Abstract

Our Universe originated with Finkelstein Iteration of Real Clifford Algebras from the Void (First Grothendieck Universe) to Cl(16) (Second Grothendieck Universe) whose BiVectors and two quarter-Spinors (++ and --) give E8 Physics and whose TriVectors give Fr3(O) String Theory leading to an Algebraic Quantum Field Theory (AQFT) that generalizes Hyperfinite II1 von Neumann factor Fock Space from 2-Periodic Complex Clifford Algebra to 8-Periodic Real Clifford Algebra to get the Third Grothendieck Universe.

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All Universes begin as Quantum Fluctuations of the Empty Set = Void by Quantum Fluctuation of Compact E8(-248) Real Form of E8 which is the First Grothendieck Universe and they all evolve according to David Finkelstein’s Iteration of Real Clifford Algebras:

<table>
<thead>
<tr>
<th>$n$</th>
<th>Structures</th>
<th>=</th>
<th>Grothendieck Universe</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\emptyset$</td>
<td>= Void</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>${\emptyset}$</td>
<td>= $\text{Cl}(0)$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\emptyset, {\emptyset}$</td>
<td>= $\text{Cl}(1)$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\emptyset, {\emptyset}, {\emptyset &amp; {\emptyset}}$</td>
<td>= $\text{Cl}(2)$</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$\bullet \bullet \bullet \bullet$</td>
<td>= $\text{Cl}(4)$</td>
<td></td>
</tr>
<tr>
<td>65,536</td>
<td>$\text{Cl}(2^4=16) = \text{Cl}(16)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^{65,536}$</td>
<td>$\text{Cl}(2^{16}=65,536) = \text{Cl}(65,536)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As the Finkelstein Iteration grows from the Void to $\text{Cl}(0)$ to $\text{Cl}(\text{Cl}(0))$ to $\text{Cl}(\text{Cl} ... n \text{ times} ... \text{Cl})$ the number of elements $n$ grows from 0 to $2^0 = 1$ to $2^1 = 2$ to $2^2 = 4$ to $2^4 = 16$ to $2^{16} = 65,536$ to $2^{65,536}$ ... and beyond ... so it is clear that $\text{Cl}(16)$ is the last stage of the process that is manageable for construction of a Physics Model based on Hereditarily Finite Sets which is the Second Grothendieck Universe.

**What Structures of $\text{Cl}(16)$ lead to a useful Physics Model?**

By 8-Periodicity of Real Clifford Algebras $\text{Cl}(16) = \text{Cl}(8) \times \text{Cl}(8)$ (where $\times =$ tensor product) so the graded structure of $\text{Cl}(16)$ is
Similarly, the Spinor structure of Cl(16) is

\[ \text{Cl}(8) \text{ Spinors} \times \text{Cl}(8) \text{ Spinors} = \text{Cl}(16) \text{ Spinors} \]

8-Periodicity tensor product

\[ \text{Cl}(8) \ 8 \text{ S}^+ + 8 \text{ S}^- \times \text{Cl}(8) \ 8 \text{ S}^+ + 8 \text{ S}^- = \]

\[ = \text{Cl}(16) \ 8x8 \text{ S}^{++} + 8x8 \text{ S}^{+-} + 8x8 \text{ S}^{-+} + 8x8 \text{ S}^{--} \]

\[ \text{Cl}(16) \text{ helicity consistent Spinors} = 64 \text{ S}^{++} + 64 \text{ S}^{--} = 128 \]

Cl(16) is M256(R) = 256 x 256 Matrix Algebra of Real Numbers.
Cl(8) is M16(R) = 16 x 16 Matrix Algebra of Real Numbers.

If the BiVectors are given an antisymmetric Bracket Product
then they form a Lie Algebra.

If the TriVectors are given a symmetric Jordan Product
then they form a Jordan Algebra.
The 120 BiVectors of Cl(16) with Lie Bracket Product form the D8 Lie Algebra which can be extended by the 128 helicity consistent spinors to form the 248-dimensional E8 Lie Algebra.
The structure of E8 gives a realistic Lagrangian (such Lagrangians, like Hamiltonians and Quaternions, were invented by W. R. Hamilton) that is over Octonionic 8D spacetime at high energies but at lower energies breaks symmetry to Quaternionic Kaluza-Klein spacetime M4 x CP2 where M4 is physical Minkowski 4D spacetime and CP2 = SU(3) / SU(2) x U(1) is Internal Symmetry Space of the Standard Model Gauge Group symmetries. The symmetry breaking produces the Higgs and Generations 2 and 3.

E8 / D8 = 128 = 64 + 64 = 8 components of 8 First Generation Fermion Particles + 8 components of 8 First Generation Fermion AntiParticles to give the Spinor Fermion terms of the Lagrangian Density

D8 / D4 x D4 = 64 = 8x8 = 8-dim spacetime for Lagrangian Base Manifold x 8 Fermion Types so that spacetime is a superposition of 8-dim spaces, one for each Fermion Type within which that Fermion Type propagates.

D4 = 28 = 16 + 12 where 16 = U(2,2) Conformal Group that gives Gravity + Dark Energy as well as a U(1) propagator phase and 12 = Ghosts in the M4 part of M4 x CP2 Kaluza-Klein of Standard Model

D4 = 28 = 12 + 16 where 12 = Standard Model Gauge Bosons and 16 = Ghosts in the CP2 part of M4 x CP2 Kaluza-Klein of Conformal U(2,2)

For details see viXra 1804.0121 and 1806.0388

/
To understand the Geometry related to the 240 E8 Root Vectors, consider that
248-dim E8 = 120-dim Spin(16) D8 + 128-dim half-spinor of Spin(16) D8
240 E8 Root Vectors = 112 D8 Root Vectors + 128 D8 half-spinors
112 D8 Root Vectors = 24 D4 (orange) + 24 D4 (yellow) + 64 (blue)
128 D8 half-spinors = 128 elements of E8 / D8

Green and Cyan dots with white centers (32+32 = 64 dots) and
Red and Magenta dots with black centers (32+32 = 64 dots)
correspond to the 128 elements of E8 / D8
Lagrangian

First D4

Second D4

E8 / D8

M4 CP2

A7+1 D8 / D4xD4

Mayer Mechanism

First Generation Fermions are represented by Octonions O:
CP2
A
B
M4

Second Generation Fermions are represented by Octonion Pairs OxO:
CP2
M4
A
B

NJL Higgs as Condensate of T T

Third Generation Fermions are represented by Octonion Triples OxOxO.
The 560 TriVectors of Cl(16) with Jordan Product form 10 copies of the 56-dim Fr3(O) Freudenthal Algebra each of which contains two copies of the 27-dim J3(O) Jordan Algebra of 3x3 Hermitian Octonion matrices and therefore contains two copies of the 26-dim traceless part J3(O)o that describes basic 26-dim String Theory.

To see this, start with the 56 TriVectors of Cl(8) with Jordan Product that form the Freudenthal Algebra Fr3(O)

\[
\begin{array}{ccc}
\text{Fr3(O) is Zorn-type matrices} \\
\text{where} \\
a,b,d,d',e,e',f,f' \text{ are Real Numbers} \\
\text{and} \\
S+,S',V,V',S-,S'- \text{ are Octonions} \\
* = \text{Conjugate} \\
\hline
\begin{array}{cccc}
d & S+ & V \\
a & S+^* & e & S- \\
V^* & S^{-*} & f \\
d' & S'+^* & V'^* \\
S'+ & e' & S'-^* \\
V' & S'- & f' \\
\end{array}
\end{array}
\]
and use the 16x16 Matrix Representation of Cl(8) to see how the 56 Cl(8) Trivector elements correspond to the 56 Fr3(O) elements.

To see how Fr3(O) gives String Theory look at one of the J3(O)o in Fr3(O)

One of the two 26D traceless J3(O)o parts of Fr3(O)

\[
\begin{align*}
\text{S}^+ &= \text{8 First-Generation Fermion Particles} \\
\text{S}^- &= \text{8 First-Generation Fermion AntiParticles} \\
\text{S}^+ &\text{ and S}^- \text{ are Orbifolded in the 26D String Theory Space leaving 26 - 16 = 10 dimensions of 8-dim V and 1-dim d and 1-dim f.} \\
d \text{ and f act to make 10-dim V+d+f a Conformal Space over 8-dim V with Octonionic symmetries Spin(1,9) = SL2(O) and Spin(0,8) = Spin(1,7) due to the Clifford Algebra isomorphism Cl(0,8) = Cl(1,7) = M16(R)}
\end{align*}
\]
When Octonionic symmetry is broken to Quaternionic
\( \text{Cl}(0,8) = \text{Cl}(1,7) = \text{M}16(\text{R}) \) is broken to \( \text{Cl}(2,6) = \text{M}8(\text{H}) \)
which contains \( \text{Cl}(2,4) = \text{M}4(\text{H}) \) with Conformal Spin(2,4) = SU(2,2)
so
the 10-dim \( V+d+f \) breaks to Cnf(2,4) + CP2
where Cnf(2,4) = 6-dim Vector Space of Conformal Cl(2,4)
and CP2 = SU(3) / SU(2) x U(1) = Compact Internal Symmetry Space
carrying the Gauge Group symmetries of the Standard Model.

By Twistor Correspondences 6-dim Vector Space of Conformal Cl(2,4)
contains 4-dim M4 Minkowski Physical Spacetime
so that our experiments see Spacetime as Kaluza-Klein M4 x CP2
and 8-dim \( V \) is effectively M4 x CP2 Kaluza-Klein.

Symmetry breaking from Octonionic to Quaternionic
also gives the Higgs and Generations 2 and 3 of Fermions,
with Higgs related to a Truth Quark Condensate by Nambu - Jona-Lasinio.

In this Physics Model, with Fermions propagating in Spacetime,
Strings are physically interpreted as World-Lines, according to
“... According to relativity, the world is a collection of processes (events)
with an unexpectedly unified causal or chronological structure. Then an
object is secondary ... [to]... a long causal sequence of processes, world
line. .. [if] we assemble these ... into chromosomelike code sequences ...
and braid and cross-link these strands to make more complex objects and
their interactions. ...[then]... The idea of the quantum jump
comes into its own, and reigns supreme, even over space and time. ...
”.

The 26D String Theory gives massless 24x24 symmetric traceless matrices
that have been misinterpreted by most physicists as a graviton
but are really the carriers of the Bohm Quantum Potential
for which Roderick Sutherland ( arXiv 1509.02442 ) has given a Lagrangian
that has been extended by Jack Sarfatti to include nonlinear Back-Reaction
that enables Penrose-Hameroff Quantum Consciousness and Free Will,
justifying Clifford’s characterization of Real Clifford Algebras as
“... mind-stuff tak[ing] the form of ... human consciousness ...".
The antisymmetric 24x24 matrices give a SO(24) symmetry related to the Monster Group that is the symmetry of each cell of the Planck-scale local Lorentz-Leech lattice structure of the 26D String theory. That symmetry describes the virtual clouds of particles / antiparticles that are created by String Theory Tachyons localized at Orbifolds of Fermions to produce the Schwinger Source structure of Fermions. The idea of Schwinger Sources as more than mere points is in David Finkelstein’s Space-Time Code ( Trieste March 1968 - Phys. Rev. 184, 1261 (25 August 1969) ) in which David said “... What is too simple about general relativity is the space-time point ... each point of space-time is some kind of assembly of some kind of thing ... Each point, as Feynman once put it, has to remember with precision the values of indefinitely many fields describing many elementary particles; has to have data inputs and outputs connected to neighboring points; has to have a little arithmetic element to satisfy the field equations; and all in all might just as well be a complete computer ...”.

Each Source should contain about $10^{27}$ particle/antiparticle pairs. Its size should be about $10^{(27/3)} \times 1.6 \times 10^{(-33)}$ cm = about $10^{(-24)}$ cm. It has geometry of Bounded Complex Domains with Shilov Boundaries and Bergman Kernels that give propagator Green’s Functions and allow calculation of force strengths and particle masses using ideas of Armand Wyler, with results as shown on the following two pages.

For details see viXra 1804.0121 and 1806.0388
Results of E8 Physics Calculations:

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations. Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about $10^{-24}$ cm.

(for calculation details see viXra 1804.0121)

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

<table>
<thead>
<tr>
<th>Particle/Force</th>
<th>Tree-Level</th>
<th>Higher-Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>e-neutrino</td>
<td>0</td>
<td>0 for $\nu_1$</td>
</tr>
<tr>
<td>mu-neutrino</td>
<td>0</td>
<td>$9 \times 10^{-3}$ eV for $\nu_2$</td>
</tr>
<tr>
<td>tau-neutrino</td>
<td>0</td>
<td>$5.4 \times 10^{-2}$ eV for $\nu_3$</td>
</tr>
<tr>
<td>electron</td>
<td>0.5110 MeV</td>
<td></td>
</tr>
<tr>
<td>down quark</td>
<td>312.8 MeV</td>
<td>charged pion = 139 MeV</td>
</tr>
<tr>
<td>up quark</td>
<td>312.8 MeV</td>
<td>proton = 938.25 MeV</td>
</tr>
<tr>
<td>muon</td>
<td>104.8 MeV</td>
<td>neutron − proton = 1.1 MeV</td>
</tr>
<tr>
<td>strange quark</td>
<td>625 MeV</td>
<td></td>
</tr>
<tr>
<td>charm quark</td>
<td>2090 MeV</td>
<td></td>
</tr>
<tr>
<td>tauon</td>
<td>1.88 GeV</td>
<td></td>
</tr>
<tr>
<td>beauty quark</td>
<td>5.63 GeV</td>
<td></td>
</tr>
<tr>
<td>truth quark (low state)</td>
<td>130 GeV</td>
<td>(middle state) 174 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(high state) 218 GeV</td>
</tr>
<tr>
<td>W+</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W−</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W0</td>
<td>98.379 GeV</td>
<td>$Z_0 = 91.862$ GeV</td>
</tr>
<tr>
<td>Mplanck</td>
<td>$1.217 \times 10^{19}$ GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs VEV (assumed)</td>
<td>252.5 GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs (low state)</td>
<td>126 GeV</td>
<td>(middle state) 182 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(high state) 239 GeV</td>
</tr>
<tr>
<td>Gravity Gg (assumed)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$(Gg)(M_{proton}^2 / M_{planck}^2)$</td>
<td>$5 \times 10^{-39}$</td>
<td></td>
</tr>
<tr>
<td>EM fine structure</td>
<td>$1/137.03608$</td>
<td></td>
</tr>
<tr>
<td>Weak Gw</td>
<td>0.2535</td>
<td></td>
</tr>
<tr>
<td>$Gw(M_{proton}^2 / (M_{W^+}^2 + M_{W^-}^2 + M_{Z^0}^2))$</td>
<td>$1.05 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Color Force at 0.245 GeV</td>
<td>0.6286</td>
<td>0.106 at 91 GeV</td>
</tr>
</tbody>
</table>

Kobayashi-Maskawa parameters for $W^+$ and $W^-$ processes are:

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>0.975</td>
<td>0.222</td>
<td>0.00249 -0.00388i</td>
</tr>
<tr>
<td>c</td>
<td>-0.222 -0.000161i</td>
<td>0.974 -0.0000365i</td>
<td>0.0423</td>
</tr>
<tr>
<td>t</td>
<td>0.00698 -0.00378i</td>
<td>-0.0418 -0.00086i</td>
<td>0.999</td>
</tr>
</tbody>
</table>

The phase angle $\Delta d_{13}$ is taken to be 1 radian.
E8 Physics: Higgs and Truth Quark = 3-Mass-State Nambu-Jona-Lasinio System:

Higgs at 125 GeV and Truth Quark at 130 GeV
Higgs at 200 GeV and Truth Quark at 174 GeV
Higgs at 250 GeV and Truth Quark at 220 GeV

Upper Left = Higgs-Truth Quark mass state phase diagram

Upper Center = CDF semileptonic histogram of 3 Truth Quark Mass States
FERMILAB-PUB-94/097E

Upper Right = D0 semileptonic histogram of 3 Truth Quark Mass States
hep-ex/9703008

Lower = CMS H → ZZ* → 4l histogram of 3 Higgs Mass States
arXiv 1804.01939

(for detailed comparison with experiments see viXra 1804.0121)
As to why there are 10 copies of Fr3(O) in the TriVectors of Cl(16):

8-dim spacetime is a superposition of 8-dim space E8 Lattices
   \{ 1E8 , iE8 , jE8 , kE8 , EE8 , IE8 , JE8 , KE8 \}
one for each Fermion Type within which that Fermion Type propagates.
26D String Theory has a 10-dim \( V^{+d+f} \) Spacetime Conformal over 8-dim \( V \)
so \( Cl(16) \) TriVector String Theory has an Fr3(O) for each of 10 dimensions.

The Superposition Structure of Spacetime is useful with respect to
propagation of Gauge Bosons from one Spacetime Brane to the next:
Gauge Bosons from 1E8, iE8, jE8, and kE8 parts
of a \( V \) give \( U(2,2) \) Conformal Gravity
Gauge Bosons from IE8, JE8, and KE8 parts
of a \( V \) give \( SU(3) \) Color Force
Gauge Bosons from EE8 part
of a \( V \) give \( U(2) \) Electroweak Force

Ghosts from one \( V \) Spacetime Superposition to the next
are defined similarly.
As to an Algebraic Quantum Field Theory (AQFT)

26D String Theory has a Real Clifford Algebra \( \text{Cl}(1,25) \) constructed from

\[
\text{Cl}(16) = \text{Cl}(8) \times \text{Cl}(8) \rightarrow \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) = \text{Cl}(24)
\]

to get to the Leech Lattice 24-dim Vector Space

Conformal Structure of 2x2 matrices with entries in \( \text{Cl}(24) \)
(Porteous, Clifford Algebras and the Classical Groups and
Lounesto and Porteous, Lectures on Clifford (Geometric) Algebras and Applications)
gives \( M(2, \text{Cl}(24)) = \text{Cl}(1,25) \) with Lorentz Leech Lattice Vector Space.

Since all the matrix entries are tensor product of 3 copies of \( \text{Cl}(0,8) \)
8-Periodicity allows formation of the tensor products of copies of \( \text{Cl}(1,25) \)

\[
\text{Cl}(1,25) \times \ldots (N \text{ times tensor product}) \ldots \times \text{Cl}(1,25)
\]

For \( N = 2^8 = 256 \) the copies of \( \text{Cl}(1,25) \) are on the 256 vertices
of the 8-dim HyperCube

For \( N = 2^{16} = 65,536 \) the copies of \( \text{Cl}(1,25) \) fill in the 8-dim HyperCube

William Gilbert’s web page says: “... The n-bit reflected binary Gray code
will describe a path on the edges of an n-dimensional cube that can be
used as the initial stage of a Hilbert curve that will fill an n-dim... cube. ...”.

As \( N \) grows, the copies of \( \text{Cl}(1,25) \) continue to fill the 8-dim HyperCube of
E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected
binary Gray code subdividing the initial 8-dim HyperCube into more and
more sub-HyperCubes. If edges of sub-HyperCubes, equal to the distance
between adjacent copies of \( \text{Cl}(1,25) \), remain constantly at the Planck
Length, then the full 8-dim HyperCube of our Universe expands as N grows to $2^{16}$ and beyond similarly to the way shown by this 3-HyperCube example for $N = 2^3, 4^3, 8^3$ from Wiliam Gilbert’s web page:

![3-HyperCube Example](image)

**Completion of Union of All Tensor Products of Cl(1,25) =
= hyperfinite AQFT = Algebraic Quantum Field Theory =
= the Third Grothendieck Universe**

The AQFT contains a copy of E8 within Cl(16) within each copy of Cl(1,25)

**The E8 is a Recipe for a Realistic Physics Lagrangian**

so the AQFT has a natural realistic Lagrangian structure.

The Vector Space of Cl(1,25) is the Spacetime of a 26D String Theory in which Strings are World-Lines of Particles

and

the Massless Symmetric Spin 2 State is the Carrier of the Bohm Quantum Potential with Sarfatti Back-Reaction

For details see viXra 1804.0121 and 1806.0388