Fuzzy Neutrosophic Alpha^m-Closed Sets in Fuzzy NeutrosophicTopological Spaces

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Abstract. In this paper, we state a new class of sets and called them fuzzy neutrosophic Alpha^m-closed sets, and we prove some theorem related to this definition. Then, we investigate the relation between fuzzy neutrosophic Alpha^m-closed sets, fuzzy neutrosophic α closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic semi closed sets and fuzzy neutrosophic pre closed sets. On the other hand, some properties of the fuzzy neutrosophic Alpha^m-closed set are given.

Keywords: fuzzy neutrosophic closed sets, fuzzy neutrosophic Alpha^m-closed sets, fuzzy neutrosophic topology.

1. **Introduction:**

The concept of fuzzy sets was introduced by Zadeh in 1965 [14]. Then the fuzzy set theory are extension by many researchers. The concept of intutionistic fuzzy sets (IFS) was one of the extension sets by K. Atanassov in 1983 [2, 3, 4], when fuzzy set give the digree of membership of an element in the sets, the intuitionistic fuzzy sets give a degree of membership and a degree of non-membership. Then, several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets, one of them was Floretin Smarandache in 2010 [7] when he developed another membership in addition to the two memberships which was defined in intuitionistic fuzzy sets and called it neutrosophic set.

The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In the last year, (2017) Veereswari [13] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

Neutrosophic topological spaces and many applications have been investigated by Salama et al. in [9-12]

In this paper, the concept of Alpha^m-closed sets in double fuzzy topological spaces [6] were developed. We discussed some new class of sets and called them fuzzy neutrosophic Alpha^m-closed sets in fuzzy neutrosophic topological spaces, and we also discussed some new properties and examples based of this defined concept.

2. Basic definitions and terminologies

Definition 2.1 [8]: A neutrosophic topology (NT, for short) on a non-empty set X is a family τ of neutrosophic subsets of X satisfying the following axioms:

i) $\emptyset_N, X_N \in \tau$.

ii) $A_1 \cap A_2 \in \tau$ for any A_1 and $A_2 \in \tau$.

iii) $\cup A_j \in \tau$ for any $\{A_j : j \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS, for short) in X. The elements in τ are called neutrosophic open sets (N-open sets for short) in X. A N-set is said to be neutrosophic closed set (N-closed set, for short) if and only if its complement is a N-open set.

Definition 2.2 [1, 13]: Let X be a non-empty fixed set. A fuzzy neutrosophic set (FNS, for short), λ_N is an object having the form $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ where the functions $\mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N} : X \to [0, 1]$ denote the degree of membership function (namely $\mu_{\lambda N}(x)$), the degree of indeterminacy function (namely $\sigma_{\lambda N}(x)$) and the degree of non-membership function (namely $\nu_{\lambda N}(x)$) respectively, of each set λ_N we have, $0 \leq \mu_{\lambda N}(x) + \sigma_{\lambda}(x) + \nu_{\lambda N}(x) \leq 3$, for each $x \in X$.

Remark 2.3 [13]: FNS $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ can be identified to an ordered triple $<x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ in $[0, 1]$ on X.

Definition 2.4[13]: Let X be a non-empty set and the FNSs λ_N and β_N be in the form:

$\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ and,

$\beta_N = \{< x, \mu_{\beta N}(x), \sigma_{\beta N}(x), \nu_{\beta N}(x) >: x \in X\}$ on X then:

i. $\lambda_N \subseteq \beta_N$ iff $\mu_{\lambda N}(x) \leq \mu_{\beta N}(x)$, $\sigma_{\lambda N}(x) \leq \sigma_{\beta N}(x)$ and $\nu_{\lambda N}(x) \geq \nu_{\beta N}(x)$ for all $x \in X$,

ii. $\lambda_N = \beta_N$ iff $\lambda_N \subseteq \beta_N$ and $\beta_N \subseteq \lambda_N$,

iii. $\underline{1}_N - \lambda_N = \{<x, \nu_{\lambda N}(x), 1 - \sigma_{\lambda N}(x), \mu_{\lambda N}(x) >: x \in X\}$

iv. $\lambda_N \cup \beta_N = \{< x, \mathrm{Max}(\mu_{\lambda N}(x), \mu_{\beta N}(x)), \mathrm{Max}(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), \mathrm{Min}(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,

v. $\lambda_N \cap \beta_N = \{< x, \mathrm{Min}(\mu_{\lambda N}(x), \mu_{\beta N}(x)), \mathrm{Min}(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), \mathrm{Max}(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,

vi. $\underline{0}_N = < x, 0, 0, 1>$ and $\underline{1}_N = <x, 1, 1, 0>$.

Definition 2.5 [13]: A Fuzzy neutrosophic topology (FNT, for short) on a non-empty set X is a family τ_N of fuzzy neutrosophic subsets in X satisfying the following axioms.

i. $\underline{0}_N, \underline{1}_N \in \tau_N$,

ii. $\lambda_{N1} \cap \lambda_{N2} \in \tau_N$ for any $\lambda_{N1}, \lambda_{N2} \in \tau_N$,

iii. $\cup \lambda_{Nj} \in \tau_N$, $\forall \{\lambda_{Nj}: j \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called fuzzy neutrosophic topological space (FNTS, for short). The elements of τ are called fuzzy neutrosophic open sets (F_N-open set, for short). The complement of F_N-open sets in the FNTS (X, τ_N) are called fuzzy neutrosophic closed sets (F_N-closed set, for short).

Definition 2.6 [13]: Let (X, τ_N) be FNTS and $\lambda_N = <x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ be FNS in X. Then, the fuzzy neutrosophic closure of λ_N (FNCl, for short) and fuzzy neutrosophic interior of λ_N (FNInt, for short) are defined by:

$FNCl(\lambda_N) = \cap \{ \beta_N : \beta_N$ is F_N-closed set in X and $\lambda_N \subseteq \beta_N \}$,

$FNInt(\lambda_N) = \cup \{ \beta_N : \beta_N$ is F_N-open set in X and $\beta_N \subseteq \lambda_N \}$.

Note that $FNCl(\lambda_N)$ be F_N-closed set and $FNInt(\lambda_N)$ be F_N-open set in X.

Further,

i. λ_N be F_N-closed set in X iff $FNCl(\lambda_N) = \lambda_N$,

ii. λ_N be F_N-open set in X iff $FNInt(\lambda_N) = \lambda_N$.

Proposition 2.7 [13]: Let (X, τ_N) be FNTS and λ_N, β_N are FNSs in X. Then, the following properties hold:

i. $FNInt(\lambda_N) \subseteq \lambda_N$ and $\lambda_N \subseteq FNCl(\lambda_N)$,

ii. $\lambda_N \subseteq \beta_N \Longrightarrow FNInt(\lambda_N) \subseteq FNInt(\beta_N)$ and $\lambda_N \subseteq \beta_N \Longrightarrow FNCl(\lambda_N) \subseteq FNCl(\beta_N)$,

iii. $Int(FNInt(\lambda_N)) = FNInt(\lambda_N)$ and $FNCl(FNCl(\lambda_N)) = FNCl(\lambda_N)$,

iv. $FNInt(\lambda_N \cap \beta_N) = FNInt(\lambda_N) \cap FNInt(\beta_N)$ and $FNCl(\lambda_N \cup \beta_N) = FNCl(\lambda_N) \cup FNCl(\beta_N)$,

v. $FNInt(\underline{1}_N) = \underline{1}_N$ and $FNCl(\underline{1}_N) = \underline{1}_N$,

vi. $FNInt(\underline{0}_N) = \underline{0}_N$ and $FNCl(\underline{0}_N) = \underline{0}_N$.

Definition 2.8 [2]: FNS λ_N in FNTS (X, τ_N) is called:

i. fuzzy neutrosophic semi-open set (FNS-open, for short) if $\lambda_N \subseteq FNCl(FNInt(\lambda_N)$,

ii. fuzzy neutrosophic semi-closed set (FNS-closed, for short) if $FNInt(FNcl(\lambda_N)) \subseteq \lambda_N$,

iii. fuzzy neutrosophic pre-open set (FNP-open, for short) if $\lambda_N \subseteq FNInt(FNCl(\lambda_N))$.

iv. fuzzy neutrosophic pre-closed set (FNP-closed, for short) if $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N$,

v. fuzzy neutrosophic α-open set (FNα-open, for short) if $\lambda_N \subseteq FNInt(FNCl(FNInt(\lambda_N)))$,

vi. fuzzy neutrosophic α-closed set (FNα-closed, for short) if $FNCl(FNInt(FNCl(\lambda_N))) \subseteq \lambda_N$.

3. Fuzzy Neutrosophic Alpha^m - Closed Sets in Fuzzy Neutrosophic Topological Spaces.

Now, the concept of fuzzy neutrosophic Alpha^m-closed set in fuzzy neutrosophic topological space is introduced, as follows:

Definition 3.1: Fuzzy neutrosophic subset λ_N of FNTS (X, τ_N) is called fuzzy neutrosophic Alpha^m-closed set (FNα^m- closed set, for short) if $FNint(FNcl(\lambda_N)) \subseteq U_N$, wherever $\lambda_N \subseteq U_N$ and U_N be FNα-open set. And λ_N is said to be fuzzy neutrosophic Alpha^m-open set (FNα^m-open set, for short) in (X, τ_N) if the complement $\underline{1}_N$-λ_N be FNα^m-closed set in (X, τ_N).

Proposition 3.2: For any FNS, the following statements satisfy:

i. Every F_N-open set is FNα-open set.

ii. Every FNα-closed set is FNα^m-closed set.

iii. Every F_N-closed set is FNα^m-closed set.

iv. Every FNS-closed set is FNα^m-closed set.

Proof:

i. Let λ_N ={<x, $\mu_{\lambda N}$(x), $\sigma_{\lambda N}$ (x), $\nu_{\lambda N}$ (x) >: x∈ X} be F_N-open set in FNTS (X, τ_N).Then, by **Definition 2.6 (ii)** we get,

λ_N = FNInt(λ_N)……(1)

And, by **Proposition 2.7 (i)** we get, $\lambda_N \subseteq$ FNCl(λ_N). But, $\lambda_N \subseteq$ FNCl(FNInt(λ_N)).

Then, FNInt(λ_N) $\subseteq$ FNInt(FNCl(FNInt(λ_N))).

Therefore, by **(1)** we get, $\lambda_N \subseteq$ FNInt(FNCl(FNInt(λ_N)))**.** Hence, λ_N be FNα-open set in (X, τ_N).

ii. Let λ_N ={<x, $\mu_{\lambda N}$(x), $\sigma_{\lambda N}$(x), $\nu_{\lambda N}$(x) >: x∈ X } be FNα-closed set in FNTS (X, τ_N).

Then, FNCl(FNInt(FNCl(λ_N))) $\subseteq \lambda_N$.

Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$.

Since, β_N be F_N-open set then, is FNα-open set by **(i)**.Then, FNInt(FNCl(λ_N)) $\subseteq$ FNInt(λ_N) $\subseteq \lambda_N \subseteq \beta_N$.

Therefore, FNInt(FNCl(λ_N)) $\subseteq \beta_N$.

Hence, λ_N be FNα^m- closed set in (X, τ_N).

iii. Let λ_N = {< x, $\mu_{\lambda N}$(x), $\sigma_{\lambda N}$(x), $\nu_{\lambda N}$(x) >: x∈ X } is F_N-closed set in FNTS (X, τ_N).Then, by **Definition 2.6 (i)**. We get, λ_N = FNcl(λ_N)**.... (1)**.

And, by **Proposition 2.7 (i)**. We get, FNint(λ_N) $\subseteq \lambda_N$**……(2)**. But,

FNint(FNcl(λ_N)) $\subseteq$ FNcl(λ_N).Then, by **(1)**. We get,

FNint(FNcl(λ_N) $\subseteq \lambda_N$. Now, let β_N is F_N-open set such that, $\lambda_N \subseteq U_N$. By, **Proposition 3.2 (i)**. If, β_N is F_N-open set. Then, is FNα-open set. Then, FNint(FNcl(λ_N) $\subseteq \lambda_N \subseteq \beta_N$. Therefore, FNint(FNcl($\lambda_N$)) $\subseteq \beta_N$.

Hence, λ_N is FNα^m-closed set in (X, τ_N).

iv. Let λ_N ={<x, $\mu_{\lambda N}$(x), $\sigma_{\lambda N}$(x), $\nu_{\lambda N}$(x) >:x∈ X } be FNS-closed set in FNTS (X, τ_N).

Then, FNInt(FNCl(λ_N)) $\subseteq \lambda_N$.

Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$

Since, β_N be F_N-open set then, is FNα-open set, by **(i)**

Then, FNInt(FNCl (λ_N)) $\subseteq \lambda_N \subseteq \beta_N$.

Therefore, FNInt(FNCl (λ_N)) $\subseteq \beta_N$.

Hence, λ_N be FNα^m- closed set in (X, τ_N).

Remark 3.3: The converse of **Proposition 3.2** is not true in general and we can show it by the following examples:

Example 3.4:

i. Let X={x} define FNSs λ_N and β_N in X as follows:

λ_N ={<x, 0.7, 0.6, 0.5>: x∈ X }, β_N ={<x, 0.8, 0.9, 0.4>: x∈ X }.

And the family τ_N = {$\underline{0}_N$, $\underline{1}_N$, λ_N, β_N } be FNT such that, $\underline{1}_N$-τ_N ={1_N, 0_N, <x, 0.5, 0.4, 0.7>, <x, 0.4, 0.1, 0.8>}. Now if ,

$\omega_N = \{<x, 0.8, 0.6, 0.5>: x \in X\}$.

Then, FNInt(ω_N) $=\{<x, 0.7, 0.6, 0.5>: x \in X\}$, FNCl(FNInt($\omega_N$)) $= \underline{1}_N$ and FNInt(FNCl(FNInt(ω_N))) $= \underline{1}_N$.

Therefore, $\omega_N \subseteq$ FNInt(FNCl(FNInt(ω_N))).

Hence, ω_N be FNα-open set. But, not F_N-open set.

ii. Let X={x} define FNSs λ_N, β_N, η_N and Ψ_N in X as follows:

$\lambda_N = \{<x, 1, 0.5, 0.7>: x \in X\}$, $\beta_N = \{<x, 0, 0.9, 0.2>: x \in X\}$,

$\eta_N = \{<x, 1, 0.9, 0.2>: x \in X\}$ and $\Psi_N = \{<x, 0, 0.5, 0.7>: x \in X\}$

And the family $\tau_N = \{\underline{0}_N, \underline{1}_N, \lambda_N, \beta_N, \eta_N, \Psi_N\}$ be FNT.such that, $\underline{1}_N$-$\tau_N = \{\underline{1}_N, \underline{0}_N, <x, 0.7, 0.5, 1>, <x, 0.2, 0.1, 0>, <x, 0.2, 0.1, 1>, <x, 0.7, 0.5, 0>\}$

Now if, $\omega_N = \{<x, 0, 0.4, 0.8>: x \in X\}$ and $U_N = \{<x, 0, 0.5, 0.7>: x \in X\}$.

Where, U_N be F_N-open set such that, $\omega_N \subseteq U_N$.

Since, U_N be F_N-open set then, is FNα-open set by **Proposition 3.2 (i)**.

Then, FNCl(ω_N) $=\{<x, 0.7, 0.5, 0>: x \in X\}$ and FNInt(FNCl(ω_N)) $=\{<x, 0, 0.5, 0.7>: x \in X\}$.

Therefore, FNInt(FNCl(ω_N)) $\subseteq U_N$.

Hence, ω_N be FNα^m- closed set.

But, FNCl(ω_N) $=\{<x, 0.7, 0.5, 0>: x \in X\}$,

FNInt(FNCl(ω_N)) $=\{<x, 0, 0.5, 0.7>: x \in X\}$ and

FNCl(FNInt(FNCl(ω_N))) $=\{<x, 0.7, 0.5, 0>: x \in X\}$

Therefore, FNCl(FNInt(FNCl(ω_N))) $\not\subseteq \omega_N$. Hence, ω_N be not FNα- closed set.

iii. Take, the example which defined in **ii**. Then, we can see ω_N be FNα^m-closed set. But, not F_N-closed set.

Take again, the example which defined in **ii**. Then, ω_N be FNα^m-closed set. But, not FNS-closed set.

Remark 3.5: The relation between FNP-closed sets and FNα^m- closed sets are independent and we can show it by the following examples.

Example 3.6: (1) Let X={x} define FNSs λ_N and β_N in X as follows:

$\lambda_N = \{<x, 0.1, 0.2, 0.4>: x \in X\}$, $\beta_N = \{<x, 0.7, 0.5, 0.2>: x \in X\}$,

And, the family $\tau_N = \{\underline{0}_N, \underline{1}_N, \lambda_N, \beta_N\}$ be FNT such that, $\underline{1}_N$-$\tau_N = \{\underline{1}_N, \underline{0}_N, <x, 0.4, 0.8, 0.1>, <x, 0.2, 0.5, 0.7>\}$.

Now if, $\omega_N = \{<x, 0.1, 0.3, 0.4>: x \in X\}$ and

$U_N = \{<x, 0.7, 0.5, 0.2>: x \in X\}$ where, U_N be F_N-open set such that, $\omega_N \subseteq U_N$.

Since, U_N be F_N-open set then, is FNα-open set by **Proposition 3.2 (i)**

Then, FNCl (ω_N) $=\{<x, 0.4, 0.8, 0.1>: x \in X\}$ and

FNInt(FNCl(ω_N)) $=\{<x, 0.1, 0.2, 0.4>: x \in X\}$. Therefore, FNInt(FNCl($\omega_N$)) $\subseteq U_N$.

Hence, ω_N be FNα^m- closed set.

But, FNInt(ω_N) $=\{<x, 0.1, 0.2, 0.4>: x \in X\}$,

FNCl(FNInt (ω_N)) $= \{<x, 0.4, 0.8, 0.1>: x \in X\}$. Therefore, FNCl(FNInt (ω_N)) $\not\subseteq \omega_N$.

Hence, ω_N be not FNP-closed set.

(2) Let X={a, b} define FNS λ_N in X as follows:

$\lambda_N = <x, (a\backslash 0.5, b\backslash 0.5), (a\backslash 0.5, b\backslash 0.5), (a\backslash 0.4, b\backslash 0.5)>$. And the family $\tau_N = \{\underline{0}_N, \underline{1}_N, \lambda_N\}$ be FNT.

Such that, $\underline{1}_N$-τ_N = {$\underline{1}_N$, $\underline{0}_N$, <x, , (a\0.4, b\0.5), (a\0.5, b\0.5) , (a\0.5, b\0.5) >}.

Now if, ω_N = <x, (a\0.5, b\0.4) , (a\0.5, b\0.5) , (a\0.6, b\0.5) > And, $U_N = \lambda_N$ be F_N-open set such that, $\omega_N \subseteq U_N$.

Since, U_N be F_N-open set then, is FNα-open set by **Proposition 3.2 (i)**

Then, FNInt (ω_N) = $\underline{0}_N$ and FNCl(FNInt(ω_N)) = $\underline{0}_N$.

Therefore, FNCl(FNInt (ω_N)) $\subseteq \omega_N$. Hence, ω_N be FNP-closed set.

But, FNCl (ω_N) = $\underline{1}_N$ and FNInt(FNCl(ω_N)) = $\underline{1}_N$.

Therefore, FNInt(FNCl(ω_N)) $\not\subseteq U_N$. Hence, ω_N be not FNα^m- closed set.

Proposition 3.7: If λ_N be FNα^m- closed set and $\lambda_N \subseteq \eta_N \subseteq$ FNInt(FNCl(λ_N)), Then, η_N be FNα^m- closed set.

Proof: Let λ_N={<x, $\mu_{\lambda N}$ (x), $\sigma_{\lambda N}$ (x), $\nu_{\lambda N}$ (x) >: x$\in$ X} be FNα^m- closed set such that,$\lambda_N \subseteq \eta_N \subseteq$ FNInt(FNCl(λ_N)).

Now let β_N be FNα- open set such that, $\eta_N \subseteq \beta_N$.

Since, λ_N be FNα^m-closed set then, we have

FNInt(FNCl(λ_N)) $\subseteq \beta_N$, where $\lambda_N \subseteq \beta_N$.

Since, $\lambda_N \subseteq \eta_N$ and $\eta_N \subseteq$ FNInt(FNCl(λ_N)) we get,

FNInt(FNCl(η_N)) $\subseteq$ FNInt(FNCl(FNInt(FNCl(λ_N)))) $\subseteq$ FNInt(FNCl(λ_N)) $\subseteq \beta_N$.

Therefore, FNInt(FNCl(η_N)) $\subseteq \beta_N$. Hence, η_N be FNα^m- closed set in (X, τ_N).

Proposition 3.8: Let (X, τ_N) be FNTS. So, the intersection of two FNα^m-closed sets be FNα^m-closed set.

Proof: Let λ_N and β_N are FNS-closed sets on FNTS (X, τ_N)

Then, FNInt(FNCl(λ_N)) $\subseteq \lambda_N$……(1)

And, FNInt(FNCl(β_N)) $\subseteq \beta_N$……(2)

Consider $\lambda_N \cap \beta_N \supseteq$ FNInt(FNCl(λ_N)) $\cap$ FNInt(FNCl(β_N))

$=$ FNInt(FNCl(λ_N) $\cap$ FNCl(β_N))

$\supseteq$ FNInt(FNCl($\lambda_N \cap \beta_N$)).

Therefore, FNInt(FNCl($\lambda_N \cap \beta_N$)) $\subseteq \lambda_N \cap \beta_N$

Now, let η_N be F_N-open set such that, $\lambda_N \cap \beta_N \subseteq \eta_N$.

Since, η_N be F_N-open set then it is FNα-open set, by **Proposition 3.2 (i).**

Then, FNInt(FNCl($\lambda_N \cap \beta_N$)) $\subseteq \lambda_N \cap \beta_N \subseteq \eta_N$.

Therefore, FNInt(FNCl($\lambda_N \cap \beta_N$)) $\subseteq \eta_N$. Hence, $\lambda_N \cap \beta_N$ be FNα^m-closed set in (X, τ_N).

Remark 3.9: The union of any FNα^m-closed sets is not necessary to be FNα^m-closed set and we can show it by the following example.

Example 3.10: Take, **Example 3.4 (ii)** if,

ω_{N1} ={<x, 0.4,0.5, 1>: x$\in$ X } and ω_{N2} ={<x, 0.2, 0, 0.8>: x$\in$ X}. And U_N ={<x, 1,0.5, 0.7>: x$\in$ X }

Then, FNCl(ω_{N1}) ={<x, 0.7, 0.5,1>: x$\in$ X } and FNInt(FNCl(ω_{N1}) = $\underline{0}_N$.

Therefore, FNInt(FNCl(ω_{N1}) $\subseteq U_N$.

Hence, ω_{N1} be FNα^m- closed set.

And, $FNCl(\omega_{N2}) = \{<x, 0.2, 0.1, 0>: x \in X\}$, $FNInt(FNCl(\omega_{N2})) = \underline{0}_N$.

Therefore, $FNInt(FNCl(\omega_{N2})) \subseteq U_N$. Hence, ω_{N2} be $FN\alpha^m$- closed set.

Therefore, $\omega_{N1} \cup \omega_{N2}$ be not $FN\alpha^m$- closed set.

Definition 3.11: Let (X, τ_N) be FNTS and $\lambda_N = <x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>$ be FNS in X. Then, the fuzzy neutrosophic Alpha^m closure of λ_N ($FN\alpha^mCl$, for short) and fuzzy neutrosophic Alpha^m interior of λ_N ($FN\alpha^mInt$, for short) are defined by:

i. $FN\alpha^mCl(\lambda_N) = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\}$,

ii. $FN\alpha^mInt(\lambda_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\}$.

Proposition 3.12: Let (X, τ_N) be FNTS and λ_N, β_N are FNSs in X. Then the following properties hold:

- **i.** $FN\alpha^mCl(\underline{0}_N) = \underline{0}_N$ and $FN\alpha^mCl(\underline{1}_N) = \underline{1}_N$,
- **ii.** $\lambda_N \subseteq FN\alpha^mCl(\lambda_N)$,
- **iii.** If $\lambda_N \subseteq \beta_N$, then $FN\alpha^mCl(\lambda_N) \subseteq FN\alpha^mCl(\beta_N)$,
- **iv.** λ_N be $FN\alpha^m$-closed set iff $\lambda_N = FN\alpha^mCl(\lambda_N)$,
- **v.** $FN\alpha^mCl(\lambda_N) = FN\alpha^mCl(FN\alpha^mCl(\lambda_N))$.

Proof:

i. by **Definition 3.11 (i)** we get,

$FN\alpha^mCl(\underline{0}_N) = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\underline{0}_N \subseteq \beta_N\} = \underline{0}_N$.

And,

$FN\alpha^mCl(\underline{1}_N) = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\underline{1}_N \subseteq \beta_N\} = \underline{1}_N$.

ii. $\lambda_N \subseteq \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\} = FN\alpha^mCl(\lambda_N)$.

iii. Suppose that $\lambda_N \subseteq \beta_N$ then,

$\cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\} \subseteq \cap\{\eta_N: \eta_N$ is $FN\alpha^m$-closed set in X and $\beta_N \subseteq \eta_N\}$. Therefore, $FN\alpha^mCl(\lambda_N) \subseteq FN\alpha^mCl(\beta_N)$.

iv. If, λ_N be $FN\alpha^m$-closed set, then

$FN\alpha^mCl(\lambda_N) = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\}$……(1)

And, by **(ii)** we get, $\lambda_N \subseteq FN\alpha^mCl(\lambda_N)$……(2) but, λ_N is necessarily to be the smallest set.

Thus, $\lambda_N = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\}$,

Therefore, $\lambda_N = FN\alpha^mCl(\lambda_N)$.

Conversely; Let $\lambda_N = FN\alpha^mCl(\lambda_N)$ by using **Definition 3.11 (i)**,we get, λ_N be $FN\alpha^m$-closed set.

v. Since, by **(iv)** we get, $\lambda_N = FN\alpha^mCl(\lambda_N)$

Then, $FN\alpha^mCl(\lambda_N) = FN\alpha^mCl(FN\alpha^mCl(\lambda_N))$.

Proposition 3.13: Let (X, τ_N) be FNTS and λ_N, β_N are FNSs in X. Then the following properties hold:

- **i.** $FN\alpha^mInt(\underline{0}_N) = \underline{0}_N$ and $FN\alpha^mInt(\underline{1}_N) = \underline{1}_N$,
- **ii.** $FN\alpha^mInt(\lambda_N) \subseteq \lambda_N$,
- **iii.** If $\lambda_N \subseteq \beta_N$, then $FN\alpha^mInt(\lambda_N) \subseteq FN\alpha^mInt(\beta_N)$,
- **iv.** λ_N be $FN\alpha^m$-open set iff $\lambda_N = FN\alpha^mInt(\lambda_N)$,

v. $FN\alpha^m Int(\lambda_N) = FN\alpha^m Int(FN\alpha^m Int(\lambda_N))$.

Proof:

i. by **Definition 3.11 (ii)** we get,

$FN\alpha^m Int(\underline{0}_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \underline{0}_N\} = \underline{0}_N$,

And, $FN\alpha^m Int(\underline{1}_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \underline{1}_N\} = \underline{1}_N$.

ii. Follows from **Definition 3.11 (ii)**.

iii. $FN\alpha^m Int(\lambda_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\}$.Since, $\lambda_N \subseteq \beta_N$ then,

$\cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\} \subseteq \cup\{\eta_N: \eta_N$ is $FN\alpha^m$-open set in X and $\eta_N \subseteq \beta_N\}$

Therefore, $FN\alpha^m Int(\lambda_N) \subseteq FN\alpha^m Int(\beta_N)$.

iv. We must proof that, $FN\alpha^m Int(\lambda_N) \subseteq \lambda_N$ and $\lambda_N \subseteq FN\alpha^m Int(\lambda_N)$.

Suppose that λ_N be $FN\alpha^m$-open set in X.

Then, $FN\alpha^m Int(\lambda_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\}$.

by using **(ii)** we get, $FN\alpha^m Int(\lambda_N) \subseteq \lambda_N$….(1)

Now to proof, $\lambda_N \subseteq FN\alpha^m Int(\lambda_N)$, we have, For all $\lambda_N \subseteq \lambda_N$, the $FN\alpha^m Int(\lambda_N) \subseteq \lambda_N$ So, we get $\lambda_N \subseteq \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\} = FN\alpha^m Int(\lambda_N)$ ……(2)

From (1) and (2) we have, $\lambda_N = FN\alpha^m Int(\lambda_N)$.

Conversely; assume that $\lambda_N = FN\alpha^m Int(\lambda_N)$ and by using **Definition 3.11 (ii)** we get, λ_N be $FN\alpha^m$-open set in X.

v. By **(iv)** we get, $\lambda_N = FN\alpha^m Int(\lambda_N)$

Then, $FN\alpha^m Int(\lambda_N) = FN\alpha^m Int(FN\alpha^m Int(\lambda_N))$.

Proposition 3.14: Let (X, τ_N) be FNTS. Then, for any fuzzy neutrosophic subsets λ_N of X.

i. $\underline{1}_N - (FN\alpha^m Int(\lambda_N)) = FN\alpha^m Cl(\underline{1}_N - \lambda_N)$,

ii. $\underline{1}_N - (FN\alpha^m Cl(\lambda_N)) = FN\alpha^m Int(\underline{1}_N - \lambda_N)$.

Proof:

i. $FN\alpha^m Int(\lambda_N) = \cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\}$, by the complement we get,

$\underline{1}_N - (FN\alpha^m Int(\lambda_N)) = \underline{1}_N - (\cup\{\beta_N: \beta_N$ is $FN\alpha^m$-open set in X and $\beta_N \subseteq \lambda_N\})$.

So, $\underline{1}_N - (FN\alpha^m Int(\lambda_N)) = \cap\{(\underline{1}_N - \beta_N)$:

$(\underline{1}_N - \beta_N)$ is $FN\alpha^m$-closed set in X and $(\underline{1}_N - \lambda_N) \subseteq (\underline{1}_N - \beta_N)\}$.

Now, replacing $(\underline{1}_N - \beta_N)$ by η_N we get,

$\underline{1}_N - (FN\alpha^m Int(\lambda_N)) = \cap\{\eta_N: \eta_N$ is $FN\alpha^m$-closed set in X and $(\underline{1}_N - \lambda_N) \subseteq \eta_N\} = FN\alpha^m Cl(\underline{1}_N - \lambda_N)$.

ii. $FN\alpha^m Cl(\lambda_N) = \cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\}$, by the complement we get,

$\underline{1}_N - (FN\alpha^m Cl(\lambda_N)) = \underline{1}_N - (\cap\{\beta_N: \beta_N$ is $FN\alpha^m$-closed set in X and $\lambda_N \subseteq \beta_N\})$.

So, $\underline{1}_N - (FN\alpha^m Cl(\lambda_N)) = \cup\{(\underline{1}_N - \beta_N): (\underline{1}_N - \beta_N)$ is $FN\alpha^m$-open set in X and

$(\underline{1}_N - \beta_N) \subseteq (\underline{1}_N - \lambda_N)\}$. Again replacing $(\underline{1}_N - \beta_N)$ by η_N we get,

$\underline{1}_N - (FN\alpha^m Cl(\lambda_N)) = \cup\{\eta_N: \eta_N$ is $FN\alpha^m$-open set in X and $\eta_N \subseteq (\underline{1}_N - \lambda_N)\} = FN\alpha^m int(\underline{1}_N - \lambda_N)$.

Proposition 3.15: Fuzzy neutrosophic interior of F_N-closed set be $FN\alpha^m$-closed set.

proof: Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ be F_N-closed set in FNTS (X, τ_N).Then, by **Definition 2.6 (i)** we get, $\lambda_N = FNCl(\lambda_N)$**.** So, $FNInt(\lambda_N) = FNInt(FNCl(\lambda_N))$……(1),

And, by **Proposition 2.7 (i)** we get,

$FNInt(\lambda_N) \subseteq \lambda_N$ ……(2)

From (1) and (2) we get, $FNInt(FNCl(\lambda_N)) \subseteq \lambda_N$

Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$.

Since, β_N be F_N-open set, then β_N is FNα-open set by **Proposition 3.2 (i)**

Therefore, $FNInt(FNCl(\lambda_N)) \subseteq \beta_N$. Hence, λ_N be $FN\alpha^m$-closed set in (X, τ_N).

Remark 3.16: The relationship between different sets in FNTS can be showing in the next diagram and the converse is not true in general.

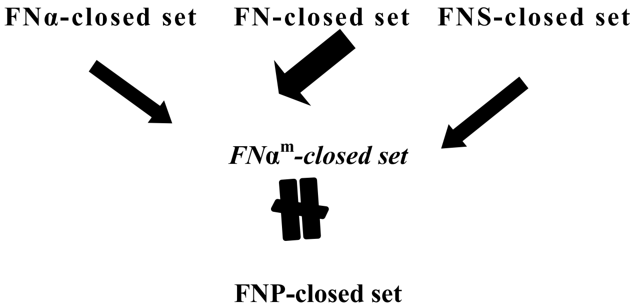

Diagram 1

Conclusion

In this paper, the new concept of a new class of sets and called them fuzzy neutrosophic Alpha^m-closed sets. we investigated the relation between fuzzy neutrosophic Alpha^m-closed sets, fuzzy neutrosophic α closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic semi closed sets and fuzzy neutrosophic pre closed sets with some properties.

References

[1] I. Arockiarani and J.Martina Jency: More on Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Topological Spaces, IJIRS, vol. 3, 2014, pp. 642-652.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala: On Some Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and System,vol. 16, 2017, pp. 16-19.

[3] K. Atanassov and S. Stoeva, Intuitionistic Fuzzy Sets, in: Polish Syrup. On Interval & Fuzzy Mathematics, Poznan, 1983, pp. 23-26.

[4] K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems,vol. 20, 1986), pp. 87-96.

[5] K. Atanassov: Review and New Results on IntuitionisticFuzzy Sets, Preprint IM- MFAIS, Sofia, 1988, pp. 1-88.

[6] Fatimah M. Mohammed, Sanna I. Abdullah and Safa, H. Obaid: (p,q)-Fuzzy α^m-Closed Sets in Double Fuzzy Topological Spaces, Diyala ournal for Pure Sciences, vol 14, Iss.1, 2018, pp.109-127.

[7] Floretin Smaradache, Neutrosophic Set: A Generalization of Intuitionistic Fuzzy Set, Journal of Defense Resourses Management, vol. 1, 2010, pp. 1-10.

[8] A. A. Salama and S. A. Alblowi: Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Mathematics, ISSN:2278-5728,Vol. 3, Iss. 4, 2012,pp. 31-35.

[9] A. A. Salama: Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology, Neutrosophic Sets and Systems, vol. 7, 2015, pp. 18-22.

[10] A.A. Salama, Florentin Smarandache, S. A. Alblowi: The Characteristic Function of a Neutrosophic Set, Neutrosophic Sets and Systems, vol. 3, 2014, pp. 14-17.

[11] A.A. Salama, Florentin Smarandache, Valeri Kromov: Neutrosophic Closed Set and Neutrosophic Continuous Functions, Neutrosophic Sets and Systems, vol. 4, 2014, pp. 4-8.

[12] A. A. Salama, Florentin Smarandache: Neutrosophic Crisp Set Theory, Neutrosophic Sets and Systems, vol. 5, 2014, pp. 27-35.

[13]Y.Veereswari:An Introduction To Fuzzy Neutrosophic Topological Spaces, IJMA,Vol.8(3), (2017), 144-149.

[14] L. Zadeh, Fuzzy Sets, Inform. and Control, vol.8, 1965, pp.338-353.

[15] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1- 22.

[16] Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, *86*, 12-29.

[17] Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems, 33(6), 4055-4066.

[18] Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. Symmetry 2018, 10, 116.

[19] Abdel-Basset, Mohamed, et al. "A novel group decision-making model based on triangular neutrosophic numbers." Soft Computing (2017): 1-15. DOI: https://doi.org/10.1007/s00500-017-2758-5

[20] Abdel-Baset, Mohamed, Ibrahim M. Hezam, and Florentin Smarandache. "Neutrosophic goal programming." Neutrosophic Sets Syst 11 (2016): 112-118.

[21] El-Hefenawy, Nancy, et al. "A review on the applications of neutrosophic sets." Journal of Computational and Theoretical Nanoscience 13.1 (2016): 936-944.

[22] Arindam Dey, Said Broumi, Le Hoang Son, Assia Bakali, Mohamed Talea, Florentin Smarandache, "A new algorithm for finding minimum spanning trees with undirected neutrosophic graphs", Granular Computing (Springer), pp.1-7, 2018

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