\epsilon_{0}}}$.

$$
\begin{equation*}
Z_{0}=\sqrt{\frac{\frac{m_{c m b} s}{8 \hbar \pi^{2} \beta}}{\frac{2 \pi^{2} \beta \hbar}{\zeta m_{c m b} c^{2} s}}} \tag{32}
\end{equation*}
$$

Simplify to obtain the following.

$$
\begin{equation*}
Z_{0}=\frac{m_{c m b} c s \sqrt{\zeta}}{4 \hbar \beta \pi^{2}} \tag{33}
\end{equation*}
$$

### 2.2 Fundamental Constants with Energy

From (6) onwards one can rewrite the potential energy between the two mass objects.

$$
\begin{equation*}
E_{G}=\frac{m_{1} m_{2}}{x} \frac{4 \hbar c}{m_{c m b}^{2}} \cdot \frac{\beta \pi^{2} l_{p}}{4 \Theta} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{34}
\end{equation*}
$$

Next, set $G_{\text {new }}$ equal to $G=\frac{l_{p}^{2} c^{3}}{\hbar}$ which comes from $E_{G}=\frac{G_{n e w} m_{1} m_{2}}{x}$ [8] [4].

$$
\begin{equation*}
\frac{l_{p}^{2} c^{3}}{\hbar}=\frac{4 \hbar c}{m_{c m b}^{2}} \cdot \frac{\beta \pi^{2} l_{p}}{4 \Theta} \frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{35}
\end{equation*}
$$

Set $\chi$ equal to all the logarithmic terms like so.

$$
\begin{equation*}
\chi=\frac{\ln \left(\frac{\Theta e^{\gamma}}{x}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \frac{\ln \left(\frac{x e^{\gamma}}{2 l_{p}}\right)}{\ln \left(\frac{\Theta e^{\gamma}}{2 l_{p}}\right)} \tag{36}
\end{equation*}
$$

Next use substitute in for $\Theta=\frac{\beta \pi^{2} \hbar^{2}}{4 m_{c m b}^{2} c^{2} l_{p}}$ into (35) [2].

$$
\begin{equation*}
\frac{l_{p}^{2} c^{3}}{\hbar}=\frac{4 \hbar c 4 m_{c m b}^{2} c^{2} \beta \pi^{2} l_{p}^{2}}{4 m_{c m b}^{2} \beta \pi^{2} \hbar^{2}} \chi \tag{37}
\end{equation*}
$$

This will result in the same form for all three fundamental constants following the same procedure in section 2.1 except $\zeta$ will be replaced with $\chi$.

$$
\begin{equation*}
c=\sqrt{\frac{1}{\frac{4 \hbar m_{c m b} \beta \pi^{2} l_{p} s}{16 \hbar m_{c m b} c^{2} \beta \pi^{2} l_{p} s \chi}}} \tag{38}
\end{equation*}
$$

$$
\begin{gather*}
\epsilon_{0}=\frac{2 \pi^{2} \beta \hbar}{\chi m_{c m b} c^{2} s}  \tag{39}\\
\mu_{0}=\frac{m_{c m b} s}{8 \hbar \pi^{2} \beta}  \tag{40}\\
Z_{0}=\frac{m_{c m b} c s \sqrt{\chi}}{4 \hbar \beta \pi^{2}} \tag{41}
\end{gather*}
$$

## 3 Discussion

The following comparison between the computed constants and the traditional CODATA values of the four constants will be analyzed [7]. The derived fundamental constants lack full numerical convergence to the measured values, however, several causes may contribute to the deviation. These could come from the fact that the gravitational constant is defined by Planck length numerically and also it is still well known that $G$ is not precisely determined experimentally. Furthermore, the cosmic particle horizon, although a good estimate by the present measurements, may have slight variations. Finally, the method of computing the high number of summations of the modes of the discrete spectrum energy eigenstates in confinement by the logarithmic approximation to determine the physical action balance may also introduce some minor numerical errors. Table 1 below shows the highlighted equations and their errors with respect to the fundamental constants.

Table 1: Comparison of Constants

| Equation <br> Number | Fundamental <br> Constant | Error <br> $\%$ |
| :---: | :---: | :---: |
| 14 | $c$ | 0.16405 |
| 16 | $\epsilon_{0}$ | 1.6894 |
| 17 | $\mu_{0}$ | 1.33792 |
| 18 | $Z_{0}$ | 1.49977 |
| 29 | $\epsilon_{0}$ | 1.69934 |
| 30 | $\mu_{0}$ | 1.33792 |
| 33 | $Z_{0}$ | 1.50458 |
| 39 | $\epsilon_{0}$ | 2.05210 |
| 40 | $\mu_{0}$ | 1.33792 |
| 41 | $Z_{0}$ | 1.67496 |

Overall the suggested constant definitions are in close convergence to the observed values. Additionally, the determined mass of the CMB corresponding to neutrinos as introduced by Sheppeard and Dungworth may involve a minor error which could accumulate into the simulated fundamental value of the new defined constants. These new results may reflect a situation where the constants are interactions within the cosmos of saturated neutrino masses. These could be related to momentum transfer and virtual photonic radiation (similar to Unruh radiation and Hawking radiation) with momentum eigenstates of a cosmic quantum oscillator.

## 4 Conclusion

An amendment to the previous definition of the magnetic and electric constant has been made which introduces a dependency to the cosmic particle horizon and the microwave background. As initially suggested by McCulloch, a discrete radiation spectrum has been used and has been estimated by a logarithmic approximation [6]. This was done to compute a symmetric modification of the physical action introduced by information horizon boundaries with the maximum distance defined by the cosmic particle background. Overall the convergence of the constants and the correctly identified units show that the method introduced could provide further insight into the nature of the universe.

## 5 Appendix

Starting with both $\mu_{0}$ and $\epsilon_{0}$ from (29) and (30), multiply both by $\frac{\pi c^{2}}{\pi c^{2}}$ to obtain the following.

$$
\begin{gather*}
\epsilon_{0}=\frac{2 \pi^{2} \beta \hbar c^{2} \pi}{\zeta m_{c m b} c^{2} c^{2} s \pi}  \tag{42}\\
\mu_{0}=\frac{m_{c m b} c^{2} s \pi}{8 \hbar \pi^{2} \beta c^{2} \pi} \tag{43}
\end{gather*}
$$

Interestingly, it was found that numerically $m_{p}^{2}=8 \hbar \pi^{2} \beta c^{2} \pi$ with an error of only $4.08 \cdot 10^{-4}$. Therefore substitute into both (42) and (43) to obtain the following.

$$
\begin{gather*}
\epsilon_{0}=\frac{m_{p}^{2}}{4 \pi \zeta m_{c m b} c^{2} c^{2} s}  \tag{44}\\
\mu_{0}=\frac{\pi m_{c m b} c^{2} s}{m_{p}^{2}} \tag{45}
\end{gather*}
$$

Finally $Z_{0}$ can also be computed in terms of Planck mass.

$$
\begin{equation*}
Z_{0}=\frac{2 \pi m_{c m b} c^{3} s \sqrt{\zeta}}{m_{p}^{2}} \tag{46}
\end{equation*}
$$

Therefore, the error of all three of these constants will almost coincide to (29), (30) and (33) in Table 1.

## References

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