Zero Expresses Non-possibility

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Abstract: In this paper, as one important property of zero, we will simply show that zero expresses non-possibility.

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1 Zero, division by zero and division by zero calculus

For the long history of division by zero, see [1, 18]. The division by zero with mysterious and long history was indeed trivial and clear as in the followings.

By the concept of the Moore-Penrose generalized solution of the fundamental equation $ax = b$, the division by zero was trivial and clear as $a/0 = 0$ in the **generalized fraction** that is defined by the generalized solution of the equation $ax = b$. Here, the generalized solution is always uniquely determined and the theory is very classical. See [4] for example.

Division by zero is trivial and clear from the concept of repeated subtraction - H. Michiwaki.

Recall the uniqueness theorem by S. Takahasi on the division by zero. See [4, 27].

The simple field structure containing division by zero was established by M. Yamada ([7]). For a simple introduction, see Okumura [16].

Many applications of the division by zero to Wasan geometry were given by H. Okumura. See [10, 11, 12, 13, 14, 15] for example.

As the number system containing the division by zero, the Yamada field structure is perfect. However, for applications of the division by zero to **functions**, we need the concept of the division by zero calculus for the sake of uniquely determinations of the results and for other reasons.

For example, for the typical linear mapping

$$W = \frac{z - i}{z + i}, \tag{1.1}$$

it gives a conformal mapping on $\{\mathbf{C} \setminus \{-i\}\}$ onto $\{\mathbf{C} \setminus \{1\}\}$ in one to one and from

$$W = 1 + \frac{-2i}{z - (-i)}, \tag{1.2}$$

we see that $-i$ corresponds to 1 and so the function maps the whole $\{\mathbf{C}\}$ onto $\{\mathbf{C}\}$ in one to one.

Meanwhile, note that for

$$W = (z - i) \cdot \frac{1}{z + i}, \tag{1.3}$$

we should not enter $z = -i$ in the way

$$[(z - i)]_{z=-i} \cdot \left[\frac{1}{z + i}\right]_{z=-i} = (-2i) \cdot 0 = 0. \tag{1.4}$$

However, in many cases, the above two results will have practical meanings and so, we will need to consider many ways for the application of the division by zero and we will need to check the results obtained, in some practical viewpoints. We referred to this delicate problem with many examples.

Therefore, we will introduce the division by zero calculus. For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n (z - a)^n + C_0 + \sum_{n=1}^{\infty} C_n (z - a)^n, \tag{1.5}$$

we **define** the identity, by the division by zero

$$f(a) = C_0. \tag{1.6}$$

Note that here, there is no problem on any convergence of the expansion (1.5) at the point $z = a$, because all the terms $(z - a)^n$ are zero at $z = a$ for $n \neq 0$.

Apart from the motivation, we define the division by zero calculus by (1.6). With this assumption, we can obtain many new results and new ideas. However, for this assumption we have to check the results obtained whether they are reasonable or not. By this idea, we can avoid any logical problems. – In this point, the division by zero calculus may be considered as a fundamental assumption like an axiom.

The division by zero calculus opens a new world since Aristotele-Euclid. See, in particular, [2] and also the references for recent related results.

On February 16, 2019 Professor H. Okumura introduced the surprising news in Research Gate:

José Manuel Rodríguez Caballero

Added an answer
In the proof assistant Isabelle/HOL we have $x/0 = 0$ for each number x. This is advantageous in order to simplify the proofs. You can download this proof assistant here: **https://isabelle.in.tum.de/**.

J.M.R. Caballero kindly showed surprisingly several examples by the system that

$$\tan\frac{\pi}{2} = 0,$$

$$\log 0 = 0,$$

$$\exp\frac{1}{x}(x = 0) = 1,$$

and others. It seems that the division by zero calculus seems to be implimented to the computer system already.

Meanwhile, on ZERO, the authors S. K. Sen and R. P. Agarwal [24] published its long history and many important properties of zero. See also R. Kaplan [3] and E. Sondheimer and A. Rogerson [26] on the very interesting books on zero and infinity. In particular, for the fundamental relation of zero and infinity, we stated the simple and fundamental relation in [23] that

The point at infinity is represented by zero; and zero is the definite complex number and the point at infinity is considered by the limiting idea
and that is represented geometrically with the horn torus model.

S. K. Sen and R. P. Agarwal [24] referred to the paper [4] in connection with division by zero, however, their understandings on the paper seem to be not suitable (not right) and their ideas on the division by zero seem to be traditional, indeed, they stated as a conclusion of the introduction of the book that:

"Thou shalt not divide by zero" remains valid eternally.

However, in [22] we stated simply based on the division by zero calculus that

We Can Divide the Numbers and Analytic Functions by Zero with a Natural Sense.

They stated in the book many meanings of zero over mathematics, deeply. For a meaning of zero, in this short paper, by means of the division by zero calculus, we would like to state clearly that

Zero Expresses Non-possibility
as a basic statement and as an additional remark of the book [24].

2 Examples

We will show the statement with typical examples.

- As the solution of the simplest equation

$$ax = b \tag{2.1}$$

  we have $x = 0$ for $a = 0, b \neq 0$ as the standard value, or the Moore-Penrose generalized inverse. This will mean in a sense, that the solution does not exist; to solve the equation (2.1) is impossible. We saw for different parallel lines or different parallel planes, their common point is the origin in [5]. Certainly they have the common point of the point at infinity and the point at infinity is represented by zero. However, we can understand also that they have no solutions, no common points, because the point at infinity is an ideal point.

- We will consider the point P at the origin with starting at the time $t = 0$ with velocity $V > 0$ and the point Q at the point $d > 0$ with velocity $v > 0$. Then, the time T of coincidence P=Q is given by

$$T = \frac{d}{V - v}.$$

  When $V = v$, we have, by the division by zero, $T = 0$. This zero represents impossibility. We have many similar examples.

- We will consider the simple differential equation

$$m\frac{d^2x}{dt^2} = 0, m\frac{d^2y}{dt^2} = -mg \tag{2.2}$$

  with the initial conditions, at $t = 0$

$$\frac{dx}{dt} = v_0 \cos\alpha, \quad \frac{dy}{dt} = v_0 \sin\alpha; \quad x = y = 0.$$

  Then, the highest high h, arriving time t, the distance d from the starting point at the origin to the point $y(2t) = 0$ are given by

$$h = \frac{v_0 \sin^2\alpha}{2g}, \quad d = \frac{v_0^2 \sin 2\alpha}{g}$$

  and

$$t = \frac{v_0 \sin\alpha}{g}.$$

  For the case $g = 0$, we have $h = d = t = 0$. We considered the case that they are infinity; however, our mathematics means zero, which shows impossibility.

- A and B start at the origin on the real positive axis with, for $t = 0$

$$\frac{d^2x}{dt^2} = a, \quad \frac{dx}{dt} = u$$

  and

$$\frac{d^2x}{dt^2} = b, \quad \frac{dx}{dt} = v,$$

with constants a, b, u, v, respectively. After the time T and at the distance X from the origin, if they meet, then we obtain the relations

$$T = \frac{2(u-v)}{b-a}$$

and

$$X = \frac{2(u-v)(ub-va)}{(b-a)^2}.$$

For the case $a = b$, we obtain the reasonable results $T = 0$ and $X = 0$.

- On the real line, we look at the point P with angle α and β at the point with a distance l from P. Then, the high of the point P is given by

$$h = \frac{l \sin\alpha \sin\beta}{\sin(\alpha - \beta)}.$$

Then, if $\alpha = \beta$, then, by the division by zero, $h = 0$.

- We fix the lines $y = d$ and $x = L$ $(d, L > 0)$. We consider the line through two points $(0, t); t > d$ and (L, d), and let D be the common point with the line and the x axis. Then, we have the identity

$$\frac{D}{L} = \frac{t}{t-d}.$$

When $t = d$, by the division by zero, from $d/0 = 0$ we have $D = 0$ which is reasonable in our new mathematics. However, from the identity

$$\frac{t}{t-d} = 1 + \frac{d}{t-d},$$

by the division by zero calculus, we have another reasonable result $D = L$.

- For fixed two vectors $OA = a$ and $OB = b$ $(a \neq b)$, we consider two vectors $OA' = a' = \lambda a$ and $OB' = b' = \mu b$ with parameters λ and μ. Then, the common point x of the two lines AB and $A'B'$ is represented by

$$x = \frac{\lambda(1-\mu)a + \mu(\lambda - 1)b}{\lambda - \mu}.$$

For $\lambda = \mu$, we should have $x = 0$, by the division by zero. However, by the division by zero calculus, we have the curious result

$$x = (1 - \mu)a + \mu b.$$

These phenomena were looked in many cases on the universe; it seems that

God does not like infinity as the numbers.

Infinity is represented by zero, and zero represents impossibility.

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