

# “Rydberg Polarons, Instanton tunnel effect and graphene spatial conformation like the quantum mirror reflex of Walter Christaller hexagonal localization by The Central Place Theory and Pseudohexagonal biotite of granodiorite type BRNO KRÁLOVO POLE”

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**Abstract:** I report a spectroscopic observation of Rydberg polarons in an atomic Bose gas. Polarons are created by excitation of Rydberg atoms as impurities in a strontium Bose–Einstein condensate. First experiments are described on the impact of additive noise on the ionization of Rb Rydberg atoms in microwave fields. Dynamical localization and its gradual destruction by noise are observed. The atoms surviving in Rydberg states are detected by ionization in a static electric field. First, we count the number of atoms initially laser excited to Rydberg states. The ionization signal includes atoms excited to the continuum as well as atoms which are excited to principal quantum numbers higher than  $n_e$ . Instanton tunnel effect is theoretically based on pseudoparticle Instanton, known like a classical solution of kinematic equations in classical theory of fields in Euclidean Spacetime. Typically events of quantum mechanics is condition of a quantum interference. Graphene is the most favourite material model with simply hexagonal structure and simply chemical composition, it’s the clear mono–layer of hexagonal polygons of carboneum with kovalent bonds and electron–electron gas. The Walter Christaller’s Theory of Central Places (hexagonal settlements structure) is the most illustrative sight to express the graphene chemical bond structure in macro–dimensions. It’s possibly to say that graphene has a structure of microcosmos, and Christaller’s Theory of Central Places is a mirror reflex of this microcosmos to extradimensional MACROCOSMOS (STRUCTURE OF HUMAN CONURBATION, CITIES, TOWNS, VILLAGES AND THEIR CONNECTIONS LIKE A NANO–CHEMICAL KOVALENT BONDS OF CARBONEUM / GRAPHENE). In the ending part of this text, the Author dedicated attention to a mafic silicar mineral

biotite with its typical hexagonal space–structure like a space–point super density of hexagonal layers of this rock–forming mineral.

**Keywords:** Rydberg polaron, Bose–Einstein Condensate, Fröhlich Hamiltonian, Graphene, Instanton tunnel effect, Walter Christaller Central Place Theory, bosons, excitation, quantum interference, spin, bosonisation, Hall’s effect, Rydberg impurity, Rydberg electron wave function, Biotite.

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## 1. Introduction

The interaction of an impurity with a deformable medium can lead to a collective response and formation of quasiparticles known as polarons, dressed by excitations of the background medium.

Polarons play important roles in conduction in ionic crystals and polar semiconductors, optical absorption of two–dimensional materials, spin current transport in organic semiconductors, and collective excitations in strongly interacting fermionic and bosonic ultracold gases.

Spectroscopic observation of Rydberg polarons formed through excitation of Sr Rydberg atoms in a Strontium Bose–Einstein Condensate (BEC), which represent a new class of impurity states beyond those typically seen in condensed matter settings. Rydberg polarons are distinguished by macroscopic occupation of bound molecular states that arise from interaction of ground–state atoms with the Rydberg electron.

## 2. Highlights

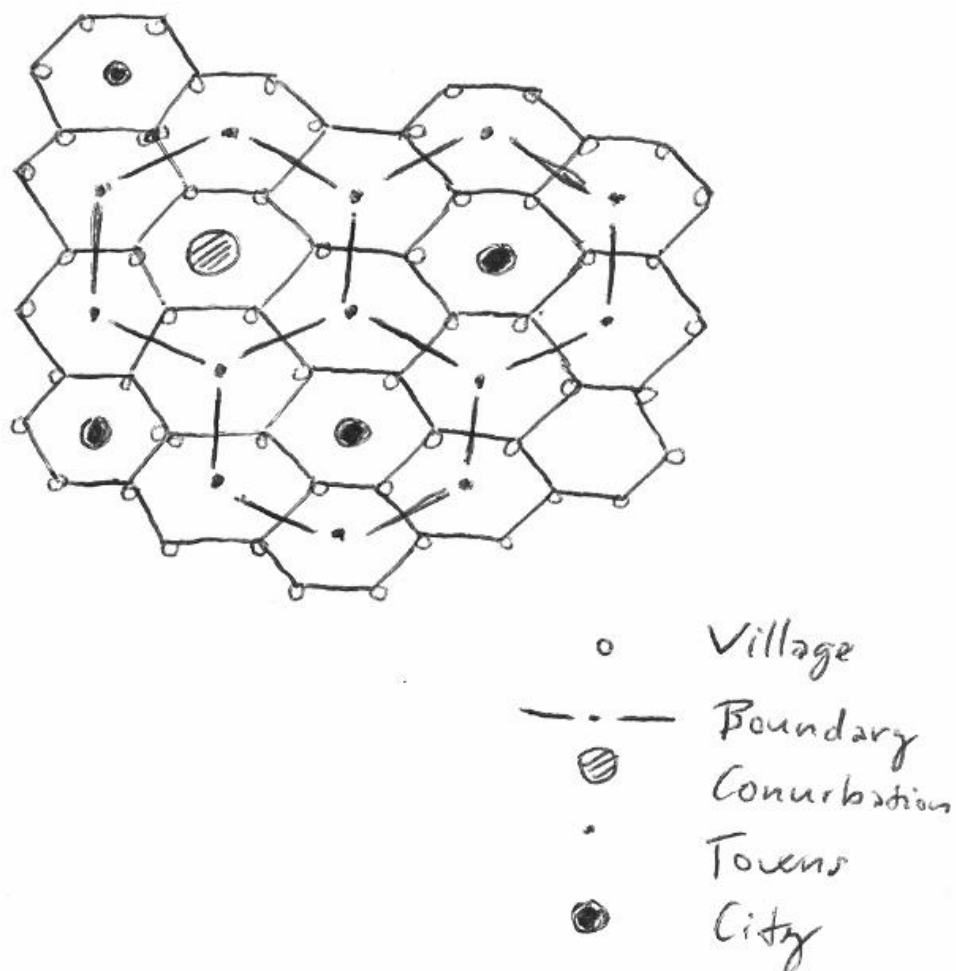


Fig. 1. Central Place Theory (2-layers graphene) respectively 2-layers concept of graphene composite with chemical kovalent bond, geographically settlement connection. Author of the sketch: Mgr. Imrich KRIŠTOF.

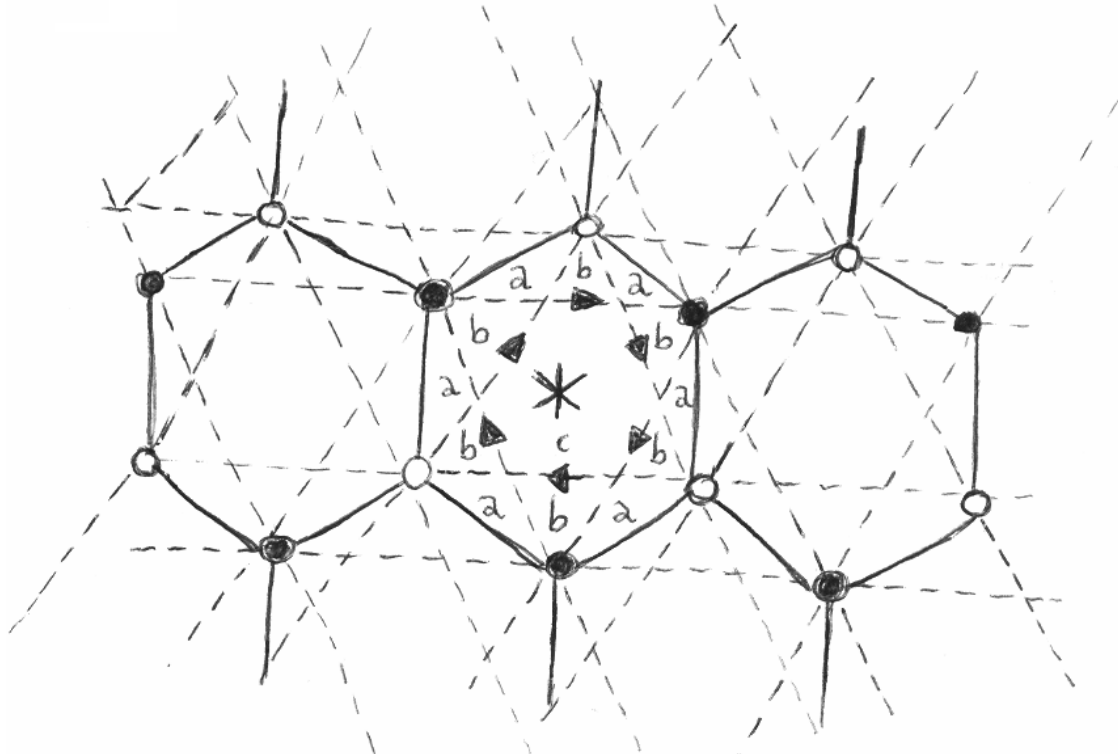


Fig. 2. Simple, graphene similar model with tight bond for topological isolator with broken symmetry to the time inversion resp. Chern isolator, which is typical with "quantum anomalous Hall's effect" in zero field. Here, a,b and c regions with magnetic fluxes in elementary cell. Author of the sketch: Mgr. I. KRIŠTOF.

### 3. Concretely Comments to the Study Problematic and the Figures to This Theme

Effects of impurity recoil, solves an extended Fröhlich Hamiltonian for an impurity in a Bose gas and accurately reproduces the observed excitation spectrum.

Comment 1: Johannes Rydberg  
Swedish physicist,  
8.11.1854, Halmstad, Sweden,  
28.12.1919 Lund, Sweden.

#### Rydberg constant

symbol  $R_\infty$  for heavy atoms or  $R_H$  for hydrogen – is a physical constant relating to atomic spectra, in the science of spectroscopy. (Rydberg formula for the hydrogen spectral series, but Niels Bohr later showed that its value could be calculated from more fundamental constants, explaining the relationship via his "Bohr model". The Rydberg constant represents the limiting value of the highest wavenumber (the inverse wavelength).

#### A Rydberg atom

is an excited atom with one or more electrons that have a very high principal quantum number.

Rydberg electron

An electron–electron repulsion term must be included in the atomic Hamiltonian.

A Rydberg Polaron

is an exotic state of matter, created at low temperatures, in which a very large atom contains other ordinary atoms in space between the nucleus and the electrons.

For the formation of this atom scientists had to combine two fields of atomic physics: Bose–Einstein condensates and Rydberg atoms.

Comment 2: Walter Christaller

(21.4.1893 – 9.3.1969)

a German–Swiss geographer, first published in 1933. This groundbreaking theory was the foundation of the study of cities, rather than simple hierarchies or simple entities. He worked for German SS–Planing and Soil Office Geography of Germany's eastern Conquest (Generalplan Ost) primarily in Czechoslovakia and Poland.

After the war he joined the Communist Party of Germany and became politically active. He was a pioneer scientist of Geography of Towns, and Tourism.

Biotite is one of the most basic rock–forming mineral, from group of phyllosilicates typical of itself mafic colour / black, brown, red–brown, green and rarely colorless.

Biotite has outstanding fissionability according basal flat (001).

The most common accessories are TITAN (Ti) and NATRIUM (Na). Chemical formula of biotite:

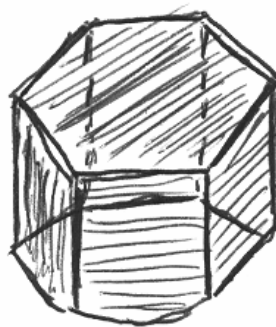
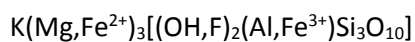
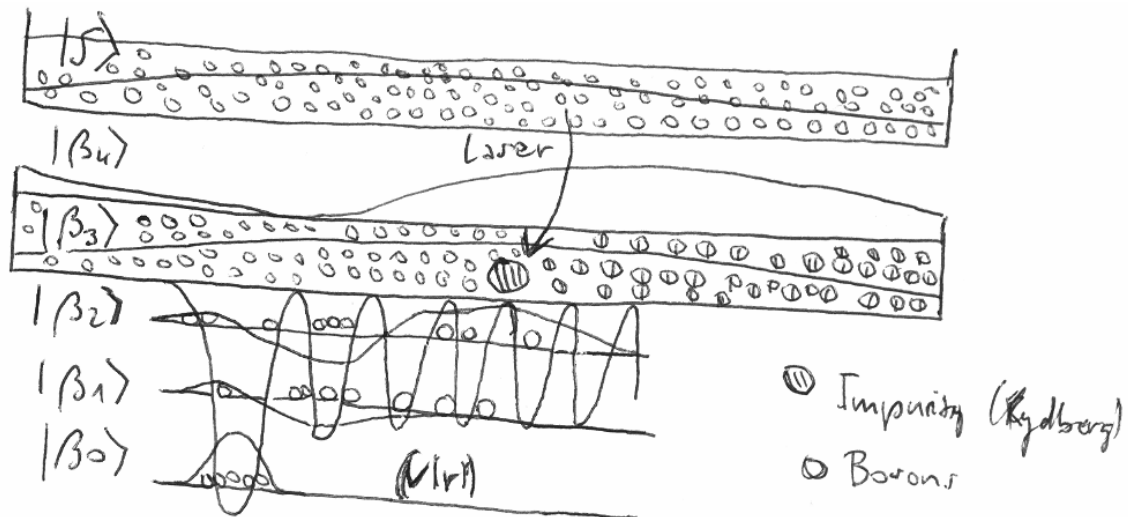


Fig. 3. Sketch of pseudo-hexagonal crystal of BIOTITE (dark mica) of isomorph line of aluminium silicate mineral flogopit–annit from relic locality Brno – Královo Pole (Brno's granodiorite). This type of crystal lattice is significant for hexagonal mineral BERYL. Author of the sketch: Mgr. I. KRIŠTOF.



*Author of the sketch: Mgr. J. KRISTOF*

Fig. 4. The formation of Rydberg and typical condensed-matter polarons. Schematic of the excitation of a Rydberg polaron in a uniform-density BEC. Laser excitation projects the system into final configurations involving atoms in bound and scattering states  $|\beta_i\rangle$ . Bound states are confined in the Rydberg potential  $V(r)$ .

Comment 3: Instanton – “tunnel events” (resp. pseudoparticle) is known from theoretical and mathematical physics. Marked a classical solution of kinematic equations with ending non-zero action in quantum mechanics and in quantum theory of fields. Exactly said it’s about a solutions of kinematic equations in classical theory of fields in Euclides spacetime. (According WIKIPEDIA)

It was shown that the spin chain with spin  $S$  in plain of easy magnetization along axis  $z$ , is an usual instanton BKT process with zero magnetization along axis  $z$  is generically present, but is exact process of quantum interference forbidden, if  $2S$  is odd ( $2k+1$ ). It expalins difference between classical statistic mechanics of transient 2D BKT and her quantum version (1+1)D. In classical 2D model is an intensity of vortex component in Boltzmann factor is real positive of fugacity, while in quantum (1+1)D model is it complex amplitude for tunneling between a topologically different configurations with a different number of circulations, and is it positive or negative real physical value in models invariant to inversion of time. It’s meaning, that between concurrent of instanton processes may state a quantum interference.

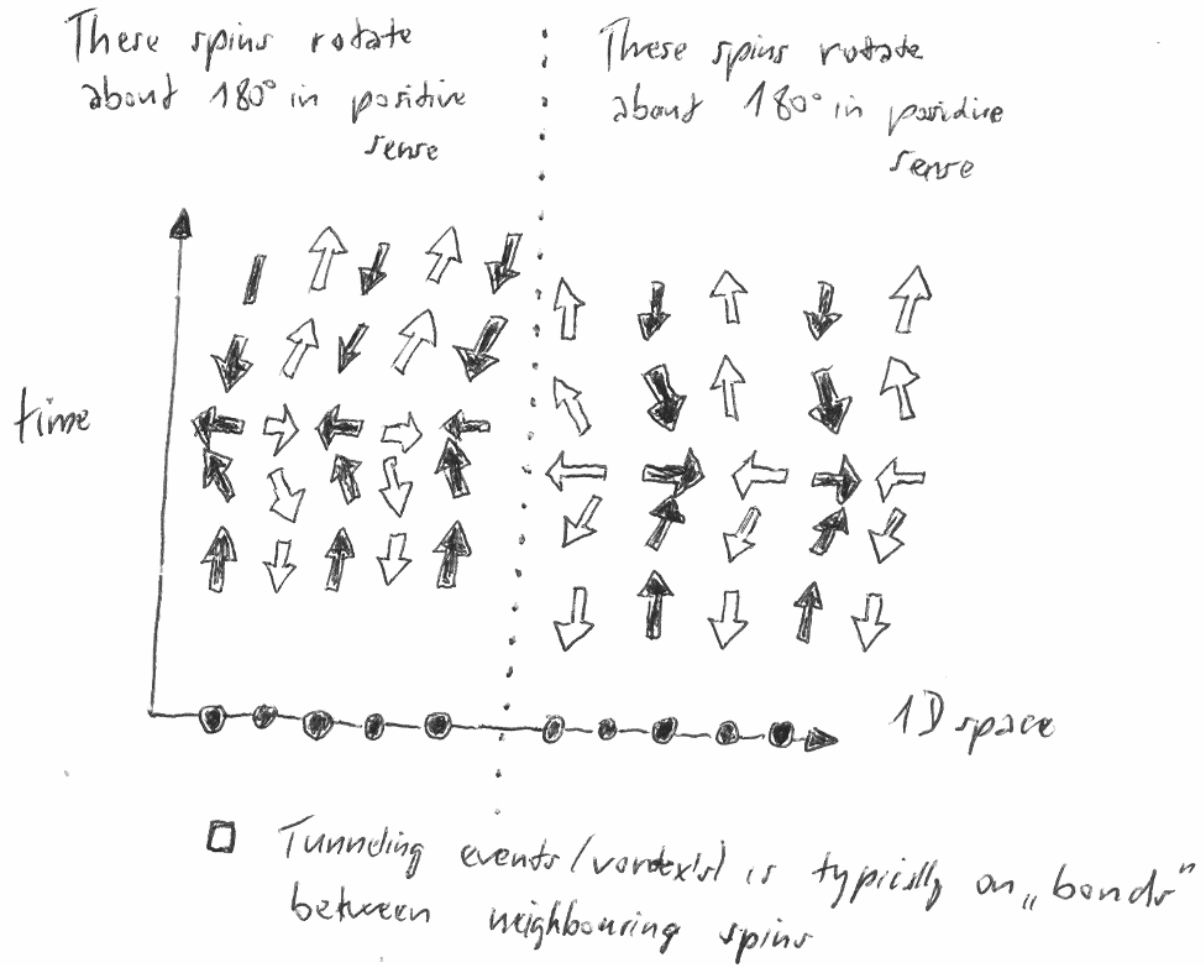


Fig. 5. An analogy of 2D vortex in spacetime with dimension (1+1) is an instanton tunnel process, where is a changing topological "number of circulation" (winding number) on an easy side of spin chain. This process is central on bond between neighboring positions, on which is local Néel's configuration during a short time interval is violate.

$$H = \sum_i \frac{1}{2} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + \lambda S_i^Z S_{i+1}^Z \quad [1]$$

$\lambda < 1$  easy plain       $\downarrow$        $\lambda > 1$  easy axis

$$H = \sum_i \frac{1}{2} (c_i^+ c_{i+1} + c_{i+1}^+ c_i) + \lambda \left( n_i - \frac{1}{2} \right) \left( n_{i+1} - \frac{1}{2} \right) \quad [2]$$

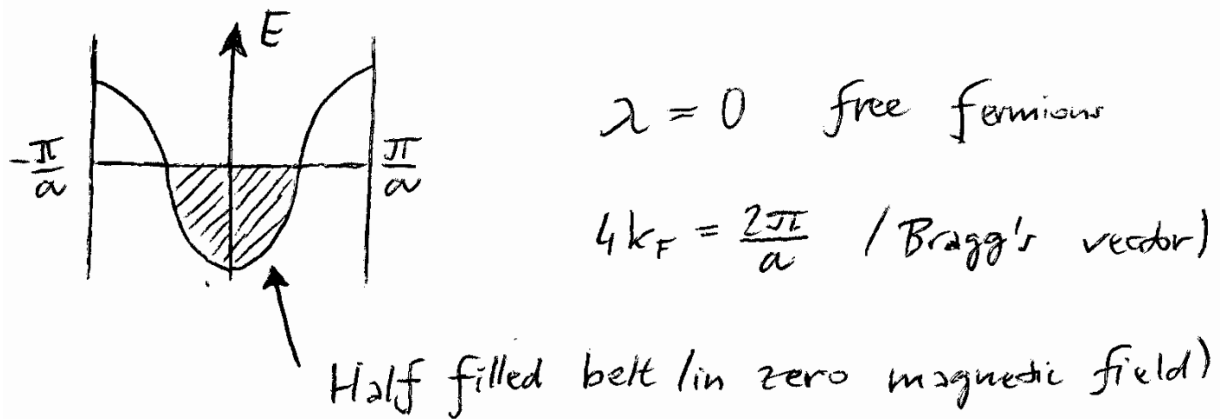


Fig. 6. Jordan–Wigner transformation map of Heisenberg chain of spin  $S = \frac{1}{2}$  with zero magnetization at half filled interaction belt of non–spin fermions, where  $4k_F$  is Bragg's vector.

J.–W. transformation mapped 1 dimensional magnet with changing between the nearest neighbors on model of non–spin fermions, which are moving in 1 Dimension.

Sin–itiro Tomonaga's dispersion with small transport of momentum could be re–solve with using Tomonaga's bosons. Sightseeing technique of "bosonisation" (representation of 1 dimensional fermions with a help of models of Tomonaga's harmonic oscillation). This technique, which explicitly formulated Schotte in his simplify description of problemma "singularity on rtg. absorption edge".

#### Eugene Paul Wigner

(17.11.1902 Budapest, Hungary – 1.1.1995 Princeton, New Jersey, U.S.A.)

Hungarian–American winner of Nobel Prize for physics. Nobel Prize gained for Theory of atomic nucleus and elementary particles, namely for discovery of basic principles of symmetry and their application in practise.

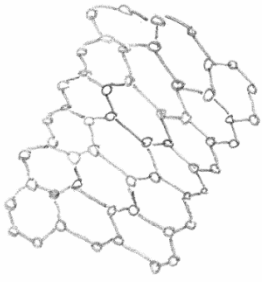
#### Sin–itiro Tomonaga

(31.3.1906 Tokio, Japan – 8.7.1979 Tokio)

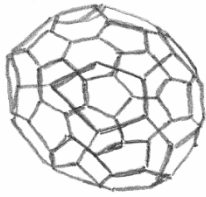
Japanese scientist, one of the most important person of Theoretical Physics, Winner of Nobel Prize 1965 for a work in Quantum electrodynamics (QED).



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graphene nanosheet



"bucky balls"  
"buckminsterfulleren"



"nano-fulleren landscape"  
"fulleren valley"  
"fulleren chreoder"

according to Richard Buckminster Fuller  
C<sub>60</sub> C<sub>50</sub> modification of carbonium

Fig. 7. Spatial conformation of graphene, fulleren and fulleren chreoder,  
respectively allotropic modification of Carbonium

Author of the sketches:  
Mgr. Imrich KRIŠTOF  
FTI 445 729 2018

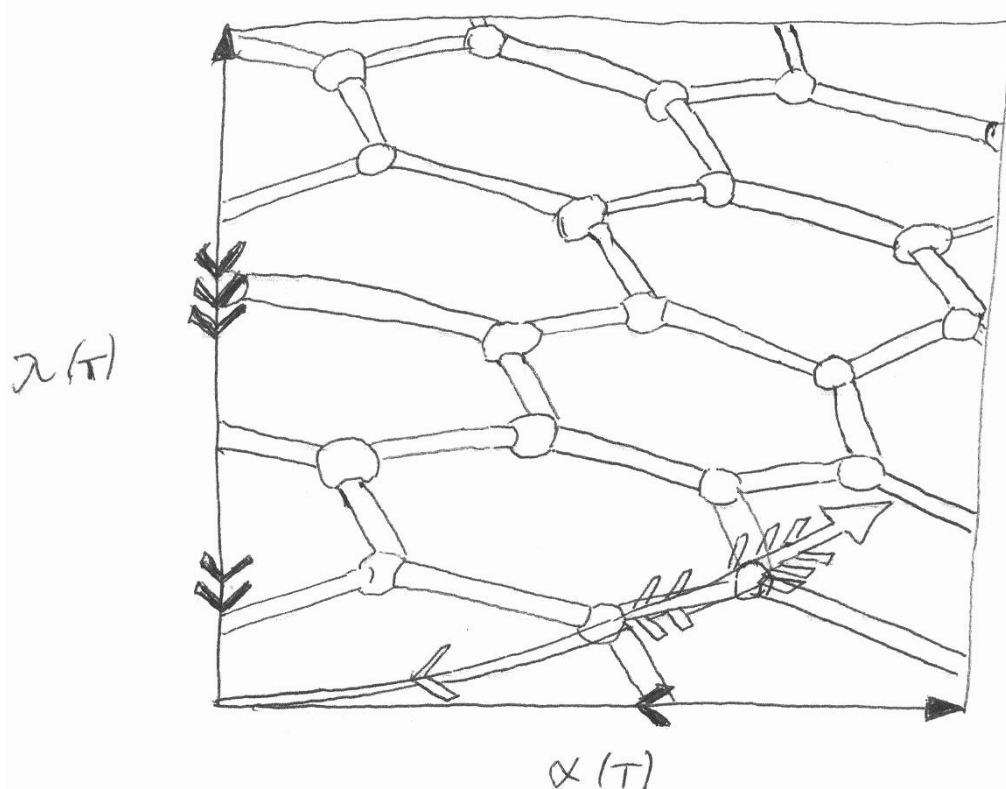


Fig. 8. The schematic “flow” of the coupling constant describing weak electron–electron interact, with decreasing temperature in graphene.

$\alpha$  parametrizes the  $\sim 1/r$ ,  $\sim 1/r$  tail of Coulomb interaction,  $\lambda$  stands collectively for the electron–electron repulsion on the lattice scale.

The density of the arrows increases with the speed of the flow (GRAPHENE – A SINGLE LAYER OF CARBON ATOMS). The popularity of graphene is rooted in the unusual nature of its low–energy excitations: near the Fermi level. This means that the electron can be described “massless” fermions, with a velocity of about 300 times less than the velocity of light, single particle states at the Fermi level, with effects of the Coulomb interaction between electrons weak.

(Simply a graphene mirror).

Illustration: Mgr. I. Krištof,  
according: Alan Stonebraker.

Electrons in graphene form a quantum liquid that is, in fact, strongly interacting, by this criterion graphene comes closer to being a “perfect fluid” than several of quantum systems that have often been labeled as strongly correlated.

Landau’s notion of a Fermi liquid as a system of interacting fermions that, at low energies, effectively behave as noninteracting quasiparticles is the central paradigm of many–body physics. Fermi liquid is to use the language of renormalization group theory.

ALONG WITH THE QUARK GLUON PLASMA AND COLD ATOM GASSES, GRAPHENE IS ESTABLISHING ITS PLACE AS A PERFECT LIQUID.

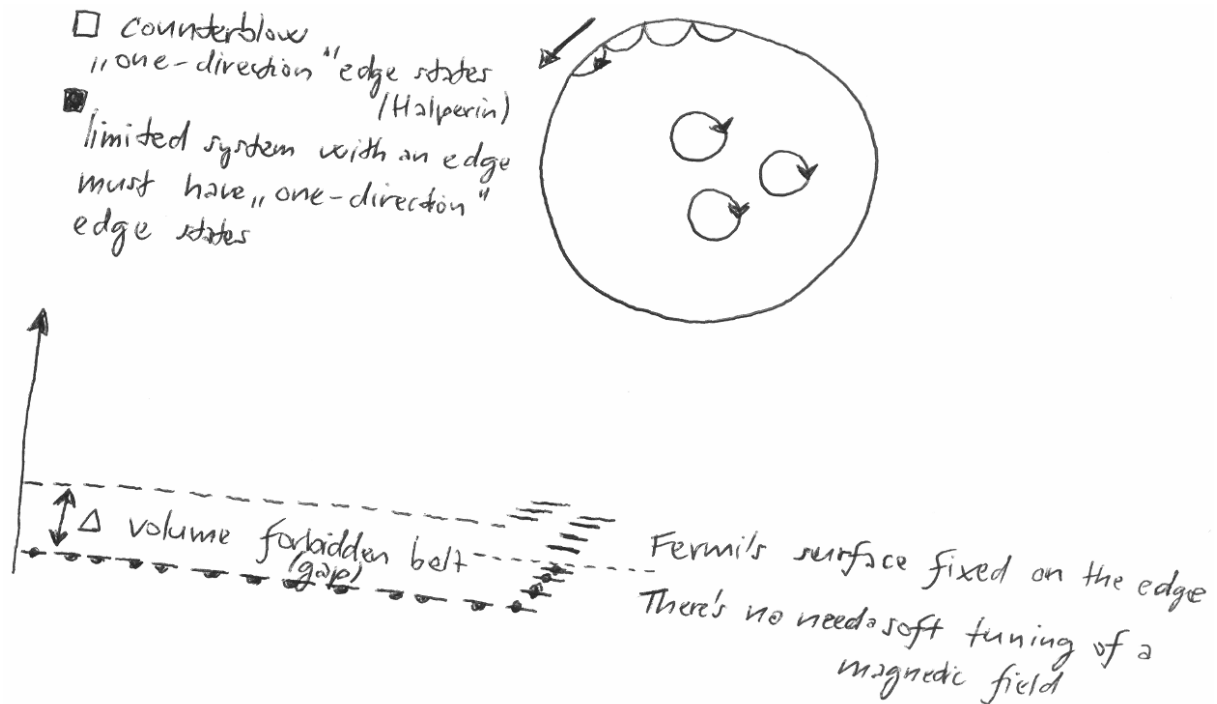


Fig. 9. A simply illustration of an energy-surfaces for integral Hall's effect, with forbidden belt of stabilized fixation of Fermi's surface on edge states without forbidden belt (gap). (Surfaces of energies are demonstrated like a functions of radius in a sample of shape of discs.)  
Author: Mgr. I. KRIŠTOF.

#### 4. Principle of Rydberg Polaron Excitation

The formation of Rydberg and typical condensed-matter polarons, can both be understood as dressing of the impurity by excitations of the bosonic bath, which entangles the impurity momentum with that of the bath excitations, but there are important differences.

Most importantly, the spectral width for a region of uniform density shows that Rydberg polarons exist not as the ground state of the many-body system but rather as a set of excited states, which is unusual in polaron physics.

This highlights the special nature of Rydberg polaron formation where no single many-body state dominates the nonequilibrium dynamics at late times.

The description of Rydberg polarons relies on multiple excitations from the Bose-Einstein Condensate (BEC) – on the order of the number of atoms within the Rydberg orbit, while other polaron states are often well described by including only single-excitation terms. The most polarons in condensed matter systems can be described fully with excitations to unbound states such as free particle or phonon modes, for Rydberg polarons it is essential to include negative-energy bound states.

Bound states are also important for Fermi and Bose Polarons consisting of ground-state impurity atoms interacting with atoms in a background Fermi gas or (BEC), respectively, contact interactions are tuned with a Feshbach resonance. Rydberg impurities introduce long-range interactions with strength tuned by changing  $n$ .

The interaction between a Rydberg atom and a bosonic medium can be described with the Born–Oppenheimer potential for a ground–state atom at a distance  $r$  from the Rydberg impurity.

$$V(r) = \frac{2\pi\hbar^2}{m_e} A_s |\Psi(r)|^2 + \frac{6\pi\hbar^2}{m_e} A_p^3 \left| \vec{\nabla} \Psi(r) \right|^2 \quad [4]$$

where  $\Psi(r)$  is the Rydberg electron wave function,  $A_s$  and  $A_p$  are the momentum–dependent s–wave and p–wave scattering lengths, and  $m_e$  is the electron mass. When  $A_s < 0$ ,  $V(r)$  can support molecular states with one or more ground–state atoms bound to the impurity. Laser excitation of an atom in the BEC creates a Rydberg impurity and projects the system into a superposition of bound and scattering states  $|\beta_i\rangle$ .

The Hamiltonian for a mobile impurity with mass  $M$  at position  $\hat{R}$  and momentum  $\hat{p}$ , interacting with a gas of bosons is

$$\hat{H} = \frac{\hbar^2 \hat{p}^2}{2M} + \sum_K \varepsilon_K \hat{a}_K^\dagger \hat{a}_K + \frac{1}{\nu} \sum_{Kq} V(q) e^{-iq\hat{R}} \hat{a}_{K+q}^\dagger \hat{a}_K \quad [5]$$

where  $\nu$  is the quantization volume, and  $\hat{a}_K$  ( $\hat{a}_K^\dagger$ ) are the annihilation (creation) operators for the bosons with dispersion  $\varepsilon_K = [(\hbar^2 K^2)/(2m)]$ , the Rydberg impurity–boson interaction where  $V(q)$  is the Fourier transform of Eq[4]. The Hamiltonian of Eq[5] is the basis for various polaron models, applying the Bogoliubov approximation and keeping only terms that are linear in bosonic field operators yields the widely studied Fröhlich Hamiltonian. However, the Fröhlich Hamiltonian does not account correctly for scattering. The impurity and boson Hilbert spaces, mapping the problem onto a purely bosonic system.

Two–photon transition using 689 nm and 319 nm light.

The 689 nm light is blue–detuned from the intermediate state by 80 MHz while the frequency of the 319 nm laser is scanned. The laser excitation is applied in a 2  $\mu$ s pulse.

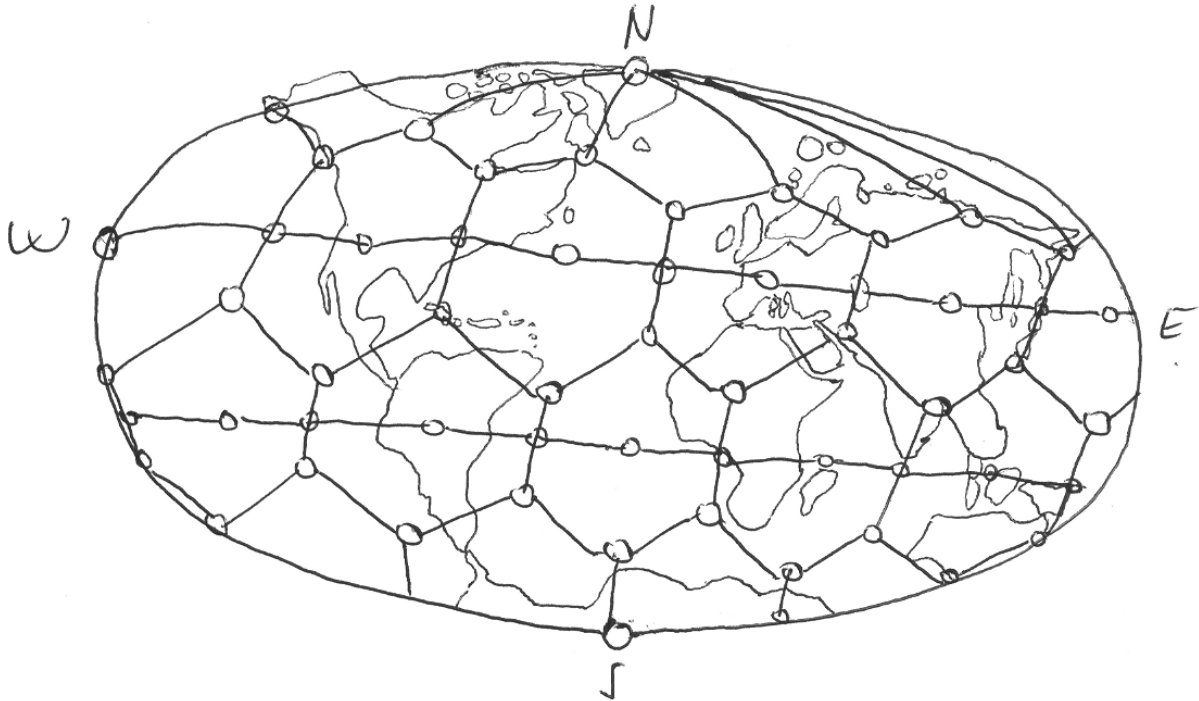


Fig. 10. A scheme illustration of the Earth like a crystal. Along the edges of a "crystal Earth" would be flown Telluric streams. Author of the sketch: Mgr. Imrich KRIŠTOF.

## 5. Conclusion

In Conclusion, our theoretical study shows that quantum coherent phenomena has a reflex mirror sense in nano–micro and macro scales. Concretely structure of graphene is compared with Christaller macro scale phenomena, respectively hexagonal structure of Central Places (villages, towns, cities, conurbations) like a geographical sight.

And finally reflex of density of a hexagonal structure like a rock–forming mafic mineral biotite like a main component of Brno's Massif Rocks – Grandiorite of Královo Pole, like a representation of applied–geology sight.

This study confirms that the identically sample of spatial conformation has a similar context across the modern natural sciences. This polysyncretic sights may lead to a complex understanding of theoretical and practical science problemma.

## 6. Acknowledgements

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