The distribution of primes, II

Ihsan Raja Muda Nasution

October 25, 2018

Abstract

In this paper, we find the axiomatic pattern of prime numbers.

MSC: 11A41
Keywords: prime numbers, distribution of primes

1 Introduction

In 1859, Riemann [Rie59] computed the distribution of primes. Our motivation is to axiomatize the structure of primes.

2 Results

These below are some patterns of number.

Let \( t_n \) denote the \( n \)th triangular number. Then

\[
 t_n = \binom{n+1}{2} \quad n \geq 1,
\]

where \( \binom{n}{k} \) is the binomial coefficients.

Let \( F_n \) be the \( n \)th Fibonacci number. Then

\[
 F_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}},
\]

where \( n \) is an integer.

Let \( B_n \) be the \( n \)th Bernoulli number. Then

\[
 B_n = (-1)^{n+1} n \zeta(1-n),
\]

where \( \zeta(1-n) \) is the Riemann zeta-function.
Postulate 2.1 (Peano Postulates). Given the number 0, the set \( \mathbb{N} \), and the function \( \sigma \). Then:

1. \( 0 \in \mathbb{N} \).
2. \( \sigma : \mathbb{N} \to \mathbb{N} \) is a function from \( \mathbb{N} \) to \( \mathbb{N} \).
3. \( 0 \notin \text{range}(\sigma) \).
4. The function \( \sigma \) is one-to-one.
5. If \( I \subset \mathbb{N} \) such that \( 0 \in I \) and \( \sigma(n) \in I \) whenever \( n \in I \), then \( I = \mathbb{N} \).

We define \( 1 = \sigma(0), 2 = \sigma(1), 3 = \sigma(2) \), etc. We have the following postulate.

Postulate 2.2. Given a prime number \( p \), \( \tau(n) \) denotes the number of positive divisors of \( n \), \( \sigma(n) \) denotes the sum of positive divisors of \( n \), and \( \Delta(n) \) denotes the number of positive divisors of \( n \) except 1 and \( n \). Then:

1. \( 2 \leq p \).
2. \( 4 \nmid p \).
3. \( 2^{\tau(p)} = 4 \).
4. \( 3 \leq \sigma(p) \).
5. \( \Delta(p) = 0 \).

References