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New Neutrosophic Sets via Neutrosophic Topological Spaces

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Abstract

In Geographical information systems (GIS) there is a need to model spatial regions with indeterminate boundary and under indeterminacy. The purpose of this chapter is to construct the basic concepts of the so-called "neutrosophic sets via neutrosophic topological spaces (NTs)". After giving the fundamental definitions and the necessary examples we introduce the definitions of neutrosophic open sets, neutrosophic continuity, and obtain several preservation properties and some characterizations concerning neutrosophic mapping and neutrosophic connectedness. Possible applications to GIS topological rules are touched upon.

Keywords

Logic, Set Theory, Topology, Neutrosophic set theory, Neutrosophic topology, Neutrosophic open set, Neutrosophic semi-open set, Neutrosophic continuous function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. In various recent papers, F. Smarandache generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). F. Smarandache also defined the notion of neutrosophic topology on the non-standard interval. Indeed, an intuitionistic fuzzy topology is not necessarily a neutrosophic topology. Also, (Wang, Smarandache, Zhang, and

Sunderraman, 2005) introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. We study in this chapter relations between interval neutrosophic sets and topology. In this chapter, we introduce definitions of neutrosophic open sets. After given the fundamental definitions of neutrosophic set operations, we obtain several properties, and discussed the relationship between neutrosophic open sets and others, we introduce and study the concept of neutrosophic continuous functions. Finally, we extend the concepts of neutrosophic topological space.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [1, 2, 3], and Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Smarandache introduced the neutrosophic components *T, I, F,* which represent the membership, indeterminacy, and non-membership values respectively, where $\rfloor^{-}0,1^{+}\lfloor$ is a non-standard unit interval. Hanafy and Salama et al. [10, 11] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following.

Definition 2.1 [24] Let *T*, *I*, *F* be real standard or nonstandard subsets of $\rfloor 0^{-},1^{+}\lfloor$, with

Sup-T=t-sup, inf-T=t-inf

Sup-I=i-sup, inf-I=i-inf

Sup-F=f-sup, inf-F=f-inf

n-sup=t-sup+i-sup+f-sup

n-inf=t-inf+i-inf+f-inf.

T, I, F are called neutrosophic components.

We shall now consider some possible definitions for basic concepts of the neutrosophic set and its operations due to Salama et al.

Definition 2.2 [23] Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$$

where $\mu_A(x), \sigma_A(x)$, and $\gamma_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A .

A neutrosophic $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ in $\rfloor 0^-, 1^+ \lfloor$ on *X*.

Remark 2.3 [23] For the sake of simplicity, we shall use the symbol

$$A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$$

for the NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.4 [4] Let $A = \langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ *a* NS *on* X , then the complement of the set $A(C(A)$ for short, maybe defined as three kinds of complements

1. $C(A) = \{\langle x, 1 - \mu_A(x), 1 - \gamma_A(x)\rangle : x \in X\}$,
2. $C(A) = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$,
3. $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$,

One can define several relations and operations between GNSS as follows:

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the NSS 0_N and 1_N [23] in X as follows:

1- 0_N may be defined as four types:

1. $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ or
2. $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$ or
3. $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$ or
4. $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

2- 1_N may be defined as four types:

1. $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$ or

2. $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$ or

3. $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$ or

4. $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

Definition 2.5 [23] Let X be a non-empty set, and GNSS A and B in the form $A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$, $B = \{x, \mu_B(x), \sigma_B(x), \gamma_B(x)\}$, then we may consider two possible definitions for subsets $(A \subseteq B)$

$(A \subseteq B)$ may be defined as

1. Type 1:

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x), \text{and}\, \gamma_A(x) \le \gamma_B(x)$ or

2. Type 1:

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x), \text{and}\, \gamma_A(x) \ge \gamma_B(x)$.

Definition 2.6 [23] Let $\{A_j : j \in J\}$ be an arbitrary family of *NSS* in X, then

1. $\cap A_j$ may be defined as two types:

-Type 1: $\cap A_j = \langle x, \underset{j\in J}{\wedge} \mu_{Aj}(x), \underset{j\in J}{\wedge} \sigma_{Aj}(x), \underset{j\in J}{\vee} \gamma_{Aj}(x)\rangle$.

-Type 2: $\cap A_j = \langle x, \underset{j\in J}{\wedge} \mu_{Aj}(x), \underset{j\in J}{\vee} \sigma_{Aj}(x), \underset{j\in J}{\vee} \gamma_{Aj}(x)\rangle$.

2. $\cup A_j$ may be defined as two types:

-Type 1: $\cup A_j = \langle x, \underset{j\in J}{\vee} \mu_{Aj}(x), \underset{j\in J}{\vee} \sigma_{Aj}(x), \underset{j\in J}{\wedge} \gamma_{Aj}(x)\rangle$.

-Type 2: $\cup A_j = \langle x, \underset{j\in J}{\vee} \mu_{Aj}(x), \underset{j\in J}{\wedge} \sigma_{Aj}(x), \underset{j\in J}{\wedge} \gamma_{Aj}(x)\rangle$.

Definition 2.7 [25] A neutrosophic topology (NT for short) and a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms

1. $0_N, 1_N \in \tau$

2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

3. $\cup G_i \in \tau$, $\forall \{G_i \mid j \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is known as neutrosophic open set (NOS for short) in X. The elements of τ are called open neutrosophic sets, A neutrosophic set F is closed if and only if it $C(F)$ is neutrosophic open [26-30].

Note that for any NTS A in (X, τ), we have $Cl(A^c) = [Int(A)]^c$ and $Int(A^c) = [Cl(A)]^c$.

Example 2.8 [4] Let $X = \{a, b, c, d\}$, and $A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$

$A = \{\langle x, 0.5, 0.5, 0.4 \rangle : x \in X\}$

$B = \{\langle x, 0.4, 0.6, 0.8 \rangle : x \in X\}$

$D = \{\langle x, 0.5, 0.6, 0.4 \rangle : x \in X\}$

$C = \{\langle x, 0.4, 0.5, 0.8 \rangle : x \in X\}$

Then the family $\tau = \{0_n, 1_n, A, B, C, D\}$ of NSs in X is neutrosophic topology on X.

Definition 2.9 [23] Let (x, τ) be NTs and $A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$ be a NS in X.

Then the neutrosophic closure and neutrosophic interior of A are defined by

1. $NCL(A) = \cap\{K : K \text{ is a NCS in X and } A \subseteq K\}$
2. $NInt(A) = \cup\{G : G \text{ is a NOS in X and } G \subseteq A\}$

It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X

1. A is in X if and only if $NCl(A)$.
2. A is NCS in X if and only if $NInt(A) = A$.

Proposition 2.10 [23] Let (x, τ) be a NTS and A, B be two neutrosophic sets in X. Then the following properties hold:

1. $NInt(A) \subseteq A$,
2. $A \subseteq NCl(A)$,
3. $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$,
4. $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$,
5. $NCL(NCL(A)) = NCL(A)$

   $NInt(NInt(A)) = NInt(A)$,
6. $NInt(A \cup B) = NInt(A) \cup NInt(B)$

   $NCl(A \cap B) = NCl(A) \cap NCl(B)$,
7. $NCl(A) \cup NCl(B) = NInt(A \cup B)$,

Definition 2.11 [23] Let $A = \{\mu_A(x), \sigma_A(x), \gamma_A(x)\}$ be a neutrosophic open sets and $B = \{\mu_B(x), \sigma_B(x), \gamma_B(x)\}$ be a neutrosophic set on a neutrosophic topological space (X, τ) then

1. A is called neutrosophic regular open iff $A = NInt(NCl(A))$.
2. If $B \in NCS(X)$ then B is called neutrosophic regular closed iff $A = NCl(NInt(A))$.

3 Neutrosophic Openness

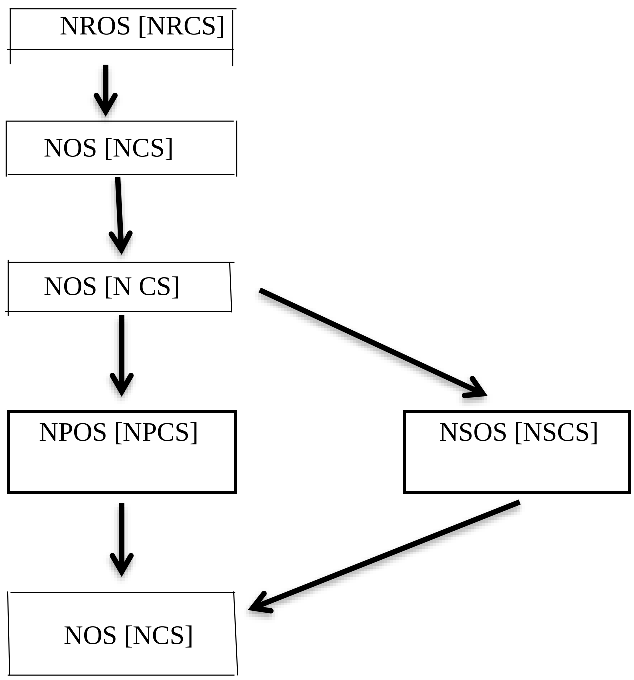
Definition 3.1 A neutrosophic set (Ns) A in a neutrosophic topology (X, τ) is called

1. Neutrosophic semiopen set $(NSOS)$ if $A \subseteq NCl(NInt(A))$,
2. Neutrosophic preopen set $(NPOS)$ if $A \subseteq NInt(NCl(A))$,
3. Neutrosophic α -open set $(N\alpha OS)$ if $A \subseteq NInt(NCl(NInt(A)))$
4. Neutrosophic β -open set $(N\beta OS)$ if $A \subseteq NCl(NInt(NCl(A)))$

An (Ns) A is called neutrosophic semi-closed set, neutrosophic α -closed set, neutrosophic pre-closed set, and neutrosophic regular closed set, respectively (NSCS, Nα CS, NPCS, and NRCS, resp.), if the complement of A is a NSOS, Nα OS, NPOS, and NROS, respectively.

Definition 3.2 In the following diagram, we provide relations between various types of neutrosophic openness (neutrosophic closedness): 0pt

Remark 3.3 From above the following implication and none of these implications is reversible as shown by examples given below



Reverse implications are not true in the above diagram. The following is a characterization of a Nα OS.

Example 3.4 Let $X = \{a, b, c\}$ and:

$A = \langle (0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5) \rangle$,

$B = \langle (0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4) \rangle$.

Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X . Define the two neutrosophic closed sets C_1 and C_2 as follows,

$C_1 = \langle (0.5,0.5,0.5),(0.6,0.5,0.5),(0.6,0.5,0.5) \rangle$,

$C_2 = \langle (0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6) \rangle$.

Then the set A is neutrosophic open set (NOs) but not neutrosophic regular open set (NROs) since $A \neq NInt(NCl(A))$, and since $A \subseteq NInt(NCl(NInt(A)))$ where the $NInt(NCl(NInt(A)))$ is equal to:

$$\langle (0.5,0.5,0.5),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$$

so that A is neutrosophic α -open set (Nα Os).

Example 3.5 Let $X = \{a,b,c\}$ and:

$A = \langle (0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,

$B = \langle (0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$, and

$C = \langle (0.5,0.5,0.5),(0.4,0.5,0.5),(0.5,0.5,0.5)\rangle$.

Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Define the two neutrosophic closed sets C_1 and C_2 as follows:

$C_1 = \langle (0.5,0.5,0.5),(0.6,0.5,0.5),(0.6,0.5,0.5)\rangle$,
$C_2 = \langle (0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$.

Then the set C is neutrosophic semi open set (NSOs), since

$C \subseteq NCl(NInt(C))$,

where $NCl(NInt(C)) = \langle (0.5,0.5,0.5),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$ but not neutrosophic α -open set (Nα Os) since $C \nsubseteq NInt(NCl(NInt(C)))$ where the $NInt(NCl(NInt(C)))$ is equal $\langle (0.5,0.5,0.5),(0.4,0.5,0.5),(0.3,0.4,0.4)\rangle$, in the sense of $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x), \text{and}\, \gamma_A(x) \le \gamma_B(x)$.

Example 3.6 Let $X = \{a,b,c\}$ and:

$A = \langle (0.4,0.5,0.4),(0.5,0.5,0.5),(0.4,0.5,0.4)\rangle$,

$B = \langle (0.7,0.6,0.5),(0.3,0.4,0.5),(0.3,0.4,0.4)\rangle$, and

$C = \langle (0.5,0.5,0.5),(0.5,0.5,0.5),(0.5,0.5,0.5)\rangle$.

Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Define the two neutrosophic closed sets C_1 and C_2 as follows:

$C_1 = \langle (0.6,0.5,0.6),(0.5,0.5,0.5),(0.6,0.5,0.5)\rangle$,

$C_2 = \langle (0.3,0.4,0.5),(0.7,0.6,0.5),(0.7,0.6,0.5)\rangle$.

Then the set C is neutrosophic preopen set *(NPOs)*, since $C \subseteq NInt(NCl(C))$, where $NInt(NCl(C)) = \langle(0.7,0.6,0.5),(0.5,0.5,0.5),(0.3,0.4,0.5)\rangle$ but not neutrosophic α -open set ($N\alpha$ *Os*) since $C\, NInt(NCl(NInt(C)))$ where the $NInt(NCl(NInt(C)))$ is equal $\langle(0,0,0),(1,1,1),(0,0,0)\rangle$.

Example 3.7 Let $X = \{a,b,c\}$ and:

$A = \langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,

$B = \langle(0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$, and

$C = \langle(0.3,0.3,0.3),(0.4,0.5,0.5),(0.3,0.4,0.4)\rangle$.

Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Define the two neutrosophic closed sets C_1 and C_2 as follows,

$C_1 = \langle(0.5,0.5,0.5),(0.6,0.5,0.5),(0.6,0.5,0.5)\rangle$,

$C_2 = \langle(0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$.

Then the set C is neutrosophic β -open set *(Nβ Os)*, since $C \subseteq NCl(NInt(NCl(C)))$, where

$NCl(NInt(NCl(A))) = \langle(0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$,

but not neutrosophic pre-open set (*NPOs*) neither neutrosophic semi-open set (*NSOs*) since $C\, NCl(NInt(C))$ where the $NCl(NInt(C))$ is equal $\langle(0.5,0.5,0.5),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$

Let (X,τ) be NTS and $A = \{A_1, A_2, A_3\}$ be a NS in X. Then the *-neutrosophic closure of A ($*-NCl(A)$ for short) and *-neutrosophic interior ($*-NInt(A)$ for short) of A are defined by

1. $\alpha NCl(A) = \cap\{K : is a NRCS\, in X\, and A \subseteq K\}$,
2. $\alpha NInt(A) = \cup\{G : G is a NROS\, in X\, and G \subseteq A\}$,
3. $pNCl(A) = \cap\{K : is a NPCS\, in X\, and A \subseteq K\}$,
4. $pNInt(A) = \cup\{G : G is a NPOS\, in X\, and G \subseteq A\}$,
5. $sNCl(A) = \cap\{K : is a NSCS\, in X\, and A \subseteq K\}$,
6. $sNInt(A) = \cup\{G : G is a NSOS\, in X\, and G \subseteq A\}$,

7. $\beta NCl(A) = \cap\{K : is\,a\,NC\beta CS\,in\,X\,and\,A \subseteq K\}$,
8. $\beta NInt(A) = \cup\{G : G\,is\,a\,N\beta OS\,in\,X\,and\,G \subseteq A\}$,
9. $rNCl(A) = \cap\{K : is\,a\,NRCS\,in\,X\,and\,A \subseteq K\}$,
10. $rNInt(A) = \cup\{G : G\,is\,a\,NROS\,in\,X\,and\,G \subseteq A\}$.

Theorem 3.8 *A Ns* A in a *NTs* (X,τ) is a $N\alpha$ *OS* if and only if it is both *NSOS* and *NPOS.*

Proof. Necessity follows from the diagram given above. Suppose that A is both a *NSOS* and a *NPOS*. Then $A \subseteq NCl(NInt(A))$, and so

$$NCl(A) \subseteq NCl(NCl(NInt(A))) = NCl(NInt(A))$$

It follows that $A \subseteq NInt(NCl(A)) \subseteq NInt(NCl(NInt(A)))$, so that A is a $N\alpha$ *OS*. We give condition(s) for a NS to be a $N\alpha$ *OS*.

Proposition 3.9 Let (X,τ) be a neutrosophic topology space NTs. Then arbitrary union of neutrosophic α -open sets is a neutrosophic α -open set, and arbitrary intersection of neutrosophic α -closed sets is a neutrosophic α -closed set.

Proof. Let $A = \{\langle x, \mu_{A_i}, \sigma_{A_i}, \gamma_{A_i}\rangle : i \in \Lambda\}$ be a collection of neutrosophic α -open sets. Then, for each $i \in \Lambda$, $A_i \subseteq NInt(NCl(NInt(A_i)))$. Its follows that

$$\bigcup A_i \subseteq \bigcup NInt(NCl(NInt(A_i))) \subseteq NInt(\bigcup NCl(NInt(A_i)))$$
$$= NInt(NCl(\bigcup NInt(A_i))) \subseteq NInt(NCl(NInt(\bigcup A_i)))$$

Hence $\bigcup A_i$ is a neutrosophic α -open set. The second part follows immediately from the first part by taking complements.

Having shown that arbitrary union of neutrosophic α -open sets is a neutrosophic α -open set, it is natural to consider whether or not the intersection of neutrosophic α -open sets is a neutrosophic α -open set, and the following example shown that the intersection of neutrosophic α -open sets is not a neutrosophic α -open set.

Example 3.10 Let $X = \{a,b,c\}$ and

$A = \langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,
$B = \langle(0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$.

Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Define the two neutrosophic closed sets C_1 and C_2 as follows,

$C_1 = \langle(0.5,0.5,0.5),(0.6,0.5,0.5),(0.6,0.5,0.5)\rangle$,
$C_2 = \langle(0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$.

Then the set A and B are neutrosophic α -open set (N α Os) but $A \cap B$ is not neutrosophic α -open set. In fact $A \cap B$ is given by $\langle(0.3,0.4,0.4),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,

and $NInt(NCl(NInt(A \cap B))) = \langle(0.5,0.5,0.5),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$,

so $A \cap B NInt(NCl(NInt(A \cap B)))$.

Theorem 3.11 Let A be a (Ns) in a neutrosophic topology space *NTs* (X,τ). If B is a *NSOS* such that $B \subseteq A \subseteq NInt(NCl(B))$, then A is a $N\alpha$ *OS*.

Proof. Since B is a NSOS, we have $B \subseteq NCl(NInt(B))$. Thus, $A \subset NInt(NCl(A)) \subseteq NInt(NCl(NCl(NInt(B)))) = NInt(NCl(NInt(B))) \subseteq NInt(NCl(NInt(A)))$. and so is a a $N\alpha$ *OS*

Proposition 3.12 In neutrosophic topology space *NTs* (X,τ), a neutrosophic α -closed *(Nα Cs) if* and only if $A = \alpha NCl(A)$.

Proof. Assume that A is neutrosophic α -closed set. Obviously,

$A \in \{B_i \mid B_i \text{ is a neutrosophic } -closed\, set\, and\, A \subseteq B_i\}$,

and also

$A = \{B_i \mid B_i \text{ is a neutrosophic } -closed\, set\, and\, A \subseteq B_i\}$,

$= \alpha NCl(A)$.

Conversely suppose that $A = \alpha NCl(A)$, which shows that

$A \in \{B_i \mid B_i \text{ is a neutrosophic } -closed\, set\, and\, A \subseteq B_i\}$.

Hence A is neutrosophic α -closed set.

Theorem 3.13 A neutrosophic set A in a NTs X is neutrosophic α -open (resp., neutrosophic preopen) if and only if for every $N\alpha Os_{p(\alpha,\beta)} \in A$, there exists a N α Os (resp., NPOs) $B_{p(\alpha,\beta)}$ such that $p(\alpha,\beta) \in B_{p(\alpha,\beta)} \subseteq A$.

Proof. If A is a N α Os (resp., NPOs), then we may take $B_{p(\alpha,\beta)} = A$ for every $p(\alpha,\beta) \in A$.

Conversely assume that for every NP $p(\alpha,\beta) \in A$, there exists a N α Os (resp., NPOs) $B_{p(\alpha,\beta)}$ such that $p(\alpha,\beta) \in B_{p(\alpha,\beta)} \subseteq A$. Then,

$$A = \bigcup \{p(\alpha,\beta) \mid p(\alpha,\beta) \in A\} \subseteq \bigcup \{B_{p(\alpha,\beta)} \mid p(\alpha,\beta) \in A\} \subseteq A,$$

and so $$A = \bigcup \{B_{p(\alpha,\beta)} \mid p(\alpha,\beta) \in A\},$$

which is a N α Os (resp., NPOs) by Proposition 3.9.

Proposition 3.14 In a NTs (X,τ), the following hold for neutrosophic α -closure:

1. $\alpha NCl(0_{\sim}) = 0_{\sim}$.
2. $\alpha NCl(A)$ is neutrosophic α -closed in (X,τ) for every Ns in A.
3. $\alpha NCl(A) \subseteq \alpha NCl(B)$ whenever $A \subseteq B$ for every Ns A and B in X.
4. $\alpha NCl(\alpha NCl(A)) = \alpha NCl(A)$ for every Ns A in X.

Proof. The proof is easy.

4 Neutrosophic Continuous Mapping

Definition 4.1 [25] Let (X,τ_1) and (Y,τ_2) be two NTSs, and let $f : X \to Y$ be a function. Then f is said to be strongly N -continuous iff the inverse image of every NOS in τ_2 is a NOS in τ_1.

Definition 4.2 [25] Let (X,τ_1) and (Y,τ_2) be two NTSs, and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each NS in τ_2 is a NS in τ_1.

Example 4.3 [25] Let $X=\{a,b,c\}$ and $Y=\{a,b,c\}$. Define neutrosophic sets A and B as follows:
$A=\langle(0.4,0.4,0.5),(0.2,0.4,0.3),(0.4,0.4,0.5)\rangle$,
$B=\langle(0.4,0.5,0.6),(0.3,0.2,0.3),(0.4,0.5,0.6)\rangle$.

Then the family $\tau_1=\{0_N,1_N,A\}$ is a neutrosophic topology on X and $\tau_2=\{0_N,1_N,B\}$ is a neutrosophic topology on Y. Thus (X,τ_1) and (Y,τ_2) are neutrosophic topological spaces. Define $f:(X,\tau_1)\to(Y,\tau_2)$
as $f(a)=b$, $f(b)=a$, $f(c)=c$.
Clearly f is N -continuous.

Now f is not neutrosophic continuous, since $f^{-1}(B)\notin\tau$ for $B\in\tau_2$.

Definition 4.4 Let f be a mapping from a NTS (X,τ) to a NTS (Y,κ). Then f is called

1. a neutrosophic α -continuous mapping if $f^{-1}(B)$ is a Nα Os in X for every NOs B in Y .
2. a neutrosophic pre-continuous mapping if $f^{-1}(B)$ is a NPOs in X for every NOs B in Y .
3. a neutrosophic semi-continuous mapping if $f^{-1}(B)$ is a NSOs in X for every NOs B in Y .
4. a neutrosophic β -continuous mapping if $f^{-1}(B)$ is a Nβ Os in X for every NOs B in Y .

Theorem 4.5 For a mapping f from a *NTS* (X,τ) to a *NTS* (Y,κ), the following are equivalent.

1. f is neutrosophic pre-continuous.
2. $f^{-1}(B)$ is *NPCs* in X for every NCs B in Y .
3. $NCl(NInt(f^{-1}(A)))\subseteq f^{-1}(NCl(A))$ for every neutrosophic set A in Y .

Proof. $(1) \Rightarrow (2)$ The proof is straightforward.

$(2) \Rightarrow (3)$ Let A be a NS in Y. Then $NCl(A)$ is neutrosophic closed. It follows from (2) that $f^{-1}(NCl(A))$ is a *NPCS* in X so that

$$NCl(NInt(f^{-1}(A))) \subseteq NCl(NInt(f^{-1}(NCl(A)))) \subseteq f^{-1}(NCl(A)).$$

$(3) \Rightarrow (1)$ Let A be a *NOS* in Y. Then $\overline{A}$ is a *NCS* in Y, and so

$$NCl(NInt(f^{-1}(\overline{A}))) \subseteq f^{-1}(NCl(\overline{A})) = f^{-1}(\overline{A}).$$

This implies that

$$\overline{NInt(NCl(f^{-1}(A)))} = NCl(\overline{NCl(f^{-1}(A))}) = NCl(NInt(\overline{f^{-1}(A)}))$$

$$= NCl(NInt(f^{-1}(\overline{A}))) \subseteq f^{-1}(\overline{A}) = \overline{f^{-1}(A)},$$

and thus $f^{-1}(A) \subseteq NInt(NCl(f^{-1}(A)))$. Hence $f^{-1}(A)$ is a *NPOS* in X, and f is neutrosophic pre-continuous.

Theorem 4.6 Let f be a mapping from a *NTS* (X, τ) to a *NTS* (Y, κ) that satisfies

$$NCl(NInt(NCl(f^{-1}(B)))) \subseteq f^{-1}(NCl(B)),$$ for every NS B in Y.

Then f is neutrosophic α -continuous.

Proof. Let B be a NOS in Y. Then B is a *NCS* in Y, which implies from hypothesis that

$$NCl(NInt(NCl(f^{-1}(\overline{B})))) \subseteq f^{-1}(NCl(\overline{B})) = f^{-1}(\overline{B}).$$

It follows that

$$\overline{NInt(NCl(NInt(f^{-1}(B))))} = NCl(\overline{NCl(NInt(f^{-1}(B)))})$$

$$= NCl(NInt(\overline{NInt(f^{-1}(B))}))$$

$$= NCl(NInt(NCl(\overline{f^{-1}(B)})))$$

$$= NCl(NInt(NCl(f^{-1}(\overline{B})))) \subseteq f^{-1}(\overline{B})$$

$$= \overline{f^{-1}(B)}$$

so that $f^{-1}(B) \subseteq NInt(NCl(NInt(f^{-1}(B))))$. This shows that $f^{-1}(B)$ is a N α OS in X . Hence, f is neutrosophic α -continuous.

Definition 4.7 Let $p(\alpha,\beta)$ be a *NP* of a *NTS* (X,τ). A NS A of X is called a neutrosophic neighborhood (*NH*) of $p(\alpha,\beta)$ if there exists a NOS B in X such that $p(\alpha,\beta) \in B \subseteq A$.

Theorem 4.8 Let f be a mapping from a NTS (X,τ) to a *NTS* (Y,κ). Then the following assertions are equivalent.

1. f is neutrosophic pre-continuous.
2. For each *NP* $p(\alpha,\beta) \in X$ and every *NH* A of $f(p(\alpha,\beta))$, there exists a *NPOS* B in X such that $p(\alpha,\beta) \in B \subseteq f^{-1}(A)$.
3. For each *NP* $p(\alpha,\beta) \in X$ and every NH A of $f(p(\alpha,\beta))$, there exists a *NPOS* B in X such that $p(\alpha,\beta) \in B$ and $f(B) \subseteq A$.

Proof. $(1) \Rightarrow (2)$ Let $p(\alpha,\beta)$ be a NP in X and let A be a NH of $f(p(\alpha,\beta))$. Then there exists a NOS B in Y such that $f(p(\alpha,\beta)) \in B \subset A$. Since f is neutrosophic pre-continuous, we know that $f^{-1}(B)$ is a NPOS in X and

$$p(\alpha,\beta) \in f^{-1}(f(p(\alpha,\beta))) \subseteq f^{-1}(B) \subseteq f^{-1}(A).$$

Thus (2) is valid.

$(2) \Rightarrow (3)$ Let $p(\alpha,\beta)$ be a NP in X and let A be a NH of $f(p(\alpha,\beta))$. The condition (2) implies that there exists a NPOS B in X such that $p(\alpha,\beta) \in B \subseteq f^{-1}(A)$ so that $p(\alpha,\beta) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (3) is true.

$(3) \Rightarrow (1)$. Let B be a NOS in Y and let $p(\alpha,\beta) \in f^{-1}(B)$. Then $f(p(\alpha,\beta)) \in B$, and so B is a NH of $f(p(\alpha,\beta))$ since B is a NOS. It follows from (3) that there exists a NPOS A in X such that $p(\alpha,\beta) \in A$ and $f(A) \subseteq B$ so that,

$$p(\alpha,\beta)\in A\subseteq f^{-1}(f(A))\subseteq f^{-1}(B).$$

Applying Theorem 3.13 induces that $f^{-1}(B)$ is a NPOS in X. Therefore, f is neutrosophic pre-continuous.

Theorem 4.9 Let f be a mapping from a *NTS* (X,τ) to a *NTS* (Y,κ). Then the following assertions are equivalent.

1. f is neutrosophic α-continuous.
2. For each *NP* $p(\alpha,\beta)\in X$ and every *NH* A of $f(p(\alpha,\beta))$, there exists a $N\alpha$ *OS* B in X such that $p(\alpha,\beta)\in B\subseteq f^{-1}(A)$.
3. For each *NP* $p(\alpha,\beta)\in X$ and every NH A of $f(p(\alpha,\beta))$, there exists a $N\alpha$ *OS* B in X such that $p(\alpha,\beta)\in B$ and $f(B)\subseteq A$.

Proof. $(1)\Rightarrow(2)$ Let $p(\alpha,\beta)$ be a NP in X and let A be a NH of $f(p(\alpha,\beta))$. Then there exists a NOS C in Y such that $f(p(\alpha,\beta))\in B\subset A$. Since f is neutrosophic α-continuous, $B=f^{-1}(C)$ is a NPOS in X and

$$p(\alpha,\beta)\in f^{-1}(f(p(\alpha,\beta)))\subseteq B=f^{-1}(C)\subseteq f^{-1}(A).$$

Thus (2) is valid.

$(2)\Rightarrow(3)$ Let $p(\alpha,\beta)$ be a NP in X and let A be a NH of $f(p(\alpha,\beta))$. Then there exists a Nα OS B in X such that $p(\alpha,\beta)\in B\subseteq f^{-1}(A)$ by (2). Thus, we have $p(\alpha,\beta)\in B$ and $f(B)\subseteq f(f^{-1}(A))\subseteq A$. Hence (3) is valid.

$(3)\Rightarrow(1)$. Let B be a *NOS* in Y and we take $p(\alpha,\beta)\in f^{-1}(B)$. Then $f(p(\alpha,\beta))\in f(f^{-1}(B))\subseteq B$, Since B is NOS, it follows that B is a NH of $f(p(\alpha,\beta))$ so from (3), there exists a Nα OS A such that $p(\alpha,\beta)\in A$ and $f(A)\subseteq B$ so that,

$$p(\alpha,\beta)\in A\subseteq f^{-1}(f(A))\subseteq f^{-1}(B).$$

Using Theorem 3.13 induces that $f^{-1}(B)$ is a $N\alpha$ *OS* in X. Therefore, f is neutrosophic α-continuous.

Combining Theorems 4.6 and 4.9, we have the following characterization of neutrosophic α-continuous.

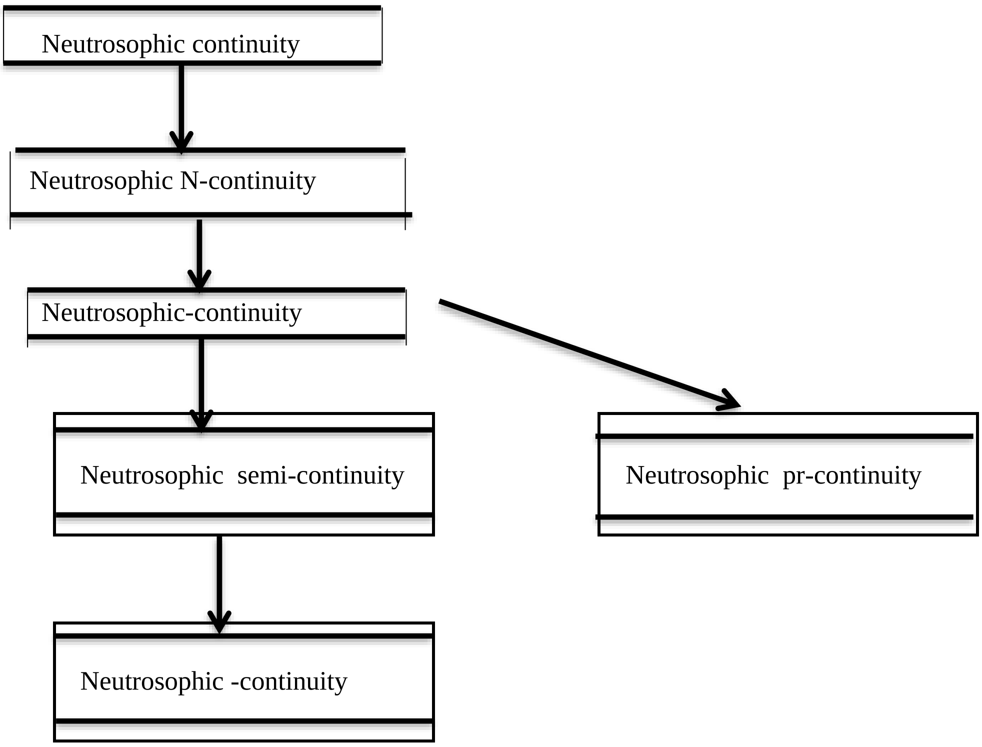
Theorem 4.10 Let f be a mapping from a *NTS* (X,τ) to a *NTS* (Y,κ). Then the following assertions are equivalent.

1. f is neutrosophic α-continuous.
2. If C is a *NCS* in Y, then $f^{-1}(C)$ is a Nα CS in X.
3. $NCl(NInt(NCl(f^{-1}(B))))\subseteq f^{-1}(NCl(B))$ for every *NS* B in Y.
4. For each *NP* $p(\alpha,\beta)\in X$ and every *NH* A of $f(p(\alpha,\beta))$, there exists a $N\alpha$ *OS* B such that $p(\alpha,\beta)\in B\subseteq f^{-1}(A)$.
5. For each *NP* $p(\alpha,\beta)\in X$ and every *NH* A of $f(p(\alpha,\beta))$, there exists a $N\alpha$ *OS* B such that $p(\alpha,\beta)\in B$ and $f(B)\subseteq A$.

Some aspects of neutrosophic continuity, neutrosophic N-continuity, neutrosophic strongly neutrosophic continuity, neutrosophic perfectly neutrosophic continuity, neutrosophic strongly *N*-continuity are studied in [25] as well as in several papers. The relation among these types of neutrosophic continuity is given as follows, where N means neutrosophic:

Example 4.11 Let $X=Y=\{a,b,c\}$. Define neutrosophic sets A and B as follows $A=\langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,
$B=\langle(0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$,
$C=\langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.5,0.5,0.5)\rangle$ and
$D=\langle(0.4,0.5,0.5),(0.5,0.5,0.5),(0.5,0.5,0.5)\rangle$. Then the family $\tau_1=\{0_N,1_N,A,B\}$ is a neutrosophic topology on X and $\tau_2=\{0_N,1_N,D\}$ is a neutrosophic topology on Y. Thus (X,τ_1) and (Y,τ_2) are neutrosophic topological spaces. Define $f:(X,\tau_1)\to(Y,\tau_2)$ as $f(a)=b$, $f(b)=a$, $f(c)=c$. Clearly f is neutrosophic semi-continuous, but not neutrosophic α-

continuous, since $f^{-1}(D)=C$ not not neutrosophic α -open set, i.e $C \nsubseteq NInt(NCl(NInt(C)))$ where the $NInt(NCl(NInt(C)))$ is equal $\langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.3,0.4,0.4)\rangle$.



The reverse implications are not true in the above diagram in general as the following example.

Example 4.12 Let $X=Y=\{a,b,c\}$ and

$A=\langle(0.4,0.5,0.4),(0.5,0.5,0.5),(0.4,0.5,0.4)\rangle$,

$B=\langle(0.7,0.6,0.5),(0.3,0.4,0.5),(0.3,0.4,0.4)\rangle$, and

$C=\langle(0.5,0.5,0.5),(0.5,0.5,0.5),(0.5,0.5,0.5)\rangle$.

Then $\tau_1=\{0_N,1_N,A,B\}$ is a neutrosophic topology on X and $\tau_2=\{0_N,1_N,C\}$ is a neutrosophic topology on Y. Thus (X,τ_1) and (Y,τ_2) are neutrosophic topological spaces. Define $f:(X,\tau_1)\to(Y,\tau_2)$ as identity function. Then f is neutrosophic pre-continuous but not neutrosophic α -continuous, since $f^{-1}(C)=C$ is neutrosophic pre open set (*NPOs*) but not neutrosophic α -open set (*N*α *Os*).

Example 4.13 Let $X=Y=\{a,b,c\}$. Define neutrosophic sets A and B as follows $A=\langle(0.5,0.5,0.5),(0.4,0.5,0.5),(0.4,0.5,0.5)\rangle$,
$B=\langle(0.3,0.4,0.4),(0.7,0.5,0.5),(0.3,0.4,0.4)\rangle$, and
$D=\langle(0.3,0.4,0.4),(0.3,0.3,0.3),(0.4,0.5,0.5)\rangle$. $\tau_1=\{0_N,1_N,A,B\}$ is a neutrosophic topology on X and $\tau_2=\{0_N,1_N,D\}$ is a neutrosophic topology on Y. Define $f:(X,\tau_1)\to(Y,\tau_2)$ as $f(a)=c$, $f(b)=a$, $f(c)=b$. Clearly f is neutrosophic β -continuous, but not neutrosophic pre-continuous neither neutrosophic semi-continuous since
$f^{-1}(D)=\langle(0.3,0.3,0.3),(0.4,0.5,0.5),(0.3,0.4,0.4)\rangle=C$ is neutrosophic β -open set (Nβ Os), since $C\subseteq NCl(NInt(NCl(C)))$,
where $NCl(NInt(NCl(A)))=\langle(0.7,0.6,0.6),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$, but not neutrosophic pre-open set (*NPOs*) neither neutrosophic semi-open set (*NSOs*) since $CNCl(NInt(C))$ where the $NCl(NInt(C))$ is equal $\langle(0.5,0.5,0.5),(0.3,0.5,0.5),(0.7,0.6,0.6)\rangle$.

Theorem 4.14 *Let* f be a mapping from NTS (X,τ_1) to *NTS* (X,τ_2). If f is both neutrosophic pre-continuous and neutrosophic semi-continuous, neutrosophic α -continuous.

Proof. Let B be an NOS in Y. Since f is both neutrosophic pre-continuous and neutrosophic semi-continuous, $f^{-1}(B)$ is both *NPOS* and *NSOS* in X. It follows from Theorem 3.8 that $f^{-1}(B)$ is a *N*α *OS* in X so that f is neutrosophic α -continuous.

5 Conclusion

In this chapter, we have introduced neutrosophic α-open sets, neutrosophic semi-open sets, and studied some of its basic properties. Also we study the relationship between the newly introduced sets namely introduced neutrosophic α-open sets and some of neutrosophic open sets that already exists. In this chapter also, we presented the basic definitions of the neutrosophic α-topological space and the neutrosophic α-compact space with some of their characterizations were deduced. Furthermore, we constructed a neutrosophic α-continuous function, with a study of a number its properties. Many different adaptations, tests, and experiments have been left for the future due to lack of time. There are some ideas that we would have liked to try during the description and the development of the neutrosophic topological space in the future work.

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References

[1] Florentin Smarandache, Neutrosophy and Neutro-sophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA(2002).

[2] Florentin Smarandache, An introduction to the Neutrosophy probability applid in Quntum Physics, International Conference on introducation Neutro-soph Physics, Neutrosophic Logic, Set, Probabil- ity, and Statistics, University of New Mexico, Gallup, NM 87301, USA2-4 December (2011).

[3] F. Smarandache, A Unifying Field in Logics: Neutro-sophic Logic. Neutrosophy, Neutrosophic Set, Neutro-sophic Probability. American Research Press, Reho-both, NM, (1999).

[4] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces, Journal computer Sci. Engineering, Vol.(2) No. (7), (2012), pp. 29-32.

[5] A.A. Salama and S.A. Alblowi, Neutrosophic set and neutrosophic topological space, ISORJ. Mathematics, Vol.(3), Issue(4), (2012), pp. 31-35.

[6] A.A. Salama and S.A. Alblowi, Intuitionistic Fuzzy Ideals Topological Spaces, Advances in Fuzzy Mathematics , Vol.(7), Number 1, (2012), pp. 51-60.

[7] A.A. Salama, and H. Elagamy, Neutrosophic Filters, International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), Vol.3, Issue(1), Mar (2013), pp. 307-312.

[8] S. A. Alblowi, A. A. Salama, and Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR), Vol.3, Issue 4, Oct (2013), pp. 95-102.

[9] I. Hanafy, A.A. Salama and K. Mahfouz, Correlation of neutrosophic Data, International Refereed Journal of Engineering and Science (IRJES), Vol.(1), Issue 2, (2012), pp. 39-43.

[10] I.M. Hanafy, A.A. Salama and K.M. Mahfouz, Neutrosophic Crisp Events and Its Probability International Journal of Mathematics and Computer Applications Research (IJMCAR) Vol.(3), Issue 1, Mar (2013), pp. 171-178.

[11] A. A. Salama, Neutrosophic Crisp Points and Neutrosophic Crisp Ideals, Neutrosophic Sets and Systems, Vol.1, No. 1, (2013) pp. 50-54.

[12] A. A. Salama and F. Smarandache, Filters via Neutrosophic Crisp Sets, Neutrosophic Sets and Systems, Vol.1, No. 1, (2013) pp. 34-38.

[13] S. A. Alblowi, A.A.Salama and Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol. 4, Issue 1, (2014) pp. 59-66.

[14] A.A. Salama and S.A. Alblowi, Intuitionistic Fuzzy Ideals Topological Spaces, Advances in Fuzzy Mathematics , Vol.(7), Number 1, (2012) pp 51- 60.Inter. J. Pure Appl. Math., 24 (2005), pp. 287-297.

[15] Debasis Sarker, Fuzzy ideal theory, Fuzzy local function and generated fuzzy topology, Fuzzy Sets and Systems 87, (1997), pp. 117-123.

[16] L.A. Zadeh, Fuzzy Sets, Inform and Control 8, (1965), pp. 338-353.

[17] K. Atanassov, intuitionistic fuzzy sets, in V.Sgurev, ed., Vii ITKRS Session, Sofia June 1983 central Sci. and Techn. Library, Bulg. Academy of Sciences (1984).

[18] K. Atanassov, intuitionistic fuzzy sets, Fuzzy Sets and Systems 20, (1986), pp. 87-96.

[19] K. Atanassov, Review and new result on intuitionistic fuzzy sets, preprint IM-MFAIS-1-88, Sofia, (1988).

[20] S. A. Alblowi, A.A.Salama and Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol. 4, Issue 1, (2014), pp. 59-66.

[21] A. A. Salama, F.Smarandache and Valeri Kroumov, Neutrosophic crisp Sets and Neutrosophic crisp Topological Spaces, Neutrosophic Sets and Systems, Vlo.(2),(2014), pp. 25-30.

[22] A. A. Salama, Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets and Possible Application to GIS Topology, Neutrosophic Sets and Systems, Vol. 7, (2015), pp. 18-22.

[23] A. A. Salama, Said Broumi, S. A. Alblowi, Introduction to Neutrosophic Topological Spatial Region, Possible Application to GIS Topological Rules, I.J. Information Engineering and Electronic Business,6, (2014), pp. 15-21.

[24] A. A. Salama, neutrosophic set- a generalization of the intuitionistic fuzzy set, University of New Mexico 705 Gurley Ave. Gallup, NM 87301, USA.

[25] A. A. Salama, Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed Set and Neutrosophic Continuous Functions Neutrosophic Sets and Systems, Vol. (4) 2014, pp. 4-8.

[26] I. M. Hezam, M. Abdel-Baset, F. Smarandache. Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem. In: Neutrosophic Sets and Systems. An International Journal in Information Science and Engineering, Vol. 10 (2015), pp. 39-45.

[27] El-Hefenawy, N., Metwally, M. A., Ahmed, Z. M., & El-Henawy, I. M. A Review on the Applications of Neutrosophic Sets. Journal of Computational and Theoretical Nanoscience, 13(1), (2016), pp. 936-944.

[28] Abdel-Baset, M., Hezam, I. M., & Smarandache, F. Neutrosophic Goal Programming. In: Neutrosophic Sets & Systems, vol. 11 (2016).

[29]Abdel-Basset, M., Mohamed, M. & Sangaiah, A.K. J Ambient Intell Human Comput (2017). DOI: https://doi.org/10.1007/s12652-017-0548-7

[30] Mohamed, Mai, et al. "Neutrosophic Integer Programming Problem." Neutrosophic Sets & Systems 15 (2017).