The simple and typical physical examples of the division by zero 1/0=0 by Ctesíbio (BC. 286-222) and E. Torricelli (1608 -1646)

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Abstract: In this paper, we give the simple and typical physical examples in order to see clearly the division by zero $1/0 = 0$.

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1 Division by zero

For the long history of division by zero, see [1, 16]. The division by zero with mysterious and long history was indeed trivial and clear as in the followings:

By the concept of the Moore-Penrose generalized solution of the fundamental equation $ax = b$, the division by zero was trivial and clear as $a/0 = 0$ in the **generalized fraction** that is defined by the generalized solution of the equation $ax = b$. Here, the generalized solution is always uniquely determined and the theory is very classical. See [2] for example.

Division by zero is trivial and clear from the concept of repeated subtraction - H. Michiwaki.

Recall the uniqueness theorem by S. Takahasi on the division by zero. See [2, 22].

The simple field structure containing division by zero was established by M. Yamada ([5]). See also Okumura [14] for the simple introduction.

Many applications of the division by zero to Wasan geometry were given by H. Okumura. See [8, 9, 10, 11, 12, 13] for example.

The division by zero opens a new world since Aristotele-Euclid. See the references for recent related results.

For the fundamental function $y = 1/x$ we did not consider any value at the origin $x = 0$, because we did not consider the division by zero $1/0$ in a good way. Many and many people consider its value by the limiting like $+\infty$ and $-\infty$ or the point at infinity as ∞. However, their basic idea comes from **continuity** with the common sense or based on the basic idea of Aristotle. – For the related Greece philosophy, see [23, 24, 25]. However, as the division by zero we will consider its value of the function $y = 1/x$ as zero at $x = 0$. We will see that this new definition is valid widely in mathematics and mathematical sciences, see ([6, 7]) for example. Therefore, the division by zero will give great impacts to calculus, Euclidian geometry, analytic geometry, complex analysis and the theory of differential equations at an undergraduate level and furthermore to our basic ideas for the space and universe.

The division by zero $1/0 = 0$ was discovered on 2014.2.2, however, the result may still not be accepted widely with old and wrong feelings; see the book [21]. Since we gave already logically mathematics on the division by zero, here we will give very good examples in order to see the division by zero $1/0 = 0$ clearly. By these examples, we will be able to understand the division by zero as a trivial one.

2 Examples

As a typical case, we recall

Ctesíbio (BC. 286-222): *We consider a flow tube with some fluid. Then, when we consider some cut with a plane with area* S *and with velocity* v *of* *the fluid on the plane, by continuity, we see that for any cut plane,* $Sv = C$*;* C *: constant. That is,*

$$v = \frac{C}{S}.$$

When S tends to zero, the velocity v tends to infinity. However, for $S = 0$, the flow stops and so, $v = 0$. Therefore, this example shows the division by zero $C/0 = 0$ clearly. Of course, in the situation, we have $0/0 = 0$, trivially.

We can find many and many similar examples, for example, in Archimedes' principle and Pascal's principle.

We will state one more example:

E. Torricelli (1608 -1647): *We consider some water tank and the initial high* $h = h_0$ *for* $t = 0$ *and we assume that from the bottom of the tank with a hole of area* A*, water is fall down. Then, by the law with a constant* k

$$\frac{dh}{dt} = -\frac{k}{A}\sqrt{h},$$

we have the equation

$$h(t) = \left(\sqrt{h_0} - \frac{kt}{2A}\right)^2.$$

Similarly, of course, for $A = 0$, we have

$$h(t) = h_0.$$

By using the Torricelli law, we obtain many similar variations.

Even the fundamental relation among velocity v, time t and distance s

$$t = \frac{s}{v},$$

we will be able to understand the division by zero

$$\frac{s}{0} = 0$$

and

$$\frac{0}{0} = 0.$$

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