Black Hole Universe and Two-potentiality of Gravity

Zbigniew Osiak

E-mail: zbigniew.osiak@gmail.com

http://orcid.org/0000-0002-5007-306X

http://vixra.org/author/zbigniew_osiak

Abstract

The acceptance of the hypothesis about two-potentiality of the stationary gravitational field made it possible to find solutions of field equations and equations of motion within the General Relativity, which in extreme case lead to compliance with the Newtonian theory of gravity both inside and outside of the mass source.

Keywords: Black Hole Universe, General Relativity, intensity of the gravitational field, potential of the gravitational field, field equations, equations of motion.

1. Introduction

In the dissertation [1] I proposed a black-hole model of the Universe. Our Universe can be treated as a gigantic homogeneous Black Hole with an anti-gravity shell. Our Galaxy, together with the solar system and the Earth, which in the cosmological scale can be considered only as a point, should be located near the center of the Black Hole Universe.

From the theory of Newtonian gravity [2] we know that the absolute value of the gravitational field intensity at the center of homogeneous ball with a constant density is equal to zero. Together with growth of distance from the center – gravitational field intensity grows linearly, reaching its maximal value on the ball surface. With further growth of distance – it decreases inversely squared. In order to obtain an analogical result in Einstein's General Theory of Relativity, it should be noted that the stationary gravitational field can be described by two potentials.

2. Two-potentiality of the stationary gravitational field

The stationary gravitational field is a two-potential field [1]:

$$\frac{\partial \mathbf{E}}{\partial t} = 0, \quad \mathrm{rot}\,\mathbf{E} = 0, \quad \mathrm{rot}\,\mathrm{grad}\,\varphi = 0 \quad \Rightarrow \quad \begin{aligned} &\mathbf{E}^{in} = \mathrm{grad}\,\varphi^{in} = -\tilde{k}\,\mathrm{grad}\,\varphi^{in}, \quad 0 \le r < R, \quad \lim_{r\to 0}\varphi^{in} = 0 \\ &\mathbf{E}^{ex} = -\mathrm{grad}\,\varphi^{ex} = -\tilde{k}\,\mathrm{grad}\,\varphi^{ex}, \quad r \ge R, \quad \lim_{r\to\infty}\varphi^{ex} = 0 \end{aligned}$$

$$E_r^{in} = -\frac{4}{3}\pi G\rho r, \quad \varphi^{in} = -\frac{2}{3}\pi G\rho r^2, \quad E_r^{ex} = -\frac{GM}{r^2}, \quad \varphi^{ex} = -\frac{GM}{r}.$$

On the ball surface we have:

$$\varphi^{in} - \varphi^{ex} = \frac{GM}{2R}, \quad \mathbf{E}^{in} - \mathbf{E}^{ex} = 0.$$

$\mathbf{E}^{in}$, $\mathbf{E}^{ex}$ – gravitational field intensity respectively inside and outside of the ball
φ^{in}, φ^{ex} – gravitational field potential respectively inside and outside of the ball
M – mass of the ball, R – radius of the ball, ρ – density

$$\tilde{k} = \begin{cases} +1 & \text{outside of the mass source} \\ -1 & \text{inside of the mass source} \end{cases}$$

3. Field equations

Einstein's gravitational field equations will be written as [4]:

$$R_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right),$$

where

$$R_{\mu\nu} = \frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}} - \frac{\partial\Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}} + \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha}, \quad \Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\sigma}\left(\frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} + \frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right),$$

$$\kappa = \frac{8\pi G}{c^4} = 2.073\cdot 10^{-43}\frac{s^2}{kg\cdot m}, \quad T \overset{df}{=} g^{\alpha\beta}T_{\alpha\beta},$$

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = ict.$$

In the case where homogeneously distributed mass in the ball area, is a source of stationary gravitational field, we postulate existence of the solution in the following form:

$$(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + g_{44}(dx^4)^2,$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \quad g^{11} = \frac{1}{g^{44}}, \quad g^{22} = \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2\sin^2\theta},$$

$$T_{\alpha\beta} = -\frac{1}{2}\rho c^2 g_{\alpha\beta}, \quad T \overset{df}{=} g^{\alpha\beta}T_{\alpha\beta} = -2\rho c^2, \quad \rho = const.$$

The divergence of the tensor ($T_{\alpha\beta}$) should be equal to zero, which actually takes place:

$$T_{\alpha\beta;\beta} = \left(-\frac{1}{2}g_{\alpha\beta}\rho c^2\right)_{;\beta} = -\frac{1}{2}\rho c^2\left(g_{\alpha\beta;\beta}\right) = 0.$$

The assumptions made allow to reduce the number of field equations to two.

$$\frac{r}{2}\frac{\partial^2 g_{44}}{\partial r^2}+\frac{\partial g_{44}}{\partial r}=-\frac{1}{2}\kappa\rho c^2 r$$
$$r\frac{\partial g_{44}}{\partial r}+g_{44}-1=-\frac{1}{2}\kappa\rho c^2 r^2$$

These equations are fulfiled when

$$0\le r<R,\quad \rho=\text{const}>0,\quad g_{44}=1-\frac{4\pi G\rho}{3c^2}r^2=1-\frac{GM}{c^2R^3}r^2=1-\frac{r_S}{2R^3}r^2,$$
$$r\ge R,\quad \rho=0,\quad g_{44}=1-\frac{2GM}{c^2 r}=1-\frac{r_S}{r},\quad r\ne r_S.$$

$$M=\frac{4}{3}\pi\rho R^3$$

R – radius of the ball in which the source mass is located

$r_S=\frac{2GM}{c^2}$ – Schwarzschild radius

The presented solutions of field equations satisfy the following boundary conditions.

$$0\le r<R,\quad \lim_{r\to 0} g_{44}=1$$
$$r\ge R,\quad \lim_{r\to\infty} g_{44}=1$$

4. Signes of right-hand sides of Poisson's equations and boundary conditions

In Newton's theory of gravity, boundary conditions for the gravitational potentials

$$0\le r<R,\quad \lim_{r\to 0}\varphi^{in}=0$$
$$r\ge R,\quad \lim_{r\to\infty}\varphi^{ex}=0$$

correspond to the following forms of Poisson's equation in spherical coordinates:

$$0\le r<R,\quad r^2\cdot\frac{\partial^2\varphi^{in}}{\partial r^2}+2r\cdot\frac{\partial\varphi^{in}}{\partial r}+\frac{1}{\sin^2\theta}\cdot\frac{\partial^2\varphi^{in}}{\partial\varphi^2}+\frac{\partial^2\varphi^{in}}{\partial\theta^2}+\text{ctg}\theta\cdot\frac{\partial\varphi^{in}}{\partial\theta}=-4\pi G\rho r^2$$
$$r\ge R,\quad r^2\cdot\frac{\partial^2\varphi^{ex}}{\partial r^2}+2r\cdot\frac{\partial\varphi^{ex}}{\partial r}+\frac{1}{\sin^2\theta}\cdot\frac{\partial^2\varphi^{ex}}{\partial\varphi^2}+\frac{\partial^2\varphi^{ex}}{\partial\theta^2}+\text{ctg}\theta\cdot\frac{\partial\varphi^{ex}}{\partial\theta}=0$$

The equivalents of these relations in General Relativity are:

$$0\le r<R,\quad \lim_{r\to 0} g_{44}=1$$
$$r\ge R,\quad \lim_{r\to\infty} g_{44}=1$$

$$0\le r<R,\quad R_{\mu\nu}=-\kappa\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right)$$
$$r\ge R,\quad R_{\mu\nu}=0$$

Signes of right-hand sides of Poisson's equations depend on assumed boundary conditions, which are connected with two-potentiality of stationary gravitational field.

5. Equations of motion and field equations in General Relativity, and two-potentiality of Newton's stationary gravitational field

Postulated by us [1], within the frames of General Relativity, equations of motion of free test particle lead to a conclusion that Newton's stationary gravitational field is a two-potential field. We will show this on an example of gravitational field, which source is a mass distributed homogeneously in the volume of a ball with radius (R).

$$\tilde{a}^{\alpha} \overset{df}{=} \left(\operatorname{sgn} ds^2\right) c^2 \frac{d^2 x^{\alpha}}{ds^2} = -\tilde{k}\left(\operatorname{sgn} ds^2\right) c^2 \, \Gamma^{\alpha}_{\mu\nu} \frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}, \quad ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \neq 0, \quad \left(\operatorname{sgn} ds^2\right) g_{\mu\nu} \leq 0$$

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = ict$$

$$(ds)^2 = g_{11}(dr)^2 + r^2 (d\theta)^2 + r^2 \sin^2\theta \, (d\varphi)^2 + g_{44}\left(dx^4\right)^2$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2\theta$$

$$\Gamma^1_{11} \frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds} = -\frac{1}{2}\frac{\partial g_{44}}{\partial r}$$

$$\tilde{a}^r = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r}$$

a^r_{in}, a^r_{ex} – radial acceleration respectively inside and outside of the ball

g^{in}_{44}, g^{ex}_{44} – metric tensor components respectively inside and outside of the ball

$$g^{in}_{44} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2 R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2, \quad 0 \leq r < R, \quad \rho = \text{const} > 0$$

$$g^{ex}_{44} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_S}{r}, \quad r \geq R, \quad r \geq R, \quad \rho = 0$$

$$\tilde{a}^r_{in} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r} = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{GM}{R^3} r, \quad 0 \leq r < R$$

$$\tilde{a}^r_{ex} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g^{ex}_{44}}{\partial r} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{GM}{r^2}, \quad r \geq R$$

$$r >> r_S, \quad v^2 << c^2, \quad r_S = \frac{2GM}{c^2}$$

$$a^r = \frac{d^2 r}{dt^2}, \quad \operatorname{sgn} ds^2 = -1, \quad \tilde{k} = \begin{cases} +1 & \text{outside of the homogeneous ball} \\ -1 & \text{inside of the homogeneous ball} \end{cases}$$

$$a^r_{in} = -\tilde{k}\,\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r} = -\frac{GM}{R^3} r, \quad 0 \leq r < R$$

$$a^{r}_{ex} = -\tilde{k}\,\frac{c^2}{2}\frac{\partial g^{ex}_{44}}{\partial r} = -\frac{GM}{r^2}, \quad r \geq R$$

Substituting in the last two formulas

$$g^{in}_{44} = 1 + \frac{2\varphi^{in}}{c^2}, \quad g^{ex}_{44} = 1 + \frac{2\varphi^{ex}}{c^2},$$

where (φ^{in}) and (φ^{ex}) are potentials of stationary gravitational field respectively inside and outside of the ball, we obtain

$$a^{r}_{in} = -\tilde{k}\,\frac{\partial \varphi^{in}}{\partial r} = \frac{\partial \varphi^{in}}{\partial r} = -\frac{GM}{R^3}r, \quad 0 \leq r < R, \quad \varphi^{in} = -\frac{GM}{2R^3}r^2, \quad \lim_{r\to 0}\varphi^{in} = 0$$

$$a^{r}_{ex} = -\tilde{k}\,\frac{\partial \varphi^{ex}}{\partial r} = -\frac{\partial \varphi^{ex}}{\partial r} = -\frac{GM}{r^2}, \quad r \geq R, \quad \varphi^{ex} = -\frac{GM}{r}, \quad \lim_{r\to \infty}\varphi^{ex} = 0$$

We will also give the definitions of potentials (φ^{in}) and (φ^{ex}) corresponding to a standard definition of gravitational potential.

$$\varphi^{in} \overset{df}{=} \int_{0}^{r<R} a^{r}_{in}\,dr = -\frac{GM}{2R^3}r^2$$

$$\varphi^{ex} \overset{df}{=} -\int_{r\geq R}^{\infty} a^{r}_{ex}\,dr = -\frac{GM}{r}$$

In field equations inside of the mass source, we will replace the time-time component (g^{in}_{44}) of metric tensor with the potential (φ^{in}).

$$\frac{r}{2}\frac{\partial^2 g^{in}_{44}}{\partial r^2} + \frac{\partial g^{in}_{44}}{\partial r} = -\frac{1}{2}\kappa\rho c^2 r$$

$$r\frac{\partial g^{in}_{44}}{\partial r} + g^{in}_{44} - 1 = -\frac{1}{2}\kappa\rho c^2 r^2$$

$$\Big\downarrow \quad g^{in}_{44} = 1 + \frac{2\varphi^{in}}{c^2}$$

$$r^2 \cdot \frac{\partial^2 \varphi^{in}}{\partial r^2} + 2r \cdot \frac{\partial \varphi^{in}}{\partial r} = -4\pi G\rho r^2$$

$$2r\frac{\partial \varphi^{in}}{\partial r} + 2\varphi^{in} = -4\pi G\rho r^2$$

Poisson's equation for the potential (φ^{in})

The first of these equations is Poisson's equation for the potential (φ^{in}) in spherical coordinates. From the classical Poisson's equation it differs only by the sign of the right-hand side. In turn, from both equations it follows that

$$r^2 \cdot \frac{\partial^2 \varphi^{in}}{\partial r^2} = 2\varphi^{in}$$

We will now analyze the second field equation for the potential (φ^{in}).

$$r \cdot \frac{\partial \varphi^{in}}{\partial r} = -2\pi G \rho r^2 - \varphi^{in}$$

$$\varphi^{in} = -\frac{2}{3}\pi G \rho r^2 = -\frac{GM}{2R^3} r^2$$

$$\frac{\partial \varphi^{in}}{\partial r} = -\frac{4}{3}\pi G \rho r = -\frac{GM}{R^3} r$$

$$a^r_{in} = -\frac{4}{3}\pi G \rho r = -\frac{GM}{R^3} r$$

$$\frac{\partial \varphi^{in}}{\partial r} = a^r_{in}$$

Equation of motion for radial component of free fall acceleration inside of the mass source

By analyzing Poisson's equation for the potential (φ^{in}) we showed that the equations of motion are contained in field equations inside of the mass source.

In field equations outside of the mass source we will replace the time-time component (g^{ex}_{44}) of metric tensor with the potential (φ^{ex}).

$$\frac{r}{2}\frac{\partial^2 g^{ex}_{44}}{\partial r^2} + \frac{g^{ex}_{44}}{\partial r} = -\frac{1}{2}\kappa \rho c^2 r$$

$$r\frac{\partial g^{ex}_{44}}{\partial r} + g^{ex}_{44} - 1 = -\frac{1}{2}\kappa \rho c^2 r^2$$

$$g^{ex}_{44} = 1 + \frac{2\varphi^{ex}}{c^2}$$

$$r \cdot \frac{\partial^2 \varphi^{ex}}{\partial r^2} + 2\frac{\partial \varphi^{ex}}{\partial r} = 0$$

$$r \cdot \frac{\partial \varphi^{ex}}{\partial r} + \varphi^{ex} = 0$$

Poisson's equation for the potential (φ^{ex})

The first of these equations is Poisson's equation for the potential (φ^{ex}) in spherical coordinates. In turn, from both equations it follows that

$$r^2 \cdot \frac{\partial^2 \varphi^{ex}}{\partial r^2} = 2\varphi^{ex}$$

We will now analyze the second field equation for the potential (φ^{ex}).

$$r \frac{\partial \varphi^{ex}}{\partial r} = -\varphi^{ex}$$

$$\varphi^{ex} = -\frac{GM}{r}$$

$$\frac{\partial \varphi^{ex}}{\partial r} = \frac{GM}{r^2}$$

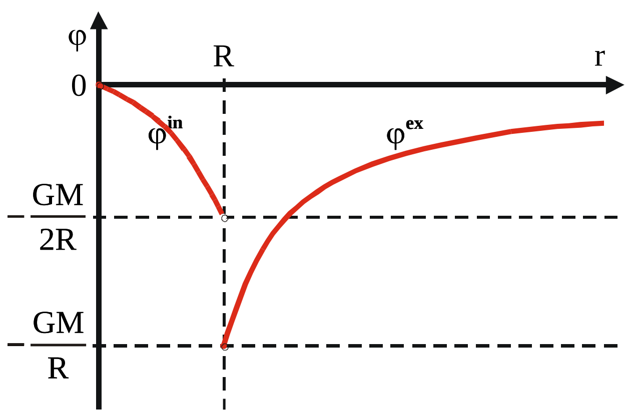
$$a^{r}_{ex} = -\frac{GM}{r^2}$$

$$\frac{\partial \varphi^{ex}}{\partial r} = -a^{r}_{ex}$$

Equation of motion for radial component of free fall acceleration outside of mass source

Analyzing Poisson's equations for the potentials (φ^{in}) and (φ^{ex}) we obtained analogical results. The equations of motion are included in the field equations.

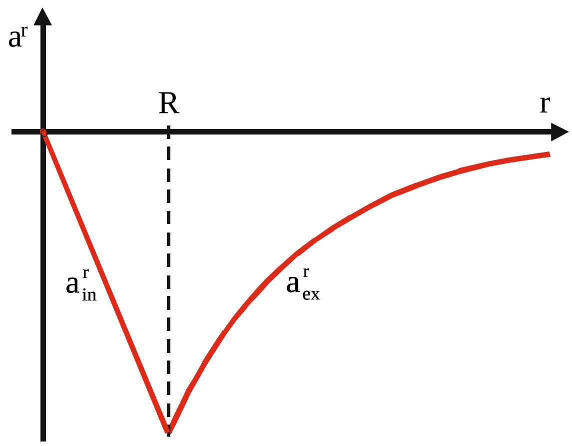
The introduction of two potentials in Newton's theory of gravity made it possible to find in the frame work of General Relativity the solutions of field equations and equations of motion that in the extreme case ($r \gg r_S$, $v^2 \ll c^2$) lead to compabillity of both theories, both inside and outside the mass source.



$$\varphi^{in} = -\frac{GM}{2R^3} r^2, \quad 0 \le r < R, \quad \lim_{r\to 0} \varphi^{in} = 0$$

$$\varphi^{ex} = -\frac{GM}{r}, \quad r \ge R, \quad \lim_{r\to\infty} \varphi^{ex} = 0$$

Diagram that shows how potentials (φ^{in}) and (φ^{ex}) depend on the distance (r) from the center of mass source in case of ($r \gg r_S$) and ($v^2 \ll c^2$).



$$a^r_{in} = -\frac{GM}{R^3}r, \quad 0 \le r < R$$

$$a^r_{ex} - \frac{GM}{r^2}, \quad r \ge R$$

Diagram that shows how radial components of free fall accelerations (a^r_{in}) and (a^r_{ex}) depend on the distance (r) from the center of mass source in case of ($r >> r_S$) and ($v^2 << c^2$).

6. Equations of motion and field equations

How should the equations of motion have to be formulated so that the unscaled radial coordinates of the four-vector acceleration were given by the following expressions?

$$\tilde{a}^r_{in} \overset{df}{=} \left(\mathrm{sgn}\, ds^2\right) c^2 \frac{d^2 r}{ds^2} = -\tilde{k}\left(\mathrm{sgn}\, ds^2\right)\frac{GM}{R^3} r = -\frac{GM}{R^3} r, \quad 0 \le r < R, \quad \tilde{k} = -1, \quad \left(\mathrm{sgn}\, ds^2\right) < 0$$

$$\tilde{a}^r_{ex} \overset{df}{=} \left(\mathrm{sgn}\, ds^2\right) c^2 \frac{d^2 r}{ds^2} = \tilde{k}\left(\mathrm{sgn}\, ds^2\right)\frac{GM}{r^2} = -\frac{GM}{r^2}, \quad r > R, \quad \tilde{k} = +1, \quad \left(\mathrm{sgn}\, ds^2\right) < 0$$

Answering this question, we will use the second of the two field equations.

$$\frac{r}{2}\frac{\partial^2 g^{in}_{44}}{\partial r^2} + \frac{\partial g^{in}_{44}}{\partial r} = -\frac{1}{2}\kappa \rho c^2 r$$

$$r\frac{\partial g^{in}_{44}}{\partial r} + g^{in}_{44} - 1 = -\frac{1}{2}\kappa \rho c^2 r^2$$

$$g^{in}_{44} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2 R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2, \quad \kappa = \frac{8\pi G}{c^4}, \quad r_S = \frac{2GM}{c^2}$$

$$\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r} = -\frac{4}{3}\pi G \rho r = -\frac{GM}{R^3} r$$

$$\tilde{a}^r_{in} \overset{df}{=} \left(\mathrm{sgn}\, ds^2\right) c^2 \frac{d^2 r}{ds^2} = -\tilde{k}\left(\mathrm{sgn}\, ds^2\right)\frac{GM}{R^3} r = -\frac{GM}{R^3} r$$

$$\tilde{a}^r_{in} = \tilde{k}\left(\mathrm{sgn}\, ds^2\right)\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r}$$

$$\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds}+\Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}=-\frac{1}{2}\frac{\partial g^{in}_{44}}{\partial r}$$

$$\tilde{a}^r_{in}=-\tilde{k}\left(\mathrm{sgn}\, ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds}+\Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right)$$

Analogical considerations for the component ($\tilde{a}^r_{ex}$) lead to a conclusion that equations of motion should have the form given below.

$$\tilde{a}^\alpha \overset{df}{=}\left(\mathrm{sgn}\, ds^2\right)c^2\frac{d^2x^\alpha}{ds^2}=-\tilde{k}\left(\mathrm{sgn}\, ds^2\right)c^2\,\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\cdot\frac{dx^\nu}{ds}$$

$$\tilde{k}=\begin{cases}+1 & \text{outside of the mass source}\\ -1 & \text{inside of the mass source}\end{cases}$$

7. New test of General Relativity

In order to show in Earth conditions the two-potentiality of the stationary gravitational field, should be measured ratio of distance passed by light to the time of flight, in a vertically positioned vacuum cylinder, right under and right above the surface of Earth (of the sea level). The difference of the squares of these measurements should be equal to the square of escape speed. This experiment would be a new test of General Relativity.

Below we will justify the desirability of the proposed experiment [1].

$$\left(\frac{dr}{dt}\right)^2_{in}-\left(\frac{dr}{dt}\right)^2_{ex}=?$$

$\left(\frac{dr}{dt}\right)_{in}$ i $\left(\frac{dr}{dt}\right)_{ex}$ – ratio of path travelled by the light to the time of travel measured respectively right under and right over Earth surface

$$\left(\frac{dr}{dt}\right)^2_{in}=c^2\left(1-\frac{r_S}{2R^3}r^2\right)^2,\ [3]$$

$$\left(\frac{dr}{dt}\right)^2_{ex}=c^2\left(1-\frac{r_S}{r}\right)^2,\ [3]$$

$$r\approx R$$

$$r_S=\frac{2GM}{c^2}$$

$$\frac{r_S}{R}<<1$$

$$\left(\frac{dr}{dt}\right)_{in}+\left(\frac{dr}{dt}\right)_{ex}\approx 2c$$

$$\frac{GM}{R} = \left(7.91\frac{km}{s}\right)^2$$

$$c = 2.99792458 \cdot 10^8 \frac{m}{s} \approx 3 \cdot 10^8 \frac{m}{s}$$

$$\left(\frac{dr}{dt}\right)_{in}^2 - \left(\frac{dr}{dt}\right)_{ex}^2 \cong \frac{2GM}{R} \qquad \left(\frac{dr}{dt}\right)_{in} - \left(\frac{dr}{dt}\right)_{ex} \cong \frac{GM}{cR} \cong 0.208\frac{m}{s}$$

8. Final remarks

The spherical coordinate system used in this paper, for (r = 0) and (θ = 0º, 180º), generates apparent singularities in expressions such as

$$g^{22} = \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2 \sin^2\theta}$$

and in the original Poisson's equation. We used this coordinate system despite the mentioned pathologies, because it is convenient in calculations.

References

[1] Zbigniew Osiak: *Anti-gravity*. viXra:1612.0062 (2016)
http://viXra.org/abs/1612.0062
[2] Zbigniew Osiak: *Black Hole Universe and Gauss' Law of Gravity.*
viXra:1806.0267 (2018), http://vixra.org/abs/1806.0267
[3] Zbigniew Osiak: *Black Hole Universe and Speed of Light.* viXra:1805.0288 (2018)
http://vixra.org/abs/1805.0288
[4] Zbigniew Osiak: *Ogólna Teoria Względności (General Theory of Relativity).*
Self Publishing (2012), ISBN: 978-83-272-3515-2, http://vixra.org/abs/1804.0178