The behavior of primes

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Abstract

In this paper, we propose the axiomatic regularity of prime numbers.

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1 Introduction

In 1859, Riemann [Rie59] showed a deep connection between non-trivial zeros of the Riemann zeta-function and the prime numbers. Our motivation is to axiomatize the structure of primes.

2 Results

These below are some patterns of number.

Let $t_n$ denote the $n$th triangular number. Then

$$t_n = \binom{n + 1}{2}, \quad n \geq 1,$$

where $\binom{n}{k}$ is the binomial coefficients.

Let $F_n$ be the $n$th Fibonacci number. Then

$$F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},$$

where $n$ is a positive integer.
Let \( B_n \) be the \( n \)th Bernoulli number. Then

\[
B_n = (-1)^{n+1}n \zeta(1-n),
\]

where \( \zeta(1-n) \) is the Riemann zeta-function.

**Postulate 2.1** (Peano Postulates). Given the number 0, the set \( \mathbb{N} \), and the function \( \sigma \). Then:

1. \( 0 \in \mathbb{N} \).
2. \( \sigma : \mathbb{N} \to \mathbb{N} \) is a function from \( \mathbb{N} \) to \( \mathbb{N} \).
3. \( 0 \not\in \text{range}(\sigma) \).
4. The function \( \sigma \) is one-to-one.
5. If \( I \subset \mathbb{N} \) such that \( 0 \in I \) and \( \sigma(n) \in I \) whenever \( n \in I \), then \( I = \mathbb{N} \).

We define \( 1 = \sigma(0), 2 = \sigma(1), 3 = \sigma(2) \), etc. Next, we propose the fundamental properties of prime numbers.

**Postulate 2.2.** Given a prime number \( p \), \( \chi(n) \) denotes the number of third positive divisors of \( n \), \( \sigma(n) \) denotes the sum of positive divisors of \( n \), and \( \Delta(n) \) denotes the number of positive divisors of \( n \) besides 1 and \( n \). Then:

1. \( 2 \leq p \).
2. \( 4 \nmid p \).
3. \( (-1)^{\chi(p)} = 1 \).
4. \( 3 \leq \sigma(p) \).
5. \( \Delta(p) = 0 \).

**References**