Proof of the Legendre’s Conjecture

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Abstract—This article solves the problem for the second time using the Formula of Disjoint Sets of Odd Numbers numbers that I proposed.

Index Terms—algorithm

I. DEFINITION OF THE LEGENDRE’S CONJECTURE

Is it true that between $n^2$ and $(n+1)^2$ there is always a prime number $(y_n)$, where $n \in \mathbb{N}^*$ and $\mathbb{N}^*$ be natural numbers without zero?

II. ALGORITHM FOR PROOF OF THE LEGENDRE’S CONJECTURE

After number 2 the sequence of primes $\{y_n\}$ enters an infinite sequence of odd numbers $\{y\}$, the formulation of the Legendre’s Conjecture must be changed to consider this sequence.

For this, if $n^2$ or $(n+1)^2$ are even numbers, they will be replaced by odd numbers $(n^2-1)$ or $((n+1)^2-1)$, respectively, which does not change the very essence of the question, since these numbers are composite.

Let the odd number $n^2 = y^2$, then the even number $(n+1)^2 = (y+1)^2 = y^2 + 2y + 1$. Let’s replace this even number in a sequence of odd numbers $\{y\}$:

$$y^2 + 2y + 1 - 1 = y^2 + 2y = y(y + 2). \quad (1)$$

Thus, let’s must consider each set:

$$\{y_n \mid y_k < y_n < y_k(y_k + 2), y_k \geq 3\}. \quad (2)$$

The number of terms in each set (2):

$$N_{y_n} = y_k - 1. \quad (3)$$

But in the sets:

$$\{y_n \mid y_k(y_k - 2) < y_n < y_k^2, y_k \geq 3\} \quad (4)$$

the number of terms is also equal to (3).

It is logical to consider (2) and (4) with respect to sets with equal $N_{y_n}$. But the segments between $y_k(y_k - 2)$ and $y_k^2$, $y_k^2$ and $y_k(y_k + 2)$ are segments in a sequence of odd numbers for which the Formula of Disjoint Sets of Odd Numbers is valid. For the entire sequence of odd numbers $\{y\}$, it has the form of the following expression:

$$Z_y = \left(0, 0 \ldots 01(1) + 33, 3 \ldots 3\%\{3y\} + \right.\!
+ \sum_{n=3}^{n \to \infty} Z_{y_n}\left(\{y_{om}y_n \mid \frac{y_n}{3} \notin \mathbb{N}^*, \ldots, \right.\!
\left.\ldots, \frac{y_{o(n-1)}}{y_{o(n-1)}} \notin \mathbb{N}^*\}\right) \to 100\%,$$

where:

$Z_y$ is of appearance of all odd numbers $y$;

the number of digits represented by $(\ldots)$ in the first two terms $\to \infty$;

$Z_{y_{om}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;

$n$ is the number of a member of a sequence of odd primes;

$y_n$ is a sequence of odd numbers with the conditions given in the formula;

$y_{o(n-1)}$ is the prime number in sequence of primes just before $y_{om}$.

For the segments (3) of (2) and (4) Formula of Disjoint Sets of Odd Numbers (5) takes the following form, where the percentage of the sets remains unchanged:

$$Z_{y_{comp}}(\{y_{comp}\}) = \left.\ldots 3%\{3y \mid y \geq 3, 3y = y_n\} + \right.\!
+ \sum_{m=3}^{m \to \infty} Z_{y_{om}}\left(\{y_{om}y_m \mid y_m \geq y_{om}, y_{om}y_m = y_n, y_{om} < N_{y_n}, \frac{y_m}{3} \notin \mathbb{N}^*, \ldots, \frac{y_m}{y_{om}(m-1)} \notin \mathbb{N}^*\}\right),$$

where:

$Z_{y_{comp}}$ is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence $\{y\}$;

$y_{comp}$ is a composite odd number in a given segment of a sequence of odd numbers $y$;

the number of digits represented by $(\ldots)$ in the first term, $\to \infty$;

$m$ is the number of a member of a sequence of odd primes;

$Z_{y_{om}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;

$y_m$ is a sequence of odd numbers with the conditions given in the formula;

$N_{y_n}$ is the number of terms in (2) or (4) (see (3)).
But since in the whole sequence of odd numbers $y$ the frequency of appearance of known sets according to (5) only tends to 100%, then in the case of (6):

$$Z_{y_{\text{comp}}} \left( \left\{ y_{\text{comp}} \right\} \right) < 100\%.$$  \hspace{1cm} (7)

That is, between $n^2$ and $(n + 1)^2$ there is always a prime number.

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REFERENCES