Extended Standardmodel

$$ESM = SU(5)_s \times SU(3)_c \times SU(2)_L \times U(1)_{y1} \times U(1)_{y2}$$

s...sense charges  
c...color charges  
L...isospin  
y...hypercharge

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Contents

< 0 >  Introduction  1
< 1 >  The idea  2
< 2 >  The Octoquintenfield  3
< 3 >  Golden – Potential GP over the Octoquintenfield  4
  < 3.1 >  Einstein – Form of the potential GP  4
  < 3.2 >  new 28 GeV particle by the GP  6
  < 3.3 >  Planck – Form of the potential  6
  < 3.4 >  GP and the 16 – Cell  7
  < 3.5 >  Combinatorial – Form of the potential  9
< 4 >  Lagrangedensity of the Octoquintenfield/Golden – Potential GP  10
< 5 >  Curveatureensors by the Octoquintenfield  14
< 6 >  Extension of the General Relativity GR to EGR  16
  < 6.1 >  the vacuumpart of the EGR  17
  < 6.2 >  Scalefactor for the accelerated expanding universe  17
< 7 >  Getting a closed form for the Extended General Relativity EGR.  18
  < 7.1 >  combinatorial dimensionless form of the EGR.  19
  < 7.2 >  Fermion spin and the EGR.  20
< 8 >  curve discussion about the Golden – Potential GP (Zeropoints aso.)  24
< 9 >  Conclusions (what is dark energy and dark matter)  26

Appendix I  Details affine Coxeter – Group E9  27
Appendix II  Deduction of $\Lambda_0^2 = \frac{4}{48^2}$  28
Appendix III  16 – Cell  24

Introduction :

This symmetries arises by the symmetries of the coxeterelement of the affine group E9  
which is the affine one point extension of the well known exceptional group E8.

Why do we consider the E9 group (more specifically the Coxeter element of this group)?

1) E9 is an affine group and thus has something to do with extension.
2) The extension is flat as the universe.
3) The action of the Coxeter elements of the group produces symmetries involving our current standard model.

The fundamentals here:
https://en.wikipedia.org/wiki/Coxeter_group  
https://de.wikipedia.org/wiki/Tree system  
http://home.mathematik.uni-freiburg.de/soergel/Skripten/XXSPIEG.pdf

Dynkin Diagram E9 (affine one point extension of group E8):
\[ \widetilde{E}_9 \]

Derivative of the symmetries of ESM from the invariants of the Coxeter elements E9.

A Coxeter element is a product of the generating reflections of E9.

For example: Coxeter element = e1,e2,e3,e4,e5,e6,e7,e8,e9

The Coxeter polynomial is the characteristic polynomial of Coxeter elements and has the form:

\[ E_9(x) = \frac{x^5 - 1}{x - 1}, \frac{x^4 - 1}{x - 1}, \frac{x^3 - 1}{x - 1} \cdot (x - 1)^2 \]

\[ E_9CS = SU(5) \times SU(3) \times SU(2) \times U(1)^2 \]

\[ E_9(x) \quad \text{characteristic polynomial of the coxeter element of E9} \]

\[ E_9(x) \text{ is a polynorn with terms of cyclotomic factors } Z_n = \frac{x^n - 1}{x - 1} \]

for \( n > 1 \) and \( (x - 1) \) for \( n = 1 \).

The cyclotomic factors are the characteristic polynomial of the \( A_{n-1} \) which is the Dynkin diagram for the \( SU(n) \) Lie group.

See more here: https://en.wikipedia.org/wiki/Special_orthogonal_group

So finally the symmetry space by the actions of the Coxeter elements is

\[ SU(5) \times SU(3) \times SU(2) \times U(1) \times U(1) \]

- Symmetric by vertical root
- Symmetric by Translation - A.e root
- Orbital or horizontal root system

Details see https://arxiv.org/abs/1312.7781 and Appendix I

\[ \text{Compact:} \]

<table>
<thead>
<tr>
<th>Lie group</th>
<th>Coxeter - Weyl</th>
<th>name</th>
<th>count bosons</th>
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<tbody>
<tr>
<td>( SU(5) \times U(1) )</td>
<td>( A_4 )</td>
<td>Replicas</td>
<td>( 5^2 - 1 = 20 + 4 = 24 )</td>
</tr>
<tr>
<td>( SU(3) \times U(1) )</td>
<td>( A_2 )</td>
<td>Gluons</td>
<td>( 3^2 - 1 = 6 + 2 = 8 )</td>
</tr>
<tr>
<td>( SU(2) \times U(1) )</td>
<td>( A_1 )</td>
<td></td>
<td>( W^+, W^-, Z_0 )</td>
</tr>
<tr>
<td>( U(1) \times U(1) )</td>
<td>( A_0 )</td>
<td>Photon</td>
<td>( 2^2 - 1 = 2 + 1 = 3 )</td>
</tr>
<tr>
<td>( U(1) \times U(1) )</td>
<td>Graviton</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

See also https://arxiv.org/pdf/1808.05090.pdf

The action of a Coxeter element on an affine root system.

Hint:

Our symmetry is a special case of the models given by

\[ SU(n_1) \times SU(n_2) \times ... \times SU(n_k) \times U(1)^{k-1} \]

If we set \( n_1 = 5, n_2 = 3, n_3 = 2 \) and \( k = 3 \) then we get our ESM.

What bring us the additional symmetries?

1. These have the potential to describe new particles.
2. These have the potential to describe the space and time.
3. These have the potential to describe gravity.
< 1 > The Idea

Light and gravitation just like photon and graviton have something in common.
Both are massless and propagate with the speed of light.

We know that light by the
Symmetry breaking 1: \( SU(2) \times U(1) \rightarrow U(1) \) is described as a mixture.
So light is a part of the electro – weak interactions.
we consider analog gravity as a result of a further symmetry breaking
Symmetry breaking 2: \( SU(5) \times U(1) \times U(1) \rightarrow U(1) \)g

Our extended standard model allows us this.

We will now like to assign our relevant \( SU(n) \)'s to division algebras (real numbers, complex numbers, ...).

\( SU(1) \rightarrow \mathbb{R} \)
\( SU(2) \rightarrow \mathbb{C} \)
\( SU(3) \rightarrow \mathbb{H} \)
\( SU(5) \rightarrow \mathbb{O} \)

This 4 division algebras can be generated by the doubling process
(see more at https://de.wikipedia.org/wiki/Verdopplungsvorfahren).

Considering the rank of the \( SU(2) = 1, SU(3) = 2, SU(5) = 4 \) then this is double as well.

There appears to be a connection between the division algebras and the \( SU(n) \) (\( n = 2, 3, 5 \)).

The connections are the rank (maximal torus) of the \( SU(n) \) and the orthogonal complex subspaces of the division algebra.

For example the quaternions have two orthogonal complex subspaces
\( a + b.i \) and \( c.j + d.k \) (a, b, c, d real).

The \( SU(3) \) has also 2 neutral elements (toris).

With this assignment we can create background fields to the \( SU(3) \) and \( SU(5) \) like it is the higgsfield for \( SU(2) \).

\( SU(1) \rightarrow \mathbb{R} \) real singlet
\( SU(2) \rightarrow \mathbb{C} \) complex doublet  higgsfield
\( SU(3) \rightarrow \mathbb{H} \) quaternionic triplet
\( SU(5) \rightarrow \mathbb{O} \) octonionic quintet  Octoquintenfeld

Therefore, we rely analogously on the Higgsfield (\( 2 \times \text{complex} = \text{doublet} \)).

\[ \phi = \begin{bmatrix} \phi^+ \\ \phi^0 \\ \end{bmatrix} = \begin{bmatrix} \phi_1^+ + i.\phi_2^+ \\ \phi_3^0 + i.\phi_4^0 \\ \end{bmatrix} \]

\(< 2 > \) the Octoquintenfield (\( 5 \times \text{Octonions} = \text{Quintet} \)).

\[ \phi = \begin{bmatrix} \phi^0 \\ \phi^R \\ \phi^F \\ \phi^C \\ \phi^O \\ \end{bmatrix} = \begin{bmatrix} \phi_0^0 + i_1.\phi_1^0 + i_2.\phi_2^0 + i_3.\phi_3^0 + i_4.\phi_4^0 + i_5.\phi_5^0 + i_6.\phi_6^0 + i_7.\phi_7^0 \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^C + i_1.\phi_1^C + i_2.\phi_2^C + i_3.\phi_3^C + i_4.\phi_4^C + i_5.\phi_5^C + i_6.\phi_6^C + i_7.\phi_7^C \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \\ \end{bmatrix} \]

This provides 40 degrees of freedom.
24 of which will be "spent" for our \( SU(5) \) tensor bosons for the 5th longitudinal spin degree of freedom
(24 Goldstone bosons swallowed over gauge transformation) thus remain 16 left.
The S, F, R, G and H charges are the 5 charges of the \( SU(5) \) analogous to the 3 color charges of \( SU(3) \) and
the 2 charges (±) of \( SU(2) \).
The letters stand for S = See, F = feeling, R = smelling G = Taste and H = Hear
Calling therefore the charges of the \( SU(5) \) sense charges.
Note: These charges have (such as the color charges of quarks with color) nothing to do with the senses,
but to give a name to the child for reference only.

We now want to look at the 16 (40 – 24 – 16) remaining degrees of freedom.
Make the following division for the 40 field components of the Octoquinten field as a physical approach:
Analogous to the Higgs potential we declare a Potential on the Octoquinten field

\[ V(\phi) = \frac{\mu^2}{2} |\phi|^2 + \frac{\lambda_3}{4} |\phi|^4 + \frac{\gamma^2}{8} |\phi|^6 \quad \text{with} \quad \phi \in \mathbb{O}^8 \]

\[ |\phi|^2 = \phi^\dagger \phi \]

\[ \gamma, \mu \in \mathbb{R} \text{ (imaginary) and} \quad \lambda \in \mathbb{R} \]

\[ \frac{\mu^2}{2} \quad \text{momentum density}^2 \quad (\text{kg})^2 = \left( \frac{\text{kg} \ m}{\text{s}} \right)^2 \]

\[ \frac{\lambda_3}{4} \quad \text{mass density}^2 \quad (\text{kg})^2 = \left( \frac{\text{kg} \ m}{\text{s}^2} \right)^2 \]

\[ \frac{\gamma^2}{8} \quad \text{spin density}^2 \quad (\text{m/s})^2 = \left( \frac{\text{kg} \ m}{\text{s}^4} \right)^2 \quad \text{Spin here is used not in the sense of} \ Spin = \text{action} \]

\[ V(\phi) = B^2 \quad \text{energy density}^2 \quad (\text{kg} \ m^2)^2 = \left( \frac{\text{kg} \ m^2}{\text{s}^2} \right)^2 \]

The coefficients of the potential come from self interactions. Therefore we make the assumption that we have the following relation:

\[ C := \frac{-\lambda_3}{\gamma^2} = \frac{4 \mu^2}{\lambda_3^2} \quad \gamma^2, \mu^2 < 0 \quad \lambda_3^2 > 0 \]

Then it follows by exact calculation that

\[ C = c^4 \varphi^2 \]

\[ c \ldots \text{speed of light} \]

\[ \varphi \ldots \text{golden ratio} = 1.6180... \]
Because of the appearance of the golden ratio and some nice properties of it we call the potential the **Golden – Potential** short GP.

The first angle which comes from the minimum of the Octoquartic potential is appr. equal to the
\[ \text{W} \text{E} \text{I} \text{N} \text{B} \text{E} \text{R} \text{G} \text{ - A} \text{N} \text{G} \text{E} \approx 28.89^\circ \]
see \(< 8 \>

Comparing the Golden – Potential GP with the standard relativistic energy (density) equation:
\[ E^2 = p^2 c^2 + m^2 c^4 \quad p = \text{momentum}; m = \text{mass} \]

Our GP has a third term and expand the equation to
\[ E^2 = p^2 c^2 + m^2 c^4 + s^2 c^6 \]

we call the third term the spin – term.

\(< 3.1 > \) **Einstein – Form**

We want that the second part of the Golden – Potential is our quadratic vacuum energy density:
\[ \frac{\lambda^2}{4} |\phi|^4 = \left( \frac{\Lambda c^4}{8\pi G} \right)^2 \left( \phi_{\text{vacuum}} c^2 \right)^2 \]

\( \Lambda \) ... cosmological constant
\( \phi_{\text{vacuum}} \) ... vacuum mass density

then with the relation \( c^4 \phi^2 = \frac{\lambda^2}{\gamma^2} = (4 \frac{\mu^2}{\chi^2})^2 \) we get the potential as

\[
V(\phi) = \frac{\Lambda c^4}{8\pi G} \left( \frac{\gamma}{2} \left| \frac{\phi}{c} \right| \right)^2 + \left( \frac{\gamma}{2} \left| \frac{\phi}{c} \right| \right)^4 - \frac{1}{2} \frac{1}{\gamma^2} \left( \frac{\gamma}{c} \right)^4
\]

**EINSTEIN – FORM**

We have 3 "spheres" where the potential vanishes.
The first in the center which is a point. We name it \( S_0 \). Then one with \( |\phi| = c - 1 \) which we name \( S_c \) and one with \( |\phi| = \sqrt{\gamma} c = \sqrt{\gamma} \) which we name \( S_{\sqrt{\gamma}} \).

Compact \( (S_0, S_c, S_{\sqrt{\gamma}}) \) for the zero subspace.

Now we want to take a look on a complex subspace of our definition range \( \mathbb{C}^{\gamma} \).
For example \( C = \{ \phi_0 + i, \phi_1 \} \)

- **particle division**
  - **particle without mass**
  - **photons**
  - **gravitons**
  - **particle with mass**
  - **flat**
  - **particle with imaginary mass**
  - **tachyons**

- **division**
  - **elliptic**
  - **hyperbolic**

Diagram:

- **golden hyperbola**
- **focus of the hyperbola**
- **\( S_0, S_c, S_{\sqrt{\gamma}} \)**
- **\( \phi_0, \sqrt{\gamma} \)**
- **\( c - 1, \sqrt{\gamma} c \)**
- **\( \phi_1, \phi_0 \)**
- **\( i, \phi_0 \)**
- **\( \phi_1, i \)**
We have 3 "cycles" $S_0, S_1, S_{/\pi}$ for the 3 different curvatures, elliptic, flat, hyperbolic on any point $\psi$ with $|\psi| = c = 1$.
The cycles build the foci points of the different curvatures.
The cycle $S_0$ which is the origin 0 is the foci point (center point) for the elliptic curvature on c.
The cycle $S_1$ is the set of foci points for the flat curvature.
The cycle $S_{/\pi}$ is the set of foci points for the hyperbolic curvature.

This can be extended to quaternionic subspaces of our definition range $O^4$.
For example $\Omega = \{v_0 + i_1v_1 + i_2v_2 + i_3v_3\}$

Then instead of cycles $S^4$ we have $S^3$ three spheres where the potential vanishes.

< 3.2 > Getting a new particle by the Octoquintenfield of $m \approx 28$ GeV mass.

A second particle can be maybe found by the first term of the Golden Potential.

Hint: The Higgs mass as calculated above comes from the second term of the Golden Potential.

$$V(\phi) = \frac{\Lambda \phi^2}{8\pi G^2} \left[ -\frac{c}{2} \left( \frac{\phi}{c} \right)^2 \right]$$
leads to

$$\frac{1}{4} m_H \sqrt{\frac{c}{2}}$$

On the quadratic term we have the factor $\frac{\sqrt{c}}{2}$.

This gives us an energycalation of $\frac{1}{4} \sqrt{\frac{c}{2}} \approx \frac{1}{4 \cdot 44714376} \approx 0.022486$

The mass of the Higgsberson is $m_H \approx 125.15 \pm 0.16$ GeV
The mass of the Octoquintenboson then is $m_\phi \approx \frac{m_H}{4 \cdot 44714376} \approx 28.1481$ GeV

This is very near to the maybe found new particle on the LHC (https://arxiv.org/abs/1808.01890).
This is not a proof just a first idea for the new particle!
We will see if the new particle will be reconquired.

< 3.3 > Planck - Form

With the relation

$$P_{\pi / c}^2 = c^4$$

we get from the Einstein - Form the Planck - Form of the GP.
PLANK – FORM

\[ V(\phi) = \frac{1}{2} \left( \frac{P_p}{2\pi} \right)^2 \left( \frac{\Lambda^2}{4} \right)^2 \left( -\frac{\phi}{c\sqrt{\phi}} \right)^4 + 2 \left( \frac{\phi}{c\sqrt{\phi}} \right)^4 - \left( \frac{\phi}{c\sqrt{\phi}} \right)^8 \]

where

\[ P_p = \frac{c^5}{\hbar G^2} \text{ Planck pressure} \]

\[ \Lambda_p^2 = \frac{\hbar^2}{c^3} \text{ Planck length}^2 \]

\[ \Lambda_p^2 \approx \frac{2.6}{10^{112}} \approx \frac{4}{48^2} \text{ dimensionless} \]

This assumption \[ \approx \Rightarrow \] is backcalculated from the
Combinatorial – Form of the Golden – Potential GP
which you can see later in this document.

On the combinatorial form the normalization factor \( N \) is very easy to explain
and it leads to this assumption by going backward to the
Planck – Form of the Golden – Potential GP

if \( \Lambda_p^2 \approx \frac{4}{48^2} \) then we can interprete

\[ N = \frac{\sqrt{5}}{2} \frac{1}{48!} \] as normalization factor

then our potential has the form

\[ V(\phi) = \left( \frac{P_p}{2\pi} \right)^2 N^4 \left( -\frac{\phi}{c\sqrt{\phi}} \right)^4 + 16 \left( \frac{\phi}{c\sqrt{\phi}} \right)^4 - 8 \left( \frac{\phi}{c\sqrt{\phi}} \right)^8 \]

In < 3.1 > we have defined the potential in a way so that the second
term of the potential is the quadratic vacuumenergysdensity on \( |\phi| = c \).

Here we can see clearly that the vacuumenergysdensity is :

\[ \sqrt{V(c)} = \frac{P_p}{2\pi \cdot 48^2}, P_p \text{ Planck pressure} \]

So we can say that we have such a low vacuumenergysdensity because we have
a lot of coordinates (48 counted) and the vacuumenergysdensity comes from
self interactions of the permutations.

For simplification we want to set \( c = \hbar = G = 1 \).

then the potential polynomial can be written in determinant form :

\[ V(\phi) = 8 \cdot \begin{vmatrix} \frac{\sqrt{5}}{2} & 1 \end{vmatrix}^4 \begin{vmatrix} 1^2 & 1^2 & 0 & \left( \frac{\phi}{c\sqrt{\phi}} \right)^2 \\ 0 & \left( \frac{\phi}{c\sqrt{\phi}} \right)^2 & 0 & 1^2 \\ 0 & 0 & \left( \frac{\phi}{c\sqrt{\phi}} \right)^2 & 0 \\ \left( \frac{\phi}{c\sqrt{\phi}} \right)^2 & 1^2 & 0 & 1^2 \end{vmatrix} \]

with \( \phi \in \mathbb{C}^6 \)

On this form we can see why the normalization factor has fourth power (4 x 4 matrix).

why

the Octoquidentinefield has \( 5 \times 8 = 40 \) (d's) degrees of freedome.
Because of some necessity we allow \( \phi \in \mathbb{C} \) if \( \phi \) is in the
zero charged line of the Octoquidentinefield.
Then we have 40 = 4 x 8 + 2 x 8 degrees of freedome on the OQF.
For every value of $|\phi|^2$ we can write

$$|\phi|^2 - \sum_{i=1}^{48} \phi_i^2 \quad \text{with} \quad \phi_i \in \mathbb{R}$$

then for every permutation of the 48 degrees of freedom (coordinates) we get the same value for $|\phi|^2$.

That is why we have the factor $\frac{1}{48!}$ in the normalization factor $N$.

where

$$\phi_1...\phi_{16} = \phi_1^0...\phi_2^0 \quad \text{and}$$
$$\phi_{17}...\phi_{24} = \phi_3^0...\phi_4^0 \quad \text{and}$$
$$\phi_{25}...\phi_{32} = \phi_5^0...\phi_6^0 \quad \text{and}$$
$$\phi_{33}...\phi_{40} = \phi_7^0...\phi_8^0 \quad \text{and}$$
$$\phi_{41}...\phi_{48} = \phi_9^0...\phi_{10}^0 \quad \text{and}$$

$S, F, R, G, H$ are the five senses – charges and $O$ is no charge.

< 3.4 > Golden Potential and the 16-Cell (Coxeter group $B_4, D_4$):

```
As seen above we can write the potential very simple as a product of
V = Norm factor × Determinant
The Determinant is like a higher dimensional "Volume".
In our case the dimension is 4.
Don't misunderstood "Volume" as real spactime-volume.
It is similar to the Determinant in the Einstein Hilbert action.
```

Now we want to go TopDown and take a look on the 16-cell $C_{16}$.

Some important known facts about the $C_{16}$:

1) count of cells = 16 tetrahedra
2) count of faces = 32 triangle
3) count of edges = 24
4) count of vertices = 8
5) every vertex has 6 edges
6) The Euler Characteristic of the 16-Cell is zero:
$$\chi = k_0 - k_1 + k_2 - k_3 = \#\text{vertices} - \#\text{edges} + \#\text{faces} - \#\text{cells} =$$
$$8 - 24 + 32 - 16 = 0$$
7) The order of the automorphism group $S_4\times S_4$ is:
$$|\text{Aut}(C_{16})| = 2^4 \times 4! = 384$$

How can we compare the 16-Cell with our Potential?

Es mentioned before our potential is like a 4-dimensional "volume".
More exact it is a sum of 3 (4) volumes.

$$V(z) = \frac{N^4}{2} (0.z^0 - 16.z^1 + 32.z^2 - 16.z^4)$$

where $z = \frac{\phi}{\sqrt{\phi}}^2$ and
$$c = G = h = 1$$
To make the potential zero we set \( z = 1 \) or equivalent \( \phi = \sqrt{r} \)

\[
V(1) = \frac{N^4}{2} \left( 0 \cdot 1^0 - 16 \cdot 1^1 + 32 \cdot 1^2 - 16 \cdot 1^3 \right) = 0 \text{ and compare it with}
\]

\[
\chi = k_0 - k_1 + k_2 - k_3 = 8 - 24 + 32 - 16 = 0 \text{ Eulc - Characteristic}
\]

we can see that both formulars alternate
and if we draw out 16 we get for the potential

\[
V(1) = N^4 \cdot \frac{1}{2} \left( -16 \cdot 1 + 32 \cdot 1^2 - 16 \cdot 1^3 \right)
\]

\[
\begin{array}{c}
\text{vertices} \\
\text{faces} \\
\text{cells}
\end{array}
\]

\[
\begin{array}{c}
8 - 24 \\
32 \\
16
\end{array}
\]

We have two disagreements

1) we have no vertices in the potential.

The vertices are added to the edges.

2) The power of the 1 which should be the dimension of the object

...does fit for the edges and the faces but not for the cells!

to 1)

How can we annihilate the vertices?

Answer: we replace it by a loop (string) and get a closed edge.

We can interpret this loops as particles.

\[
\begin{array}{c}
\text{or}
\end{array}
\]

\[
\begin{array}{c}
\text{or}
\end{array}
\]
the cells can be easily extended to fourth dimension on a convex graph.

2 - dimension example:

So finally the potential shows 1 - dimensional, 2 - dimensional and 4 - dimensional stimulated objects.

Interpretation:
1 - dimension objects are particles
We have two types of 1 - dimensional object. Closed and open ones.

\[ V(1) = N^3 \cdot \frac{1}{2}\left(-16 \cdot 1^1 + 32 \cdot 1^2 - 16 \cdot 1^4\right) \]

With the other zero point (\( \phi = 0 \)) we can create the same particle graph \( C_6 \).

Then is \( \frac{1}{\phi} \)

\[ V(2) = V\left(\frac{1}{\phi}\right) = N^3 \cdot \frac{1}{2}\left(-16 \cdot \left(\frac{1}{\phi}\right)^1 + 32 \cdot \left(\frac{1}{\phi}\right)^2 - 16 \cdot \left(\frac{1}{\phi}\right)^4\right) \]

\(< 3.5 > \) Combinatorial - Form

We have the strange factor \( \frac{1}{2} \) in front of the bracket.
How can we interpret this factor?

We can draw this factor inside the brackets and set finally

\[ N = \sqrt[18]{\frac{1}{2}} \]

then we get finally the

**COMBINATORIAL - FORM**

\[ V(\phi) = N^3 \cdot \left(-8 \cdot \left(\frac{\phi}{\sqrt{\phi}}\right)^2 + 16 \cdot \left(\frac{\phi}{\sqrt{\phi}}\right)^4 - 8 \cdot \left(\frac{\phi}{\sqrt{\phi}}\right)^8\right) \]

for the potential.

In mathematics this is called a generating function.
2) the red $-8$ (edges).

As seen above the $8$ comes from $-8 = 4 - 12$.

4 particles in the diagonal $\rightarrow$ zero curvature tensor

12 particles $\rightarrow$ 3) the blue $16$ (faces).

This is the count of the $16 = 4 + 2.6$ fields of the first curvature tensor 
$(\approx$ metric tensor $)$ in diagonal $\rightarrow$

4) the green $8$ (cells).

This is the count of the $8 - 2 + 2.3$ fields of the second curvature tensor 
$(\approx$ spin metric tensor $)$ in diagonal $\rightarrow$

Details to this curvature tensor see later:

Some analytics on the Golden – Potential

without absolute values and $\phi \in \mathbb{C}$ see $\langle 8 \rangle$

$$V(\phi) = N^4 \left( -8 \left( \frac{\phi}{\sqrt{\phi}} \right)^2 + 16 \left( \frac{\phi}{\sqrt{\phi}} \right)^4 - 8 \left( \frac{\phi}{\sqrt{\phi}} \right)^8 \right)$$

$$N = \frac{\sqrt{\phi}}{2} \frac{1}{18^4}$$

<table>
<thead>
<tr>
<th>Zeropoints</th>
<th>momentum density $^2$</th>
<th>mass density $^2$</th>
<th>spin density $^2$</th>
<th>$\times \frac{1}{48^4}$</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\sum = 0$</td>
</tr>
<tr>
<td>$\pm 1$</td>
<td>$-\frac{\phi^2}{2}$</td>
<td>1</td>
<td>$-\frac{1}{2\phi^2}$</td>
<td>$\sum = 0$</td>
</tr>
<tr>
<td>$\pm \sqrt{\phi}$</td>
<td>$-\frac{\phi^2}{2}$</td>
<td>$\phi^2$</td>
<td>$-\frac{\phi^2}{2}$</td>
<td>$\sum = 0$</td>
</tr>
<tr>
<td>$\pm i\phi$</td>
<td>$\frac{\phi^3}{2}$</td>
<td>$\phi^4$</td>
<td>$-\frac{\phi^6}{2}$</td>
<td>$\sum = 0$</td>
</tr>
<tr>
<td>quadratic sum $= 0$</td>
<td>$\sum = 0$</td>
<td>$\sum = 4\phi^2$</td>
<td>$\sum = -4\phi^2$</td>
<td>$\sum = 0$</td>
</tr>
</tbody>
</table>

Some possible deductions by $\Lambda l_p^2 = 4 \times 48^2$

We want to write this way $(\frac{48^2}{2})^2 = (\frac{\Lambda}{2 l_p})^2 = \frac{\Lambda^2}{4 l_p^2}$ with $\Lambda = \frac{2}{\sqrt{3}}$

1) Entropy in the universe $= 48^2$ UOE

We know from Bekenstein Hawking that $\Lambda l_p^2$ is one unit of entropy short UOE.

Setting $c = \hbar = G = 1$ then $l_p = 1$ then

$$\frac{48^2}{4} = \Lambda \quad \text{then} \quad S_{\text{universe}} = \frac{\Lambda^2}{4} = 48^2 \approx 1,541 \times 10^{122}$$

is the count of Entropy in the universe.

Later in $\langle 6.1 \rangle$ we will see that $\Lambda$ belongs to a double Clifford – Torus.

We can use this 2-dimensional flat object as the Entropy object for the universe.

This object can give an answer to the holographical principle.
2) Mass in the universe = \( \frac{48!}{2} \cdot m_p \)

thinking that the universe is like a black hole we get by the
Bekenstein Hawking Entropy and \( c - h - G - k - 1 \):

\[ S_{\text{universe}} = (2 \cdot M_{\text{universe}})^2 \quad S_{\text{Entropy}, M_{\text{Mass}}} \]

Then with the result above it follows that:

\[ M_{\text{universe}} = \frac{48!}{2} \cdot m_p \approx 0.75 \times 10^{30} \text{ GeV} \quad \text{with } m_p \text{...Planck mass} \]

< 4 > Lagrangedensity of the Octoquintenfield/Golden - Potential

Hint:
I do the same steps as shown in this cooking recipe for the Higgsfield.
https://www.lsw.uni-heidelberg.de/users/mammen/HDhiggs.pdf

Splitting of the Octoquintenfield

for charged tensorbosons

for neutral tensorbosons

for twisting and expanding universe

Similar to the SU(3) Vectorbosons which are named Gluons
we name our SU(5) Tensorbosons Repetions.
The force between different charged Repetions is repulsive because they are
tensorbosons (2nd - order).

Similar to the Higgsfield we assign our Repetions to the
Octoquintenfield by the following scheme.
The numbers are the sense charges (see < 2 >).

5 = See
4 = Feeling
3 = Smelling
2 = Taste
1 = Hear
similar to the higgsfield where the vacuum expectation is

\[ \phi_{\text{vac}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

the vacuum expectation of the Octoquintenfield is (green, yellow, orange)

\[ \phi_{\text{vac}} = \nu \cdot \begin{pmatrix} 0 + i_1 + i_2 + i_3 \\ 1 + 0 + i_2 + i_3 \\ 1 + i_1 + 0 + i_3 \\ 1 + i_1 + i_2 + 0 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix} \]

where \( \nu = \nu_1 \) is the minimum of the Golden Potential GP and \( i_1, i_2 \) and \( i_3 \) are the imaginary quaternions.

As mentioned above we assume that the left 4 charged bosons decomposed to electrons, neutrinos and quarks and couple to the higgsfield instead. Then the vacuum expectation changes to:

\[ \phi_{\text{vac}} = \nu \cdot \begin{pmatrix} 0 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix} \]

**STEP 1**: Lorentz invariant Lagrangedensity for the Octoquintenfield

\[ \mathcal{L}_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi) \]

with \( \phi \in \mathbb{O}^5 \) Octonions

The potential \( V \) is shown in \(<3>\)
\[ \mathcal{T}_{ij} \]

Generators of the SU(5)

\[
\begin{array}{|cccc|cccc|cccc|}
\hline
i & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

\[ W - \text{Boson scheme} \]

\[
\begin{pmatrix}
W_{11} & W_{12} & W_{23} & W_{14} & W_{15} \\
W_{21} & W_{22} & W_{24} & W_{25} \\
W_{31} & W_{32} & W_{33} & W_{34} \\
W_{41} & W_{42} & W_{43} & W_{44} \\
W_{51} & W_{52} & W_{53} & W_{54} \\
\end{pmatrix}
\]

hint: \[ W_{ij} = W_{ji} \]

We take a look on the symmetry

SU(5) \times U(1) \times U(1)

\[ W_{ij}, B^0, B^1 \]

calculate covariant derivation

\[ D_{\mu}\phi = \left( \partial_{\mu} + \frac{ig}{2} \tau_{ij} W_{ij}^\mu + \frac{ig}{2} \left( Id_{\mu} B^0 + \frac{ig}{2} \left( Id_{\mu} B^1 \right) \phi \right) \right) \]

\[ \tau_{ij}, W_{ij}^\mu = \begin{pmatrix}
H_{11} & [H_{11} - i H_{12}] & [H_{11} - i H_{13}] & [H_{11} - i H_{14}] & [H_{11} - i H_{15}] \\
H_{21} + i H_{22} & [H_{22} + i H_{23}] & [H_{22} + i H_{24}] & [H_{22} + i H_{25}] \\
H_{31} - i H_{33} & [H_{31} - i H_{33}] & [H_{31} - i H_{34}] & [H_{31} - i H_{35}] \\
H_{42} - i H_{42} & [H_{42} - i H_{42}] & [H_{42} - i H_{42}] & [H_{42} - i H_{42}] \\
H_{53} & [H_{53}] & [H_{53}] & [H_{53}] \\
\end{pmatrix} \]

\[ \tau_{ij}, W_{ij}^\mu = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix} \]

and for example

\[ W_{12} = \frac{W_{11} + i W_{12}}{\sqrt{2}} \]

The boson which changes the charge from 1 (heav) to 2 (taste).
Then

\[
D_\mu \phi_{\text{vac}} = \frac{\nu}{2} \cdot i \cdot g \cdot \begin{pmatrix}
\sqrt{2} W_{1}^0 & \sqrt{2} W_{3}^0 & \sqrt{2} W_{1}^1 & \sqrt{2} W_{3}^1 \\
\sqrt{2} W_{2}^0 & \sqrt{2} W_{5}^0 & \sqrt{2} W_{2}^1 & \sqrt{2} W_{5}^1 \\
\sqrt{2} W_{1}^0 & \sqrt{2} W_{3}^0 & \sqrt{2} W_{1}^1 & \sqrt{2} W_{3}^1 \\
\sqrt{2} W_{2}^0 & \sqrt{2} W_{5}^0 & \sqrt{2} W_{2}^1 & \sqrt{2} W_{5}^1 \\
\end{pmatrix} \cdot \begin{pmatrix}
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
\end{pmatrix}
\]

\[
+ \frac{\nu}{2} \cdot i \cdot \begin{pmatrix}
g' B^0 + g'' B^1 & 0 & 0 & 0 \\
0 & g' B^0 + g'' B^1 & 0 & 0 \\
0 & 0 & g' B^0 + g'' B^1 & 0 \\
0 & 0 & 0 & g' B^0 + g'' B^1 \\
\end{pmatrix} \cdot \begin{pmatrix}
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
1 + i_1 + i_2 + i_3 \\
\end{pmatrix}
\]

then

\[
(D^\mu \phi_{\text{vac}})^\dagger (D_\mu \phi_{\text{vac}}) = \frac{\nu^2}{4} \left[ 4(g W'^{00} + g' W'^{01} + g'' W'^{11} + g'' W'^{11}) + 4(g W'^{00} + g' W'^{01} + g'' W'^{11} + g'' W'^{11}) + 4(g W'^{00} + g' W'^{01} + g'' W'^{11} + g'' W'^{11}) + 4(g W'^{00} + g' W'^{01} + g'' W'^{11} + g'' W'^{11}) \right] + \text{something} + \text{something} + \text{something} + \text{something} + \text{something} + \text{something} + \text{something} + \text{something}
\]

\[\text{hint: } W'^{ij} = W'^{ij}_\mu \text{ and } B^0 = B^0_\mu \text{ and } B^1 = B^1_\mu\]

take the result of the Higgsfield we expect something like that:

\[
(D^\mu \phi_{\text{vac}})^\dagger (D_\mu \phi_{\text{vac}}) = \frac{\nu^2}{8} \cdot (g^2.(W'^{00})^2 + g^2.(W'^{01})^2 + (g^2 + g'^2).Z_\mu Z^\nu + 0.A_\mu.A^\nu)
\]

We have a lot of summands so we first want to take a look on the diagonal elements of the covariant derivation.

In the Higgsfield theory we get as result the massive Z − Bosons and the Photon as a mixing of neutral W and B bosons.

We calculate the expression which is a symmetric bilinear form:

\[
\begin{pmatrix}
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
\end{pmatrix}
\begin{pmatrix}
8g^2 & 4g^2 & 4g^2 & 4g^2 \\
4g^2 & 8g^2 & 4g^2 & 4g^2 \\
4g^2 & 4g^2 & 8g^2 & 4g^2 \\
4g^2 & 4g^2 & 4g^2 & 8g^2 \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
W'^{00} & W'^{01} & W'^{11} & B^0 & B^1 \\
\end{pmatrix}
\begin{pmatrix}
20g^2 & 20g^2 & 20g^2 \\
20g^2 & 20g^2 & 20g^2 \\
20g^2 & 20g^2 & 20g^2 \\
20g^2 & 20g^2 & 20g^2 \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]

\[
\text{linearly independent} \quad \text{linearly dependent}
\]

and compare it with the red area of the dynamic lagrange part.

Someone can easy proof that is identical.

Then with diagonalizing the Momentumdensity − Matrix we get the following result:
\[ \sqrt{g'^2 + g''^2} = g \]

\[ \theta_K = \text{Mixing angle} \]

\[ 9_G = \text{mass density} \quad \text{Gravitation charge} \]

\[ \alpha_G = \frac{g^2}{4\pi} = \frac{(R - \phi_{\text{max}})^2 \sin(2 \theta_K)}{16\pi} \approx \frac{1}{140,711} \]

\[ \alpha_{\text{em}} = \frac{e^2}{4\pi} \approx \frac{1}{137.036} \]

Then the graviton and the $\Lambda$ - Boson is a mixing:

\[ \begin{pmatrix} \Lambda_{\mu} \\ \phi_{\mu} \end{pmatrix} = \begin{pmatrix} \cos(\theta_K) & -\sin(\theta_K) \\ \sin(\theta_K) & \cos(\theta_K) \end{pmatrix} \begin{pmatrix} B_\mu \\ B_\nu \end{pmatrix} \]

\[ < 5 > \text{ Curvature tensors by the Octoquinten field} \]

\[ \text{We allow } \phi \in \mathbb{C} \]

The construction comes from multiplications (symmetric to the diagonal)
by 2 degrees of freedom (complex subspaces).
With this construction the tensor is symmetric in the diagonal.
\[ \phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \end{bmatrix} = \begin{bmatrix} \phi^G_1 + i_1 \phi^G_2 + i_2 \phi^G_3 + i_3 \phi^G_4 + i_4 \phi^G_5 + i_5 \phi^G_6 + i_6 \phi^G_7 + i_7 \phi^G_8 \\ \phi^R_1 + i_1 \phi^R_2 + i_2 \phi^R_3 + i_3 \phi^R_4 + i_4 \phi^R_5 + i_5 \phi^R_6 + i_6 \phi^R_7 + i_7 \phi^R_8 \\ \phi^F_1 + i_1 \phi^F_2 + i_2 \phi^F_3 + i_3 \phi^F_4 + i_4 \phi^F_5 + i_5 \phi^F_6 + i_6 \phi^F_7 + i_7 \phi^F_8 \\ \phi^S_1 + i_1 \phi^S_2 + i_2 \phi^S_3 + i_3 \phi^S_4 + i_4 \phi^S_5 + i_5 \phi^S_6 + i_6 \phi^S_7 + i_7 \phi^S_8 \end{bmatrix} \]

\[ C_{em} = \frac{c}{G \hbar}. \]

10 independent fields.

Remark:
for \( \phi = c \) we get as curvature the plank curvature
which is the reciprocal of the plank area.
The value of the curvature is:
\[ 0.34 \times 10^{70} \cdot \frac{1}{m^2} \]

Second CURVATURE TENSOR from the Octoquintenfield (generates a spinpotential)
The construction comes from multiplications by 4 degrees of freedom (quaternionic subspaces).
With this construction the tensor is symmetric in both diagonals.

\[ \phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \end{bmatrix} = \begin{bmatrix} \phi^G_1 + i_1 \phi^G_2 + i_2 \phi^G_3 + i_3 \phi^G_4 + i_4 \phi^G_5 + i_5 \phi^G_6 + i_6 \phi^G_7 + i_7 \phi^G_8 \\ \phi^R_1 + i_1 \phi^R_2 + i_2 \phi^R_3 + i_3 \phi^R_4 + i_4 \phi^R_5 + i_5 \phi^R_6 + i_6 \phi^R_7 + i_7 \phi^R_8 \\ \phi^F_1 + i_1 \phi^F_2 + i_2 \phi^F_3 + i_3 \phi^F_4 + i_4 \phi^F_5 + i_5 \phi^F_6 + i_6 \phi^F_7 + i_7 \phi^F_8 \\ \phi^S_1 + i_1 \phi^S_2 + i_2 \phi^S_3 + i_3 \phi^S_4 + i_4 \phi^S_5 + i_5 \phi^S_6 + i_6 \phi^S_7 + i_7 \phi^S_8 \end{bmatrix} \]
\[ C_{\text{spin}} = \left( \frac{c}{G \cdot \hbar} \right)^2. \]

\[
\begin{align*}
A &= \phi^a_0 \cdot \phi^b_1 \cdot \phi^c_2 \cdot \phi^d_3 \\
B &= \phi^a_0 \cdot \phi^b_2 \cdot \phi^c_1 \cdot \phi^d_3 \\
C &= \phi^a_0 \cdot \phi^b_3 \cdot \phi^c_1 \cdot \phi^d_2
\end{align*}
\]

5 independent fields A, B, C and two in the diagonal (blue and yellow).

So finally we get three derivation – or curvature tensors of the Golden – Potential for twisted spacetime excitation

\[
\begin{align*}
\phi_1 \text{ unit is speed m/s} \\
\phi_1 &\text{ speed of light} \\
G &\text{ Gravitational constant} \\
\hbar &\text{ Planck constant} \\
K_p &\text{ Planck area} \\
\text{the curvature excitation} : \\
\phi_i^2 &= \Delta \cdot \frac{G \cdot \hbar}{c} \quad i = 0, 1, 2, 3
\end{align*}
\]

< 6 > Extension of the General Relativity GR by the second curvature tensor
The Golden – Potential GP has two symmetric curvature tensors.
This motivates us to extend the Einstein equation.
I think this shows that the GR (General Relativity) has to be extended
by an imaginary part (spinpart) to be a consistent quantumtheorie.
So finally we expect something like \( GR + i \cdot GR' \) where \( GR' \) is the spinpart.

with the two curvature tensors \( C_{\text{em}} \) and \( C_{\text{spin}} \) we can define following equation:

**EGR Extended General Relativity**

\[
g \cdot \text{Real}(C_{\text{em}}) + g^2 \frac{i}{\sqrt{2}} \frac{1}{\Lambda} C_{\text{spin}} = \frac{8 \pi G}{c^4} (T_{\mu \nu} + \frac{i}{\sqrt{2}} S_{\mu \nu})
\]

where the real part is the GR and the imaginary part \( GR' \)

**GR...General Relativity**
**GR'...Spinextension of GR**
\( \varphi \)...golden ratio
\( \Lambda \)...cosmological constant
\( l_p \)...Plancklength

\[
g = \begin{cases} 
\frac{\Lambda l_p^2}{\varphi^2} = \frac{4}{48^2} \approx \frac{2.6}{10^{112}} & \text{for the vacuum} \\
1 & \text{else} 
\end{cases}
\]

**Hint:**
The small value of \( g \) shows the cosmological constant problem or vacuum catastrophe.
\[
g = \frac{\Lambda l_p^2}{\varphi^2} = \frac{4}{48^2} \approx \frac{2.6}{10^{112}} \text{ for the vacuum}
\]

the operator \( \text{Real}(A) \) is defined by

\[
\text{Real}( \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} ) = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}
\]

the reversing \( \text{Real}^{-1} \) is :

\[
\text{Real}^{-1}( \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} ) = \begin{pmatrix} a_{0,0} & a_{1,0} & a_{2,0} & a_{3,0} \\
a_{0,1} & a_{1,1} & a_{2,1} & a_{3,1} \\
a_{0,2} & a_{1,2} & a_{2,2} & a_{3,2} \\
a_{0,3} & a_{1,3} & a_{2,3} & a_{3,3} \end{pmatrix}
\]

\( i_1, i_2, i_3 \)...imaginary quaternions

more detailed with the two curvature tensors of the Octoquinten field:

\[
\frac{8 \pi G}{c^4} (T_{\mu \nu} + \frac{i}{\sqrt{2}} S_{\mu \nu}) = g \cdot \text{Real}(C_{\text{em}}) + g^2 \frac{i}{\sqrt{2}} \frac{1}{\Lambda} C_{\text{spin}} = \\
= \frac{g \cdot c}{G \cdot \hbar} \begin{pmatrix} \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 \\
\text{sym.} & -\phi_0^0 \phi_0^0 & -\phi_0^0 \phi_0^0 & -\phi_0^0 \phi_0^0 \\
\text{sym.} & \text{sym.} & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 \\
\text{sym.} & \text{sym.} & \text{sym.} & \phi_0^0 \phi_0^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \frac{c}{\Lambda \cdot G \cdot \hbar} \begin{pmatrix} \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 \\
\text{sym.} & -\phi_0^0 \phi_0^0 & -\phi_0^0 \phi_0^0 & -\phi_0^0 \phi_0^0 \\
\text{sym.} & \text{sym.} & \phi_0^0 \phi_0^0 & \phi_0^0 \phi_0^0 \\
\text{sym.} & \text{sym.} & \text{sym.} & -\phi_0^0 \phi_0^0 \phi_0^0 \phi_0^0 \end{pmatrix}
\]

10 different products

5 different products

sym. and the red products are redundant

\[
\text{generates Poincare group } \mathbb{R}^{1,3} \rtimes O(1, 3) = 2 \times S^1 \times S^1 \times SU(2) = 2 \times \mathbb{T}^2 \times SU(2) \text{ with } \mathbb{T}^2 = \text{Torus}
\]

\( \mathbb{T}^2 \)...Clifford – Torus
This flat torus is a subset of the unit 3 – sphere \( S^3 \).
The Clifford torus divides the 3 – sphere into two congruent solid tori.
The Clifford – Torus embedded in \( S^3 \) becomes a minimal surface.

The second curvature tensor \( C_{\text{spin}} \) is determined by the first curvature tensor \( C_{\text{em}} \) because its components are a mix of the components of \( C_{\text{em}} \).

\(< 6.1 >\) The vacuum part of the extended Einstein equation then is :
with
\[ \phi_0 = \phi_1 = \phi_2 = \phi_3 = c \quad \text{speed of light and other } \phi \text{'s are zero and} \]
\[ g = \Lambda \frac{l_p^2}{l_p^4} \quad \text{for the vacuum} \]
then
\[ \text{Vacuum Energydensity} = \frac{c^4}{8\pi G} [\Lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \frac{i}{\varphi\sqrt{2}} \Lambda \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}] \]
\[ = T_{\text{up}}^2 \quad T_{\text{down}}^2 \]
\[ \text{generates flat expanding spacetime} \]
\[ \begin{pmatrix} 1 \\ C_- \\ C_+ \\ \text{Spinor} \end{pmatrix} \]
\[ \text{generates } 2 \times \text{spinning Torus } T_{\Lambda}^2 = S^1 \times S^1 \]
\[ \text{flat Clifford Torus} \]
\[ \text{generates flat expanding and twisting vacuum} \]
This is in accordance with the Golden Potential GP on \( \varphi = c \)

\[ V(c) = \frac{\Lambda c^4}{8\pi G} \left( \frac{1}{2} \frac{1}{\varphi^2} \right) \left( \begin{pmatrix} \varphi_0^2 + \frac{\varphi_1^2}{2\varphi} & \frac{\varphi_0 \varphi_1}{\sqrt{2}\varphi} & \frac{\varphi_0}{\sqrt{2}\varphi} & 1 \\ \sqrt{\varphi} & \sqrt{\varphi} & \sqrt{\varphi} & \sqrt{\varphi} \end{pmatrix} \right) = \text{Energydensity}^2 = 0 \]

\[ \frac{8\pi G}{c^4} (T_{\mu \nu} + \frac{i}{\sqrt{2}\varphi} S_{\mu \nu}) = \Lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \frac{i\Lambda}{\varphi\sqrt{2}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \]

\(< 6.2 > \quad \text{Scale factor for the accelerated expanding Universe by our assumption} \)

To make it simple we are thinking about an universe without radiation and mass. This means only the vacuum energy density is acting. Then the Hubble constant is really constant.

\[ H = \sqrt{\frac{\left( \frac{2}{3} \Lambda \right)}{3 \left( \frac{c}{a(t)} \right)^2}} \]

\[ a(t) \propto e^{Ht} = e^{\sqrt{\frac{2}{3} \Lambda} t} \]

then with assumption \( \Lambda l_p^2 = \frac{4}{48t^2} \)
we get finally

\[ a(t) \propto e^{Ht} = e^{\sqrt{\frac{4}{3} \frac{4}{48t^2} l_p^2}} \]

< 7 > Getting a closed form for the Extended General Relativity EGR.
we know that our energy - momentum curvature tensor

\[ \text{Real}(C_{em}) = R_{\mu \nu} - \frac{R}{2} g_{\mu \nu} + \lambda g_{\mu \nu} \]  and that

\[ C_{\text{spin}} \] is defined by multiplication of tensors elements of \( C_{em} \)

The question now is how can we express \( C_{\text{spin}} \) analogous to \( C_{em} \) above as terms of Riemann - Geometric?

For that we define the operator for \( 4 \times 4 \) matrices or tensors :

\[
\mathcal{T} = T^T + \gamma T
\]

Transposing in the big diagonal from top right to bottom left, and then in the small diagonals

\[
\begin{pmatrix}
  a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
  a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
  a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
  a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  a_{0,0} & a_{0,3} & a_{0,2} & a_{0,1} \\
  a_{1,0} & a_{1,3} & a_{1,2} & a_{1,1} \\
  a_{2,0} & a_{2,3} & a_{2,2} & a_{2,1} \\
  a_{3,0} & a_{3,3} & a_{3,2} & a_{3,1}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  a_{3,3} & a_{2,3} & a_{1,3} & a_{0,3} \\
  a_{3,2} & a_{2,2} & a_{1,2} & a_{0,2} \\
  a_{3,1} & a_{2,1} & a_{1,1} & a_{0,1} \\
  a_{3,0} & a_{2,0} & a_{1,0} & a_{0,0}
\end{pmatrix}
\]

\[ A \leftrightarrow A^\mathcal{T} \]

and a special simple matrices multiplication

\[ C = A \otimes B \]

with

\[
c_{i,j} = \begin{cases} 
  +a_{i,j} b_{i,j} & \text{if } i = j \\
  -a_{i,j} b_{i,j} & \text{if } i \neq j 
\end{cases}
\]

then it is easy to see that

\[
(A + B) \otimes C = A \otimes C + B \otimes C
\]

\[(A^\mathcal{T}) \otimes B = A \otimes B^\mathcal{T} \]

with

\[
C_{em} = \frac{c}{G \hbar} \begin{pmatrix}
  c_{0,0} & i_1 \phi_0^0 & i_2 \phi_0^0 & i_3 \phi_0^0 \\
  c_{0,0} & -\phi_0^0 & -\phi_0^0 & -\phi_0^0 \\
  c_{0,0} & -\phi_0^0 & -\phi_0^0 & -\phi_0^0 \\
  c_{0,0} & -\phi_0^0 & -\phi_0^0 & -\phi_0^0
\end{pmatrix}
\]

and

\[
C_{\text{spin}} = \left( \frac{c}{G \hbar} \right)^2 \begin{pmatrix}
  -\phi_0^0 & \phi_0^1 & \phi_0^2 & \phi_0^3 \\
  \phi_0^1 & -\phi_0^0 & -\phi_0^1 & -\phi_0^2 \\
  \phi_0^2 & \phi_0^1 & -\phi_0^0 & -\phi_0^1 \\
  \phi_0^3 & \phi_0^2 & \phi_0^1 & -\phi_0^0
\end{pmatrix}
\]

it follows that

\[
C_{\text{spin}} = \text{Real}(C_{em}) \cdot \text{Real}(C_{em}) \mathcal{T}
\]
with
\[ \text{Real}(C_{\text{em}}) = R_{\mu \nu} - \frac{R}{2} g_{\mu \nu} + \Lambda g_{\mu \nu} = G_{\mu \nu} + \Lambda g_{\mu \nu} = K_{\mu \nu} \]
and
\[ \text{Real}(C_{\text{em}}) + \frac{i}{\varphi \sqrt{2}} \cdot \frac{1}{4\pi} C_{\text{spin}} = \frac{8 \pi G}{c^4} \cdot (T_{\mu \nu} + \frac{i}{\varphi \sqrt{2}} S_{\mu \nu} ) \]
we get the final compact result for the extension of General Relativity by

\[
\boxed{K_{\mu \nu} + \frac{i}{\varphi \sqrt{2}} \cdot \frac{1}{4\pi} K_{\mu \nu} \bar{K}_{\nu \mu} = \frac{8 \pi G}{c^4} \cdot (T_{\mu \nu} + \frac{i}{\varphi \sqrt{2}} S_{\mu \nu} )}
\]
with
\[ S_{\mu \nu} = \frac{1}{\Lambda} T_{\mu \nu} \cdot T_{\rho \sigma} ... \text{Spintensor} \]
\[ K_{\mu \nu} = R_{\mu \nu} - \frac{R}{2} g_{\mu \nu} + \Lambda g_{\mu \nu} \]
\[ \bar{K}_{\mu \nu} = R_{\mu \nu} - \frac{R}{2} g_{\mu \nu} + \bar{\Lambda} g_{\mu \nu} \]
\[ \psi \ldots \text{golden ratio} \]
The real part is the known General Relativity.
The imaginary part is the Spinextension of GR.

\textbf{Hint:} The Energy–Stress tensor is still symmetric with or without Spin!

< 7.1 > Showing a combinatorial dimensionless form of the EGR

With the relation shown in < 3.5 > and Appendix III,
\[ \Lambda_i^2 = \frac{4}{48 \pi^2} \]
we can write:
\[ \frac{1}{\Lambda} = i_p^2 \cdot \frac{48 \pi^2}{4} \]
Then our formula in < 6 > can be written to:
\[ \frac{8 \pi G}{c^4} \cdot (T_{\mu \nu} + \frac{i}{\varphi \sqrt{2}} S_{\mu \nu} ) = g \cdot \text{Real}(C_{\text{em}}) + g^2 \cdot \frac{i}{\varphi \sqrt{2}} \cdot \frac{1}{4\pi} C_{\text{spin}} \]
\[ g = \begin{cases} \Lambda_i^2 & \text{for the vacuum} \\ 1 & \text{else} \end{cases} \]

\textbf{Hint: instead of Real}(C_{\text{em}}) \text{ we write short } C_{\text{em}} \text{ and keep it in mind!}

\[ \frac{8 \pi G}{c^4} \cdot (T_{\mu \nu} + \frac{i}{\varphi \sqrt{2}} S_{\mu \nu} ) = g \cdot C_{\text{em}} + g^2 \cdot \frac{i}{\varphi \sqrt{2}} \cdot \frac{48 \pi^2}{4} \cdot C_{\text{spin}} \]
\[
\frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{\sqrt{2}} S_{\mu\nu}) = g \frac{c}{G\hbar} V_{cm} + g^2 \frac{i}{\sqrt{2}} \frac{48\hbar^2}{c^2} \cdot \frac{1}{c^2} V_{spin}
\]

then with
\[
\frac{c}{G\hbar} = \frac{1}{c^2 \beta^2}
\]

\[
\frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{\sqrt{2}} S_{\mu\nu}) = g \frac{1}{c^2 \beta^2} V_{cm} + g^2 \frac{i}{\sqrt{2}} \frac{48\hbar^2}{c^2} \cdot \frac{1}{c^2} V_{spin}
\]

then with
\[
P_p = \frac{c^2}{G^2 \hbar} \text{ Planck pressure and } g = 1 \text{ for not vacuum}
\]

we get finally
\[
\frac{8\pi G}{P_p} (T_{\mu\nu} + \frac{i}{\sqrt{2}} S_{\mu\nu}) = \frac{1}{c^2} V_{cm} + \frac{i}{\sqrt{2}} \frac{1}{8\pi^2} \frac{1}{c^2} V_{spin}
\]

\[N = \frac{\sqrt{2}}{2.48!}\]

dimensionless Combinatorial - Form of the EGR

< 7.2 > Spin and possible proofing of the Extended General Relativity EGR

As noted in < 5 > the second curvaturesensor \(C_{spin}\)

is responsible for the Spin.

Obviously the \(C_{spin}\) is directly connected to the energy – momentum curvaturesensor \(C_{em}\).

So in principle the Extended General Relativity short EGR could be proofed because the spin of a particle changes the Energy – Momentum – Tensor.

First we have to show how the spin of a particle acts on the \(C_{spin}\).

For that we have to take a closer look on the \(C_{spin}\).

As seen in < 5 > the structure is :

\[
EGR = 4 + 6 = 10 \text{ freedoms} + 2 + 3 = 5 \text{ freedoms}
\]

Now how can we embed dirac – fermions with spin \(\frac{1}{2}\) into the EGR?

For that we want to assign the structure of the \(C_{spin}\) in the following way to fermions:

Then a resting electron excites the \(C_{spin}\) and the \(C_{em}\) as follow:
\[ \frac{8 \pi G}{c^2} \left( T_{\mu \nu} + \frac{i}{\sqrt{2}} S_{\mu \nu} \right) = \frac{\nabla}{G \hbar} \left( \begin{array}{c} \text{Minkowski} \\ \text{double Torus} \end{array} \right) \]

Then complete in the EGR:

\[ \psi = \frac{i}{\sqrt{2}} \begin{pmatrix} 1 \\ \phi \end{pmatrix} \]

This pressure to a spacetidion should be provable in principle!

We allow \( \phi \in \mathbb{C} \)

\[ \langle 8 \rangle \quad \text{Some important points of the Golden Potential} \]

To get the maxima, minima and the zeros of the potential we have to substitute
\( z = \phi^2 \) it is enough (because of symmetry) to take a look on the positive \( \phi 's \).
and solve the cubic equations in the bracket

\[ V(\sqrt{5}) = \frac{\hbar^2}{5} \left( \frac{\mu^2}{2} + \frac{\lambda^2}{4} z + \frac{\gamma^2}{8} z^3 \right) \]

We will make it short and write the results.
First the Zero-points:

\[ z_1 = \frac{1}{2} + i \frac{\sqrt{3}}{2} \quad \text{and} \quad \epsilon_2 = -\frac{1}{2} - i \frac{\sqrt{3}}{2} \]

Then the Zero-points are

\[ \phi_1 = i \cdot 0.995400169 \times \sqrt{\frac{8 C}{3}} \]
\[ \phi_2 = 0.615196099 \times \sqrt{\frac{8 C}{3}} \]
\[ \phi_3 = 0.782542290 \times \sqrt{\frac{8 C}{3}} \]
\[ \phi_1 - i.e. \varphi = i.0,995409160 \times \sqrt[3]{\frac{8C}{3}} - i.0 \sin(84,507759190) \times \sqrt[3]{\frac{8C}{3}} \]

\[ \alpha_{e.\varphi} = 84,507759190^\circ \]

\[ \phi_2 = e = 0,615196699 \times \sqrt[3]{\frac{8C}{3}} = \sin(37.966214178) \times \sqrt[3]{\frac{8C}{3}} \]

\[ \alpha_e = 37.966214178^\circ \]

\[ \phi_3 = c.\sqrt{2} = 0,78254229 \times \sqrt[3]{\frac{8C}{3}} = \sin(128,506061932) \times \sqrt[3]{\frac{8C}{3}} \]

\[ \alpha_{c.\sqrt{2}} = 128,506061932^\circ \]

Then the Maxima and the Minima:

\[ z_1 = u + i.e = -\sqrt[3]{\frac{C}{3}} \left( \sqrt[3]{\frac{\sqrt{27} - i \sqrt{27}}{64}} + \sqrt[3]{\frac{\sqrt{27} + i \sqrt{27}}{64}} \right) \]

\[ z_2 = e_1.u + e_2.e \]

\[ z_3 = e_2.u + e_1.e \]

Finally we have two positive results:

\[ z_{\text{max}} = 0,235475630 \times 2 \sqrt[3]{\frac{C}{3}} \text{ and} \]

\[ z_{\text{max}} = 0,725352944 \times 2 \sqrt[3]{\frac{C}{3}} \]

and one negative:

\[ z_3 = -(z_{\text{max}} + z_{\text{min}}) \]

Then because of \( z = \phi^2 \)

\[ \phi_{\text{min}} = 0,483091360 \times \sqrt[3]{\frac{4C}{3}} \text{ and} \]

\[ \phi_{\text{max}} = 0,8516765489 \times \sqrt[3]{\frac{4C}{3}} \]

In cubic equations the real zero points comes from the \( \cos(\alpha) \) or from \( \sin(90-\alpha) \) of angles (see https://en.wikipedia.org/wiki/Cubic_function).

Then for \( \phi_{\text{min}} \) we get an angle \( \alpha_{\text{min}} \):

\[ \phi_{\text{min}} = 0,483091360 \times \sqrt[3]{\frac{4C}{3}} = \sin(28,894160846) \times \sqrt[3]{\frac{4C}{3}} \]

\[ \alpha_{\text{min}} = 28,894160846 \text{ degrees is very near to the Weinberg angle} \]

\[ \sin^2(\alpha_{\text{min}}) = \sin^2(28,894160846) = 0,23475630 \]

with Cardanic formular and so on we can express \( \alpha_{\text{min}} \) by:

\[ \alpha_{\text{min}} = \arcsin \left( -\cos \left( \arccos \left( \frac{-\sqrt{3}}{3} \right) + \pi \right) \right) \approx 28,9^\circ \]

and for \( \phi_{\text{max}} \) we get an angle:

\[ \phi_{\text{max}} = 0,8516765489 \times \sqrt[3]{\frac{4C}{3}} = \sin(121,60508985) \times \sqrt[3]{\frac{4C}{3}} \]

\[ \alpha_{\text{max}} = 121,60508985 \text{ degrees} \]

\[ \phi_{\text{min}} = 0,483091360 \times \sqrt[3]{\frac{4C}{3}} \approx 0,660464.c \text{ c...speed of light} \]

\[ \phi_{\text{max}} = 0,8516765489 \times \sqrt[3]{\frac{4C}{3}} \approx 1,164134.c \]

In cubic equations the real zero points comes from the \( \cos(\alpha) \) or from \( \sin(90-\alpha) \) of angles (see https://en.wikipedia.org/wiki/Cubic_function).

Geometric interpretation of the roots (zeropoints) in cubic equations with 3 real zeropoints.
graphical zeropoints of the derivation of the
(radicated $\phi^2 = z$) Golden -- Potential GP

$R = \phi^2 \sqrt{\frac{4}{3}}$

$\phi...$golden ratio

*Hint*: On our special Golden -- Potential GP the zeropoints (spheres) comes from a pentagon.

< 9 > Conclusions

Dark Energy comes by definition from the Golden -- Potential (the second term in the potential).

Dark Matter could be the $W$, $Z$ Bosons and the particles by the $SU(5)$ Symmetry
(adjoint and fundamental presentation).
Understanding the action of the Coxeter element.
As mentioned on the beginning of the paper a Coxeter element is a composition of the generating reflections of the reflection group.

In our case the affine group E9 (the one point extension of E8) has 9 such generating reflections e1, e2, e3, e4, e5, e6, e7, e8, e9.
A reflection ei is a reflection on the hyperspace of the root αi.
So the Coxeter element is the composition of the root reflections.
In our case the roots are vectors in the euclidean space $\mathbb{R}^9$ and the Coxeter element is a map on this space.

We want to visualize it by the simple example $\tilde{A}_2$

The action of $v = s_1 u t$ the Coxeter element on chamber $C_0$: $C_0 \rightarrow vC_0$

This shows that the action of the Coxeter element in this case moves the chamber along the red line $L_v$ and then reflect it on $L_v$.
Doing the action twice then we move the chamber the double way.

This symmetries can be described by the Coxeter polynomial which is the characteristic polynomial of the action (map) of the Coxeter element which is an affine map.
This affine maps are well studied so i will write only the results.

In our special example $\tilde{A}_2$ the Coxeter polynomial is

$$f_2(x) = (x + 1)(x - 1)^2 = \frac{x^2 - 1}{x - 1} \cdot (x - 1)^2$$

Eigenvalues:
- $\lambda_0 = -1$
- $\lambda_1 = 1$

Eigenvectors:
- $v_0$
- $v_1$

$\lambda_0 = -1$ is the eigenvalue by this cyclotomic factor and is the eigenvalue of the so called horizontal root (system). This produces the reflection on the red line.

$\lambda_1 = 1$ is one eigenvalue of this factor and is the eigenvalue of the so called vertical root (system). Vertical because the root is orthogonal to the horizontal root (system).

$\lambda_0 = -1$ is one eigenvalue of this factor and produces the translation on the red line.

With the eigenvector $v_0$ which will be reflected by the Coxeter element action we have a simple root for the Lie algebra $su(2)$.
And with the eigenvectors $v_0$ and $v_1$ we have roots for $u(1)$.

So in summary the Coxeter element generates the symmetric $SU(2) \times U(1) \times U(1)$
An analogue for E9 which is \( E_9 \) the coxeterelement generates the symmetric composition \( SU(3) \times SU(3) \times SU(2) \times U(1) \times U(1) \).

**APPENDIX II**

Our target is to show that
\[
\Lambda_{sp}^2 = \frac{16}{\varphi} N^2 = \frac{16}{\varphi} \left( \frac{\sqrt{5} - 1}{2} \right)^2 = \frac{4}{48^2} \approx \frac{2.6}{10^{122}} \quad \varphi \text{ - golden ratio}
\]

For that we start with the Combinatorial – Form and going back to the Einstein – Form of the Golden – Potential GP

On the combinatorial form we have set \( c = h = G = 1 \).

\[
V(\phi) = N^4 \left( -8 \left| \frac{\phi}{\sqrt{\varphi}} \right|^2 + 16 \left| \frac{\phi}{\sqrt{\varphi}} \right|^4 - 8 \left| \frac{\phi}{\sqrt{\varphi}} \right|^6 \right)
\]

We will take this back and get
\[
V(\phi) = \left( \frac{P_p}{2\pi} \right)^2 N^4 \left( -8 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^2 + 16 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^4 - 8 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^6 \right)
\]

With the relation
\[
P_{mp}^2 = \frac{c^4}{G}
\]
we get
\[
V(\phi) = \left( \frac{c^4}{2\pi G} \right)^2 N^4 \left( -8 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^2 + 16 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^4 - 8 \left| \frac{\phi}{c\sqrt{\varphi}} \right|^6 \right)
\]

Then
\[
V(\phi) = \left( \frac{\Lambda c^4}{2\pi G} \right)^2 N^4 \left( -8 \left| \frac{\phi}{\sqrt{\varphi}} \right|^2 + 16 \left| \frac{\phi}{\sqrt{\varphi}} \right|^4 - 8 \left| \frac{\phi}{\sqrt{\varphi}} \right|^6 \right)
\]

Then
\[
V(\phi) = \left( \frac{\Lambda c^4}{8\pi G} \right)^2 \left( \frac{\varphi^2}{\varphi} \right)^2 N^4 \left( -\frac{\varphi^2}{2} \left| \frac{\phi}{c} \right|^2 + \left| \frac{\phi}{c} \right|^4 - \frac{1}{2\varphi^2} \left| \frac{\phi}{c} \right|^6 \right)
\]

Comparing with the Einstein – Form this must be 1.
Then
\[
\Lambda_{sp}^2 = \frac{4}{48^2}
\]

QED.

**APPENDIX III**

1) The 16 – Cell

On the 16 – Cell each vertizes is connected by an edge to all other vertizes except the opposite one!

We have 4 disjunct such pairs which are not connected by an edge.

![Diagram](image)

Now exchanging this points which are not connected is an automorphism
(for example p1 with p1 line) because p1 has the same connections as p1 line.

So at all we generate \( 2^4 = 16 \) automorphisms by this actions because we can say for all 4 pairs
0 means pair IS NOT exchanged and 1 for pair IS exchanged.

So every binary code like \( (0, 1, 0, 0) \) is an automorphism.
But this are not all automorphism. Independent from that we can permute p₁, p₂, p₃, p₄ when we simultaneously permute their opposite points.

For example:

\[
\begin{align*}
\overline{p₁} & \rightarrow p₂ \\
\overline{p₄} & \rightarrow \overline{p₂}
\end{align*}
\]

This give us $4! = 24$ automorphisms independent from the 16 automorphisms before. So at all we get $16 \times 24 = 384$ automorphisms.

With this we can divide the $\text{Aut}(C_{16})$ into $\text{AutOpposite}(C_{16})$ and $\text{AutFakt}(C_{16})$ so that $\text{Aut}(C_{16}) = \text{AutOpposite}(C_{16}) \times \text{AutFakt}(C_{16}) = 16 \times 4!$.

$\text{AutOpposite}(C_{16})$ are the Automorphisms of the 16—Cell $C_{16}$ which comes from exchanging the opposite vertices (points).

$\text{AutFakt}(C_{16})$ are the Automorphisms of the 16—Cell $C_{16}$ which comes from permutating $p_1, p_2, p_3, p_4$ vertices (points) as described above.

$\text{AutFakt}(C_{16})$ is simply the permutation—group $\text{Sym}(4) = S_4$.

Embedding (projection) the 16—Cell into the quaternionic subgroups of the Octoquaternion field:

See also Quaternion group $Q_8$.

The 16—Cell $C_{16}$ can be seen as a so called 4—4 duopyramid

more here https://en.wikipedia.org/wiki/DuopyramidExample16-cell
16 – Cell as a duopyramid with special embedding in the quaternions.

The orange base of the duopyramid is on the sphere \( S_c \) with \( c = 1 \) and the blue base on the sphere \( S_{\sqrt{c}} \).

How long are the edges of this special 16–Cell?

The coordinates of the 8 vertices are:
\((\pm 1, 0, 0, 0),(0, \pm 1, 0, 0)\) and
\((0, 0, \pm \sqrt{c}, 0),(0, 0, 0, \pm \sqrt{c})\)

Length of the orange edges are : \( \sqrt{2} \)
Length of the blue edges are : \( \sqrt{2} \sqrt{c} \)
Length of the black edges are : \( \varphi \)

**Kepler triangle**

\[
A = \frac{a + b}{2} \\
H = \frac{2ab}{a + b} \\
G = \sqrt{ab}
\]

\[A^2 = G^2 + H^2\]

An interesting point is:
For positive real numbers \( a \) and \( b \), their arithmetic mean \( A \), geometric mean \( G \), and harmonic mean \( H \) are the lengths of the sides of a right triangle \( \Leftrightarrow \) that triangle is a Kepler triangle.
\( a = b \varphi^3 \) \( \varphi \)...golden ratio.