50,000 years ago - Africans understand IFA equivalent to Real Clifford Algebras

36,000 Years Ago - From Africa by Nile to Giza where Pyramids + Sphinx encode E8 Physics

The Layers of the Great Pyramid correspond to the 256 elements of Cl(8) which is the Real Clifford Algebra that is the basis of 8-Periodicity and whose 8 Vectors and 28 BiVectors and 16 Spinors form 52-dimensional Lie Algebra F4.

The two F4 embedded in tensor product Cl(8) x Cl(8) = Cl(16) form the Lie Algebra E8.

The Root Vectors of E8 encode a realistic Classical Lagrangian.

The Completion of the Union of All Tensor Products of Cl(16) with embedded E8 form a realistic Algebraic Quantum Field Theory (AQFT) that is analogous to the Fock Space Hyperfinite II1 von Neumann factor that is based on the 2-Periodicity of Complex Clifford Algebras.
Abstract

E8 Physics of viXra 1804.0121 comes from Ancient Africa.

Table of Contents

Title ... page 1
Abstract and Table of Contents ... page 2

Straight OUTTA AFRICA ... page 3
  Africa - Cellular Automata ... page 6
  Africa - Llull - Cambridge ... page 9
  Third Age of Scholasticism ... page 11
  Microtubule Bohm Quantum Consciousness ... page 12

240 E8 Root Vectors Physical Interpretation ...page 18
  Fermions ... page 20
  Spacetime ... page 22
  Standard Model + GravityDE Ghosts ... page 24
  GravityDE + Standard Model Ghosts ... page 26

E8 Lagrangian ... page 27
  Three Generations of Fermions ... page 30

E8 Physics Calculation Results Summary ... page 31

Nambu - Jona-Lasinio Truth Quark-AntiQuark Condensate Higgs ... page 32

Fermilab see 3 Truth Quark Mass States - CMS sees 3 Higgs Mass States ... page 36

Schwinger Sources, Hua Geometry, Wyler Force Strengths ... page 38
  African Origin of Indra’s Net ... page 43

Wyler Force Strength and Mass Calculation Details ... page 46
  Force Strengths ... page 49
  Higgs mass ... page 55
  Weak Boson masses and Weinberg Angle ... page 57
  Fermion masses ... page 60
  Kobayashi-Maskawa Parameters ... page 72
  Neutrino masses ... page 82
  Proton-Neutron mass difference ... page 87
  Pion as Sine-Gordon Breather ... page 88
  Planck mass ... page 93

Conformal Gravity+Dark Energy and DE : DM : OM ... page 94

World-Line String Bohm Quantum Theory ... page 104
Algebraic Quantum Field Theory (AQFT) ... page 118
Massless Spin-2 carrier of Bohm Quantum Potential ... page 119
Penrose-Hameroff Quantum Consciousness ... page 120
Straight OUTTA AFRICA

50,000 years ago - Africans understand IFA equivalent to Real Clifford Algebras
IFA = 256 Odu = 16x16 = 2^8 = Cl(8) Clifford Algebra
16 Orishas = 16 dim of Spinors of Cl(8)

36,000 Years Ago - National Geographic Genographic -
M168 - YAP - M96 - M35 Humans go Straight Outta Africa
following Vega as North Star up the Nile to Giza and Mediterranean

where they built two large Pyramids and the Sphinx

Each Pyramid represented a copy of Cl(8) with graded structure
256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 16 x 16
so that each contained a copy of 52-dim F4 = 8 + 28 + 16
The Sphinx represented the tensor product Cl(8) x Cl(8) = Cl(16) containing E8
248-dim E8 structure came from the two F4 in the two Cl(8) of the two Pyramids
It is hard to find ancient written history of Africa back to 36,000 years ago. It was only after Alexander the Great (356 - 323 BC) founded Alexandria in Egypt that the historian Manetho, working under Ptolemy I (reign 323-283 BC) who had been a Macedonian General under Alexander, wrote a chronology of Ancient Egypt / Africa. Here is my view of that chronology:

36,525 years ago -
Vega was North Star
Geminga Supernova Shock Wave hit Earth
Late Wisconsin Glaciation
Manetho’s period of Rule of Gods (13,900 years) began

22,625 years ago -
last Glacial Maximum of Earth began
Manetho’s period of Rule of Demigods (5,212 years) began

17,413 years ago -
last Glacial Maximum of Earth ended
Manetho’s period of Rule of Spirits of the Dead (5,813 years) began

11,600 years ago -
Vega was North Star
Younger Dryas cold snap, with temperatures 14 degrees C below present-day, after which the Vela X supernova was seen on Earth and the Taurid/Encke comet fragmented and a very sudden (50 years or so) warming event ended the Ice Age and marked the start of the HOLOCENE AGE of warm climate and glacial retreat
Maneto’s Rule of Mortal Humans on Earth (still present) began

The total period of the Mantho’s Ice Age Civilization prior to the Rule of Mortal Humans is therefore about 36,525 - 11,600 = 24,925 years, which is approximately the Earth precession period of 26,000 years and the travel time of a light beam from the center of our Galaxy to our Sun, about 25,000 light years distance. It may have been the time of construction of the Giza Complex of the Sphinx and the Pyramids, including the Great Golden Pyramid. The Ice Age Civilization may have been more spiritual and less crudely technological than the present Rule of Mortal Humans.

Advanced African Ice Age Civilization building Pyramids and Sphinx 36,000 years ago would be somewhat like the Marvel (1966) fictional African nation Wakanda that “... passes itself off as a small, poor Third World nation of humble herdsmen, using an advanced holographic projection shroud around its borders to hide the advanced technological civilization within ...”. (Wikipedia)
however
the Advanced African Ice Age Civilization is not only hidden in Space like Wakanda but is also hidden in Time.
The Advanced African Ice Age Civilization would also be confirmation of what Terence McKenna said May 1993 OMNI magazine:
“... From 75,000 to about 15,000 years ago, there was a kind of human paradise on Earth. ... Community, loyalty, altruism, self-sacrifice -- all these values that we take to be the basis of humanness -- arose at the time in a situation in which the ego was absent ... My scenario, if true, has enormous implications.

For 10,000 years, with the language and social skills of angels, we've pursued an agenda of beasts and demons. Human beings created an altruistic communal society; then ... we've had nothing ... except ... all tooth-and-claw dominance ...

For the last 500 years, Western culture has suppressed the idea of disembodied intelligences -- of the presence and reality of spirit. ... You can be a New York psychotherapist or a Yoruba shaman, but these are just provisional realities you're committed to out of conventional or local customs. ... The world is not a single, one-dimensional, forward-moving, causal, connected thing, but some kind of interdimensional nexus. ... Entities there are completely formed. There's no ambiguity about the fact that these entities are there ...

On one level I call them self-transforming machine elves; half machine, half elf. They are also like self-dribbling jeweled basketballs, about half that volume, and they move very quickly and change. And they are, somehow, awaiting. ... They are teaching something. Theirs is a higher dimensional language that condenses as a visible syntax. For us, syntax is the structure of meaning; meaning is something heard or felt. In this world, syntax is something you see. There, the boundless meanings of language cause it to overflow the normal audio channels and enter the visual channels. They come bouncing, hopping toward you, and then it's like -- all this is metaphor; they don't have arms -- it's as though they reach into their intestines and offer you something. They offer you an object so beautiful, so intricately wrought, so something else that cannot be said in English, that just gazing on this thing, you realize such an object is impossible. ... The object generates other objects ... Ordinarily language creates a system of conventional meanings based on pathways determinate by experience. ... [this is] a place where the stress is on a transcending language.

Language is a tool for communication, but it fails at its own game because it's context-dependent. Everything is a system of referential metaphors. ...

If you get something from outside the metaphorical system, it doesn't compute. ... Something in an unseen dimension is acting as an attractor for our forward movement in understanding ... It's a point in the future that affects us in the present. ...

Our model that everything is pushed by the past into the future, by the necessity of causality, is wrong. There are actual attractors ahead of us in time -- like the gravitational field of a planet. Once you fall under an attractor's influence, your trajectory is diverted ... It's an extradimensional source of immense caring ...

If history goes off endlessly into the future, it will be about scarcity, preservation of privilege, forced control of populations, the ever-more-sophisticated use of ideology to enchain and delude people.

5
We are at the breakpoint.
It's like when a woman comes to term.
At a certain point, if the child is not severed from the mother
and launched into its own separate existence,
toxemia will set in and create a huge medical crisis ...
When a species prepares to depart for the stars, the planet will be shaken to its core.
All evolution has pushed for this moment, and there is no going back.
What lies ahead is a dimension of such freedom and transcendence,
that once in place, the idea of returning to the womb will be preposterous.
We will live in the imagination.
We will quickly become unrecognizable to our former selves
because we're now defined by our limitations:
the laws of gravity;
the need to eat, excrete, and make money.
We have the will to expand infinitely into pleasure, caring, attention,
and connectedness. ...".

Ron Eglash (in his book "African Fractals" (Rutgers 1999) and on his web site) says: “... a historical path for base-2 calculation ... begins with African divination, runs through the geomancy of European alchemists, and is finally transformed into binary calculation, where it is now applied in every digital circuit ...”.

Raymond Aschheim (email May 2015) said, about Cellular Automata (CA): “... An elementary CA is defined by the next value (either 0 or 1) for a cell, depending on its ... value, and the ... value of it[s] left and of it[s] right neighbor cell (it is one dimensional, and involve only the first neighbors, and the cell itself) ... So the next value depends [on] 3 bits ... eight possible combination of three bits, and for each ... combination... the next value is either zero or one.
So the[re] are 256 ... CAs ...”.

Since due to Real Clifford 8-periodicity any Real Clifford Algebra Cl(8N)) can be seen as the tensor product of N copies of Cl(8), any Real Clifford Algebra has fundamental structure of Cl(8) = Cl(1,7) = 16x16 real matrix algebra so Cellular Automata correspondence with Cl(8) means that any Real Clifford Algebra can be described by Cellular Automata so Clifford Algebra E8 physics can also be seen in terms of Cellular Automata.
For example consider the 28 Cl(8) BiVector grade-2 Cellular Automata:

These $1 + 12 + 3 = 16$ grade-2 Cellular Automata correspond to propagator phase, Conformal Lie Algebra Root Vectors, and Conformal Lie Algebra Cartan Subalgebra.

The Conformal Group $\text{Spin}(2,4) = \text{SU}(2,2)$ gives Gravity+Dark Energy by the MacDowell-Mansouri mechanism. $\text{U}(2,2) = \text{U}(1) \times \text{SU}(2,2)$ also contains the propagator phase.

These $1 + 3 + 8 = 12$ grade-2 Cellular Automata correspond to $\text{U}(1), \text{SU}(2), \text{SU}(3)$ of the Standard Model.
The 256 Elementary Cellular Automata correspond to the 256-dim Cl(8) Clifford Algebra with graded structure 1 8 28 56 (35+35=70) 56 28 8 1

The 8 Vectors have clear physical interpretation as 8-dim Spacetime.

The 28 BiVectors have clear physical interpretation as Gauge Bosons or Ghosts of Standard Model (12) and Gravity+ Dark Energy(16)

The 1 scalar, 1 pseudoscalar, and 7+7=14 of grade 4 have physical interpretation as 8 +half-spinors and 8 -half-spinors

The 8+28+8+8 = 52 with fixed physical interpretation form 52-dim F4.

The remaining 256 - 8 - 28 - 8 - 8 = 204 Cl(8) Cellular Automata are not bound to any physical interpretation but are available to carry information.

When Cl(16) is formed from the tensor product Cl(8) x Cl(8) the two F4 in Cl(8) go to 1x28 + 8x8 + 28x1 = 120 D8 BiVectors and (8+8) x (8+8) = 256 D8 Spinors all of which inherit clear physical interpretations leaving 65,536 - 120 - 256 = 65,160 Cl(16) elements available to carry information either in Lorentz Leech Lattice Spacetime Cells of Our Conscious Universe or in 40-micron Microtubules of Human Quantum Consciousness.

All of the 120 D8 BiVectors and 128 = half of the D8 Spinors form 248-dim E8 which has fixed physical interpretation inherited from the F4 in Cl(8) so 248-dim E8 and the other 128 half-Spinors are fixed structure markers in Cl(16) that do not carry information.
Ron Eglash (in his book "African Fractals" and on his web site) also says:
... Following the introduction of geomancy to Europe by Hugo of Santalla in
twelfth-century Spain ... European geomancers ... Ramon Lull ... and others ... persistently replaced the deterministic aspects of the system with chance. By mounting the 16 figures on a wheel and spinning it, they maintained their society's exclusion of any connections between determinism and unpredictability ...

Anthony Bonner in his book The Art and Logic of Ramon Llull (Brill 2007)
( unless otherwise stated illustrations herein are adapted from that book ) said:
“... Llull wanted to make the Art “general to everyone” ...
“a religiously neutral universal science” ... for Lull the Art is not enclosed in its own shell, but ... can even be adapted to “many other principles of science” ... 
Ramon Llull’s Y and Z Figures

are analogous to the binary structure of IFA
Ramon Llull’s Wheels A and X

have 16 vertices and 120 lines connecting pairs of vertices, corresponding to the 16 vectors of the Real Clifford Algebra Cl(16) and the 120 bivectors of Cl(16) that generate the 120-dim D8 Lie Algebra in the 248-dim E8 Lie Algebra with E8 / D8 = 64 + 64 Fermion Particles + AntiParticles representing 64 + 64 of E8 Maximal Contraction 28 + 64 + (A7+R) + 64 + 28
By 8-Periodicity of Real Clifford Algebras $\text{Cl}(16) = \text{tensor product } \text{Cl}(8) \times \text{Cl}(8)$
so the 16 vectors of $\text{Cl}(16) = 1 \times 8 + 8 \times 1$ where $8 = 8$ vectors of $\text{Cl}(8)$
and 8 of the 16 Wheel A vertices are the 8 blue vertices of Wheel X
and the other 8 Wheel A vertices are the 8 red vertices of Wheel X.

28 = 1 x 28 of the 120 D8 bivectors connect red vertices with red vertices
and represent the D4 Lie Algebra acting on the red 8-dim Cl(8) vector space
and 12 Standard Model Gauge Bosons plus 16 Gravity+Dark Energy Ghosts
representing 28 of E8 Maximal Contraction $28 + 64 + (A7+R) + 64 + 28$

64 = 8 x 8 of the 120 D8 bivectors connect red vertices with blue vertices
and represent A7+R of E8 Maximal Contraction $28 + 64 + A7+R + 64 + 28$

28 = 28 x 1 of the 120 D8 bivectors connect blue vertices with blue vertices
and represent the D4 Lie Algebra acting on the blue 8-dim Cl(8) vector space
and 16 Gravity+Dark Energy Gauge Bosons plus 12 Standard Model Ghosts
representing 28 of E8 Maximal Contraction $28 + 64 + (A7+R) + 64 + 28$
Around 1300 Scholasticism was being developed at the University of Paris, then the world’s leading University, and Cambridge and Oxford Universities which were getting organized based on Paris.

Doctor Illuminatus = Ramon Llull (1232-1315) produced a system of Logic and a mathematical Art based on what is now known as the Clifford Algebra $\text{Cl}(16)$ and the 120 dimensional Lie algebra $\text{Spin}(16)$. 700 years ago the details of that mathematics were not known, nor was it known that the math structure of the Art gives a realistic representation of E8 Physics of the Standard Model and Gravity+Dark Energy along with its Algebraic Quantum Field Theory. (see viXra 1804.0121)

Doctor Subtilis = John Duns Scotus (1266-1308) developed Llull’s system of Logic into sophisticated Scholasticism, but did not have the math and physics knowledge to show that the mathematical Art of Doctor Illuminatus gives a realistic physics model.

A Second Scholasticism began in 1540 with Ignatius Loyola under Pope Paul III who founded the Jesuits, but, without the ability to experimentally measure the relative strengths of the forces of the Standard Model and Gravity and the relative masses of the elementary fermion particles and to compare those observations with the physics model of Llull’s mathematical Art, by 1700 Scholasticism had been displaced by the Enlightenment of Descartes et al.

Now that we can do such experiments and make such observations we can, based on ideas Straight OUTTA AFRICA, use E8 Physics as a foundation for a Third Scholasticism.

Ron Eglash (in his book "African Fractals" and on his web site) also says: “... European geomancers ... maintained their society's exclusion of any connections between determinism and unpredictability ... The Africans, on the other hand, seem to have emphasized such connections ...[with]]... a "trickster" god, one who is both deterministic and unpredictable. ... The fractal settlement patterns of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. ... Euro-American cultures are ... "top-down" organization.

Precolonial African cultures included ... societies that are organized "bottom-up" rather than "top-down". ... African architecture tends to be fractal because that is a prominent design theme in African culture ... most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather they utilized an adaptive flexibility

African traditions of decentralized decision making could ... be combined with new information technologies, creating new forms that combine democratic rule with collective information sharing ...".
The 256-dim Cl(8) x 256-dim Cl(8) = 65,536-dim Cl(16) Clifford Algebra structure is also present in Microtubules = 40 micron size aggregates of 65,536 tubulin dimers that are the basis of Penrose-Hameroff Bohm Potential Quantum Consciousness.

Assembly of 65,536 tubulins into a 40-micron microtubule can be seen to be analogous to the 256 x 256 tensor product Cl(8) x Cl(8) where one 256-dim Cl(8) represents Standard Model U(1) SU(2) SU(3) with one 52-dim F4 related to the CP2 = SU(3) / SU(2)xU(1) of Kaluza-Klein M4 x CP2 and the other Cl(8) represents Conformal Gravity+Dark Energy with the other F4 related to the Minkowski M4 of Kaluza-Klein M4 x CP2.

The E8 of Cl(16) only uses 248 of the 65,536 elements so that 65,188 of them are available for Quantum Consciousness thought processes.
The Second Pyramid slope is of a 3-4-5 Right Triangle representing the Standard Model with Gauge Groups U(1) SU(2) SU(3) It represents CP2 of Kaluza-Klein M4 x CP2
The Great Pyramid slope is of a Golden Ratio Right Triangle representing Conformal Gravity + Dark Energy with Gauge Group $\text{Spin}(2,4) = SU(2,2)$

It represents M4 of Kaluza-Klein $M4 \times CP2$

Further,
the layers (courses) of the Great Pyramid correspond to the 256 elements of $\text{Cl}(8)$
The Sphinx represents 65,536-dim $\text{Cl}(16)$ containing 248-dim $\text{E}_8$ as the tensor product combination of
the 256-dim $\text{Cl}(8)$ containing 52-dim $\text{F}_4$ related to $\text{CP}^2$ of $M_4 \times \text{CP}^2$
and
the 256-dim $\text{Cl}(8)$ containing 52-dim $\text{F}_4$ related to $M_4$ of $M_4 \times \text{CP}^2$
The following image summarizes how the Sphinx represents the Cl(16) combination of the two large Cl(8) Pyramids and also the 65,536-element 40 micron Microtubules of Bohm Quantum Consciousness.
Since the 48 Root Vectors of $F_4 = 24$ vertices of 24-cell + 24 vertices of dual 24-cell
the 240 Root Vectors of $E_8$ are made up of
120 Root Vectors of $H_4 = 24$ $F_4$ 24-cell vertices + 96 $F_4$ dual 24-cell edges of $CP^2$
120 Root Vectors of $H_4 = 24$ $F_4$ 24-cell vertices + 96 $F_4$ dual 24-cell edges of $M_4$

Now we have explained the Title Page Image:

The following pages describe the physics of $E_8$ Root Vectors and further physics details
240 E8 Root Vectors Physical Interpretation

248-dim Lie Group E8 has 240 Root Vectors arranged on a 7-sphere S7 in 8-dim space.

Since it is hard to visualize points on S7 in 8-dim space, I prefer to represent the 240 E8 Root Vectors in 2-dim / 3-dim space as in this 2D representation by Ray Aschheim (see Appendix - Mathematica CDF)
To understand the Geometry related to the 240 E8 Root Vectors, consider that
248-dim E8 = 120-dim Spin(16) D8 + 128-dim half-spinor of Spin(16) D8

**240 E8 Root Vectors = 112 D8 Root Vectors + 128 D8 half-spinors**

**112 D8 Root Vectors** = 24 D4 (orange) + 24 D4 (yellow) + 64 (blue)

**128 D8 half-spinors** = 128 elements of E8 / D8

Green and Cyan dots with white centers (32+32 = 64 dots) and
Red and Magenta dots with black centers (32+32 = 64 dots)
correspond to the 128 elements of E8 / D8.

240 = 24 + 24 + 64 + 64 + 64
The 64 Green and Cyan Root Vectors represent half of the First Generation Fermions of E8 / D8. The White Centers of their dots indicate that they are Particles.

Their physical interpretations are
The 64 Red and Magenta Root Vectors represent the other half of the First Generation Fermions of E8 / D8. The Black Centers of their dots indicate that they are AntiParticles.

Their physical interpretations are
Spacetime, Unimodular Gravity, and Strong CP

The 64 Blue Root Vectors of the space $D8 / D4 \times D4$ represent 8D Spacetime and its symmetries such as 8 position x 8 momentum and the $A7 = SL(8,R)$ of Unimodular Gravity that is in the Maximal Contraction Heisenberg Algebra of $E8$ with structure $28 + 64 + (A7+1) = 64 + 28$.

(see Rutwig Campoamor-Stursberg in "Contractions of Exceptional Lie Algebras and SemiDirect Products" (Acta Physica Polonica B 41 (2010) 53-77)

The 4x16 = 64 blue correspond to the 64-dim symmetric space $D8 / D4 \times D4 = Gr(8,16)$ Grassmannian = set of RP7 in RP15

They are related by Triality to the 64 + 64 Fermion Components of $E8 / D8$

Creation-Annihilation Operators for 8-dim spacetime x 8-dim momentum space are the 64-dim grade-0 part of the $E8$ Maximal Contraction generalized Heisenberg Algebra $h_{92} \times A7 = 28 + 64 + ((SL(8,R)+1) + 64 + 28$

Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the metric tensor ... break[s up] ... into the conformal structure represented by a conformal metric ... with det = -1 and a four-volume element ... at each point of space-time ... [that]... may be the remnant, in the ... continuum limit, of a more fundamental discrete quantum structure of space-time itself ...".
In the Initial and Inflation Octonionic Phases of Our Universe
the 64 generators of D8 / D4 x D4 act as an Octonionic Conformal Structure
where Spin(0,8) of Cl(0,8) does rotations of 8-dim Octonion Space and
Spin(2,8) = Spin(1,9) = SL(2,O) of Cl(2,8) = Cl(1,9) = M(32,R) = M(2,Cl(0,8))
indicates a 10-dim Conformal Spacetime within 26-dim String Theory
and an 8-volume element at each point of Octonion Space indicates a fundamental
discrete structure of an underlying 26-dim String Theory in which
Strings = World-Lines and a spin-2 particle carries Bohm Quantum Potential.

Green, Schwartz, and Witten, in "Superstring Theory" vol. 1, describe 26D String Theory
saying ".... The first excited level ... consists of ... the ground state ... tachyon ...
and ... a scalar ... 'dilaton' ... and ... SO(24) ... little group of a ... massless particle ...
and ... a ... massless ... spin two state ...."

Unimodular SL(8,R) Gravity effectively describes a generalized checkerboard
of 8-dim SpaceTime HyperVolume Elements and, with respect to Cl(16) = Cl(8)xCl(8),
is the tensor product of the two 8v vector spaces of the two Cl(8) factors of Cl(16).
If those two Cl(8) factors are regarded as Fourier Duals,
then 8v x 8v describes Position x Momentum in 8-dim SpaceTime.

In the Post-Inflation Quaternionic Phase of Our Universe
8-dim Octonionic Spacetime splits into (4+4)-dim M4 x CP2 Kaluza-Klein Spacetime
M4 underlies a 6-dim Conformal Spacetime of Spin(2,4) = SU(2,2)
where Spin(2,4) = BiVectors of Cl(2,4) = M(4,H) = 4x4 Quaternion Matrices
CP2 = SU(3) / SU(2) x U(1)
carries the Gauge Groups of the Standard Model

Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said: "... Because of the existence
of ... QCD ... instantons the quantized theory contains a dimensionless parameter φ ( 0 < φ < 2π ) not explicit
in the classical lagrangian. ... the quantum dynamics of ... unimodular gravity ... may lead to
the relaxation of φ to φ = 0 ( mod π ) without the need ... for a new particle ... such as the axion ....".
The 24 Orange Root Vectors of the D4 of E8 Standard Model + Gravity Ghosts are on the Horizontal X-axis. The 4 Cartan Subalgebra elements of D4 of E8 Standard Model + Gravity Ghosts correspond to half of the 8 Cartan Subalgebra elements of E8.

**In the Initial and Inflation Octonionic Phases of Our Universe**
the 24+4 = 28 generators of D4 of E8 Standard Model + Gravity Ghosts act as a Spin(8) Gauge Group rotating all 8 Fermion types into each other.

In the Post-Inflation Quaternionic Phase of Our Universe
8-dim Octonionic Spacetime splits into (4+4)-dim M4 x CP2 Kaluza-Klein Spacetime
8 generators in the Orange Box represent the 8 Root Vectors of the Standard Model Gauge Groups SU(3) SU(2) U(1).
Their 4 Cartan Subalgebra elements correspond to the 4 Cartan Subalgebra elements of D4 of E8 Standard Model + Gravity Ghosts and to half of the 8 Cartan Subalgebra elements of E8.

The other 24-8 = 16 Orange Root Vectors represent Ghosts of 16D U(2,2) which contains the Conformal Group SU(2,2) = Spin(2,4) that produces Gravity + Dark Energy by the MacDowell-Mansouri mechanism.

Standard Model Gauge groups come from CP2 = SU(3) / SU(2) x U(1) (as described by Batakis in Class. Quantum Grav. 3 (1986) L99-L105)
Electroweak SU(2) x U(1) is gauge group as isotropy group of CP2.
SU(3) is global symmetry group of CP2 but due to Kaluza-Klein M4 x CP2 structure of compact CP2 at every M4 spacetime point, it acts as Color gauge group with respect to M4.
“... The ghost and the gauge field:
The single lines represent a local coordinate system
of a principal fiber bundle of base space-time.
The double lines are 1 forms.
The connection of the principle bundle $w$ is assumed to be vertical.
Its contravariant components $\Phi$ and $X$ are recognized, respectively,
as the Yang-Mills gauge field and the Faddeev-Popov ghost form ...”.

Steven Weinberg in The Quantum Theory of Fields Volume II Section 15.7 said:
“... there is a beautiful geometric interpretation of the ghosts and the BRST symmetry ...
The gauge fields $A_a^u$ may be written as one-forms $A_a = A_a^u \, dx^u$, where $dx_\mu$
are a set of anticommuting c-numbers. ... This can be combined with the ghost to
compose a one-form $A_a = A_a^u + w_a$ in an extended space.
Also, the ordinary exterior derivative $d = dx^u \, d/dx^u$ may be combined with the BRST
operator $s$ to form an exterior derivative $D = d + s$ in this space,
which is nilpotent because $s^2 = d^2 = sd + ds = 0$ ...”.

The 24-8 = 16 D4 of CP2 Root Vectors represent Ghosts of U(2,2) Conformal Gravity.
The 24 Yellow Root Vectors of the D4 of E8 Gravity + Standard Model Ghosts are on the Vertical Y-axis.
The 4 Cartan Subalgebra elements of D4 of E8 Gravity + Standard Model Ghosts correspond to half of the 8 Cartan Subalgebra elements of E8.

**In the Initial and Inflation Octonionic Phases of Our Universe**
the 24+4 = 28 generators of D4 of E8 Gravity + Standard Model Ghosts act as a Spin(6) Gauge Group rotating all 8 dimensions of Octonionic Spacetime into each other.

**In the Post-Inflation Quaternionic Phase of Our Universe**
8-dim Octonionic Spacetime splits into (4+4)-dim M4 x CP2 Kaluza-Klein Spacetime
12 generators in the Yellow Box represent the 12 Root Vectors of the Conformal Gauge Group SU(2,2) = Spin(2,4) of Conformal Gravity + Dark Energy
The 4 Cartan Subalgebra elements of SU(2,2) x U(1) = U(2,2) correspond to the 4 Cartan Subalgebra elements of D4 of E8 Gravity + Standard Model Ghosts and to the other half of the 8 Cartan Subalgebra elements of E8.

The other 24-12 = 12 Yellow Root Vectors represent Ghosts of 12D Standard Model whose Gauge Groups are SU(3) SU(2) U(1)

Gravity and Dark Energy come from D4 Conformal Subgroup SU(2,2) = Spin(2,4)

SU(2,2) = Spin(2,4) has 15 generators:

1 Dilation representing Higgs Ordinary Matter

4 Translations representing Primordial Black Hole Dark Matter

10 = 4 Special Conformal + 6 Lorentz representing Dark Energy
(see Irving Ezra Segal, "Mathematical Cosmology and Extragalactic Astronomy" (Academic 1976))

The basic ratio Dark Energy : Dark Matter : Ordinary Matter = 10:4:1 = 0.67 : 0.27 : 0.06
When the dynamics of our expanding universe are taken into account, the ratio is calculated to be 0.75 : 0.21 : 0.04
**E8 Lagrangian**

248-dim $E_8 = 120$-dim $D_8 + 128$-dim $E_8 / D_8$

128-dim $E_8 / D_8 = 64$-dim 8 components of 8 First-Generation Fermion Particles  
+  
64-dim 8 components of 8 First-Generation Fermion AntiParticles

120-dim $D_8 = 28$-dim $D_{4sm} + 28$-dim $D_{4gde} + 64$-dim ( $D_8 / D_{4sm} \times D_{4gde}$ )

28-dim $D_{4sm} = \text{Spin}(8)$ contains $\text{SU}(4)$ contains Color Force $\text{SU}(3)$ of Standard Model  
28-dim $D_{4gde} = \text{Spin}(4,4)$ contains $\text{SU}(2,2) = \text{Spin}(2,4)$ Conformal Group that gauges  
by MacDowell-Mansouri to produce Einstein-Hilbert Gravity plus DE  
DE = Dark Energy for Universe Expansion by I. E. Segal $\text{SU}(2,2)$ Conformal Gravity
64-dim (D8 / D4sm x D4gde) Bosonic term SL(8,R)+1 = Unimodular Gravity in 8-dim

SL(8,R)+1 = A7+1 is the grade 0 part of the Heisenberg-type Algebra that is
the Maximal Contraction h92 x A7 (x = semidirect product) of E8 with graded structure

28 + 64 + (A7+1) + 64 + 28

which is the Creation / Annihilation algebra

grades -2 and 2 for D4sm and D4gde
grades -1 and 1 for E8 / D8 Fermion AntiParticle and Particle Components
grade 0 for 8-dim Octonionic Spacetime Position and Momentum

To build a Lagrangian for E8 Physics with E8 inside Cl(16) so that E8 = D8 + E8 / D8
start with a Lagrangian Density with these terms:

Fermion terms =
= 64-dim 8 components of 8 Particles + 64-dim 8 components of 8 AntiParticles

Gauge Boson and Ghost terms = D8 = D4sm + D4gde + ( A7+1 = SL(8,R)+1 )

To find the Base Manifold Spacetime over which to integrate the Lagrangian Density:
1 - The Fermion term components are consistent with 8-dim Base Manifold Spacetime
2 - The 64-dim Bosonic term SL(8,R)+1 describes Unimodular Gravity in 8-dim
So: the E8 Physics Lagrangian (at high energies) is

\[ \int \text{D4sm + D4gde + SL(8,R)+1 + Fermion Terms} \]

8D Octonionic Spacetime

There are two terms that act as Gravity:
SL(8,R)+1 Unimodular on 8D Octonionic Spacetime
and
D4gde Conformal SU(2,2) on 4D Quaternionic Spacetime
The Initial Octonionic Lagrangian, through Inflation, of E8 Physics is

End of Inflation and Quaternionic Structure
Octonionic symmetry of 8-dim spacetime is broken at the End of Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2

CP2 = SU(3) / SU(2)xU(1) gives Standard Model SU(3) x SU(2) x U(1)
( Batakis mechanism )

Decomposition to M4 x CP2 Kaluza-Klein gives Higgs
( Mayer-Trautman mechanism )

and
In Kaluza-Klein M4 x CP2 there are 3 possibilities for a fermion represented by an Octonion O basis element to go from point A to point B:

1 - A and B are both in M4: First Generation Fermion whose path can be represented by the single O basis element so that First Generation Fermions are represented by Octonions O.

2 - Either A or B, but not both, is in CP2: Second Generation Fermion whose path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O basis element so that Second Generation Fermions are represented by Octonion Pairs OxO.

3 - Both A and B are in CP2: Third Generation Fermion whose path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O, so that Third Generation Fermions are represented by Octonion Triples OxOxO.
Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

**Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04**

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about $10^{-24}$ cm.

<table>
<thead>
<tr>
<th>Particle/Force</th>
<th>Tree-Level</th>
<th>Higher-Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>e-neutrino</td>
<td>0</td>
<td>0 for $\nu_1$</td>
</tr>
<tr>
<td>mu-neutrino</td>
<td>0</td>
<td>$9 \times 10^{-3}$ eV for $\nu_2$</td>
</tr>
<tr>
<td>tau-neutrino</td>
<td>0</td>
<td>$5.4 \times 10^{-2}$ eV for $\nu_3$</td>
</tr>
<tr>
<td>electron</td>
<td>0.5110 MeV</td>
<td></td>
</tr>
<tr>
<td>down quark</td>
<td>312.8 MeV</td>
<td>charged pion = 139 MeV</td>
</tr>
<tr>
<td>up quark</td>
<td>312.8 MeV</td>
<td>proton = 938.25 MeV</td>
</tr>
<tr>
<td>muon</td>
<td>104.8 MeV</td>
<td>neutron - proton = 1.1 MeV</td>
</tr>
<tr>
<td>strange quark</td>
<td>625 MeV</td>
<td></td>
</tr>
<tr>
<td>charm quark</td>
<td>2090 MeV</td>
<td></td>
</tr>
<tr>
<td>tauon</td>
<td>1.88 GeV</td>
<td></td>
</tr>
<tr>
<td>beauty quark</td>
<td>5.63 GeV</td>
<td></td>
</tr>
<tr>
<td>truth quark (low state)</td>
<td>130 GeV</td>
<td>(middle state) 174 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(high state) 218 GeV</td>
</tr>
<tr>
<td>W+</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W-</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W0</td>
<td>98.379 GeV</td>
<td>$Z_0 = 91.862$ GeV</td>
</tr>
<tr>
<td>Mplanck</td>
<td>1.217x10^{-19} GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs VEV (assumed)</td>
<td>252.5 GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs (low state)</td>
<td>126 GeV</td>
<td>(middle state) 182 GeV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(high state) 239 GeV</td>
</tr>
<tr>
<td>Gravity Gg (assumed)</td>
<td>1</td>
<td>$5 \times 10^{-39}$</td>
</tr>
<tr>
<td>(Gg)(Mproton^2 / Mplanck^2)</td>
<td>5/137.03608</td>
<td></td>
</tr>
<tr>
<td>EM fine structure</td>
<td>1/137.03608</td>
<td></td>
</tr>
<tr>
<td>Weak Gw</td>
<td>0.2535</td>
<td></td>
</tr>
<tr>
<td>Gw(Mproton^2 / (Mw^+2 + Mw^-2 + Mz0^2))</td>
<td>1.05 x 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>Color Force at 0.245 GeV</td>
<td>0.6286</td>
<td>0.106 at 91 GeV</td>
</tr>
</tbody>
</table>

Kobayashi-Maskawa parameters for W+ and W- processes are:

| d         | 0.975          |
| s         | 0.222          |
| b         | 0.00249 -0.00388i |
| c         | -0.222 -0.000161i |
| t         | 0.00698 -0.000378i |

The phase angle d13 is taken to be 1 radian.
forms a Higgs-Tquark NJL-type system with 3 Mass States
The Green Dot where the White Line originates in our Ordinary Phase is the **Low-mass state of a 130 GeV Truth Quark and a 125 GeV Higgs**.

The 130 GeV Tquark mass is also predicted by Connes’s NCG (NonCommutative Geometry) by the formula $Mt = \sqrt{8/3} M_w$
The Cyan Dot where the White Line hits the Triviality Boundary leaving the Ordinary Phase is the **Middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV**. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say: "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top \( m_t \) and the Higgs \( m_H \) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[ Kaluza-Klein type ]... dimension... D=8 ... \( m_t = 172-175 \text{ GeV} \) and \( m_H=176-188 \text{ GeV} \)."

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model ( Perseus Books 1999 ) at pages 175-176: "... The Higgs quartic coupling has a complicated scale dependence. It evolves according to \( d \lambda / d t = \left( \frac{1}{16 \pi^2} \right) \beta_{\lambda} \) where the one loop contribution is given by \( \beta_{\lambda} = 12 \lambda^2 - ... - 4 H \). The value of \( \lambda \) at low energies is related [to] the physical value of the Higgs mass according to the tree level formula \( m_H = v \sqrt{2 \lambda} \) while the vacuum value is determined by the Fermi constant ... for a fixed vacuum value \( v \), let us assume that the Higgs mass and therefore \( \lambda \) is large. In that case, \( \beta_{\lambda} \) is dominated by the \( \lambda^2 \) term, which drives the coupling towards its Landau pole at higher energies. Hence the higher the Higgs mass, the higher \( \lambda \) is and the closer the Landau pole to experimentally accessible regions.

This means that for a given (large) Higgs mass, we expect the standard model to enter a strong coupling regime at relatively low energies, losing in the process our ability to calculate. This does not necessarily mean that the theory is incomplete, only that we can no longer handle it ... it is natural to think that this effect is caused by new strong interactions, and that the Higgs actually is a composite ...

The resulting bound on \( \lambda \) is sometimes called the triviality bound. The reason for this unfortunate name (the theory is anything but trivial) stems from lattice studies where the coupling is assumed to be finite everywhere; in that case the coupling is driven to zero, yielding in fact a trivial theory. In the standard model \( \lambda \) is certainly not zero. ..."
The Magenta Dot at the end of the White Line is the **High-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs**. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model: "... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ...

BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale \[ \Lambda \] ...[ 10^{19} \text{ GeV} ]...

there must be a certain matching scale \( \Lambda_{\text{Matching}} \) such that the perturbative picture (BHL) is valid for \( \mu < \Lambda_{\text{Matching}} \), while only the nonperturbative picture (MTY) becomes consistent for \( \mu > \Lambda_{\text{Matching}} \)...

However, thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect \( m_t = m_t(\text{BHL}) = ... = 1/(\sqrt{2}) \ y_{\text{bart}} \) within 1-2\%, where \( y_{\text{bart}} \) is the quasi-infrared fixed point given by \( \beta(y_{\text{bart}}) = 0 \) in ...

the one-loop RG equation ...

The composite Higgs loop changes \( y_{\text{bart}}^2 \) by roughly the factor \( N_c/(N_c +3/2) = 2/3 \) compared with the MTY value, i.e., \( 250 \text{ GeV} \rightarrow 250 \times \sqrt{2/3} = 204 \text{ GeV} \), while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by \( m_t = 218 +/- 3 \text{ GeV} \), at \( \Lambda = 10^{19} \text{ GeV} \).

The Higgs boson was predicted as a tbar-t bound state with a mass \( M_H = 2m_t \) based on the pure NJL model calculation. Its mass was also calculated by BHL through the full RG equation ...

the result being ... \( M_H / m_t = 1.1 \) at \( \Lambda = 10^{19} \text{ GeV} \) ...

... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate. The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition. BHL essentially incorporates \( 1/N_c \) sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at \( 1/N_c \)-leading order. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".
Fermilab has seen all 3 Truth Quark Mass States:

At the LHC, CMS has seen all 3 Higgs Mass States:

CMS at arXiv 1804.01939 released a histogram in the Higgs $\rightarrow$ ZZ$^*$ $\rightarrow$ 4l channel for the 35.9 fb-1 of 2015-2016 LHC Run2 data that shows all 3 Higgs Mass States
The log scale for event number used by CMS makes the Higgs peaks look small. The peaks appear more realistic using a linear scale for event number:
Schwinger Sources, Hua Geometry, and Wyler Calculations

Fock “Fundamental of Quantum Mechanics” (1931) showed that Quantum requires Linear Operators “… represented by a definite integral [of a]... kernel ... function ...”.

Schwinger (1951 - see Schweber, PNAS 102, 7783-7788) “… introduced a description in terms of Green’s functions, what Feynman had called propagators ... The Green’s functions are vacuum expectation values of time-ordered Heisenberg operators, and the field theory can be defined non-perturbatively in terms of these functions ...[which]... gave deep structural insights into QFTs; in particular ... the structure of the Green’s functions when their variables are analytically continued to complex values ...”.

Wolf (J. Math. Mech 14 (1965) 1033-1047) showed that the Classical Domains (complete simply connected Riemannian symmetric spaces) representing 4-dim Spacetime with Quaternionic Structure are:
- \( S^4 \) = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity
- \( CP^2 \) = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force
- \( S^2 \times S^2 \) = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force
- \( S^1 \times S^1 \times S^1 \times S^1 \) = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon (1 copy for each of the 4 covariant components of the Photon)

Hua “Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains” (1958) showed Kernel Functions for Complex Classical Domains and calculated compact volumes (such as Euclidean spacetime) whose ratios correspond to ratios of measures of noncompact spaces (such as hyperbolic signature spacetime).

Here \( M = \) Spacetime Structure and \( D = \) Gauge Domain and \( Q = \) Shilov Boundary of \( D \):

<table>
<thead>
<tr>
<th>Force</th>
<th>( M )</th>
<th>( Vol(M) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity</td>
<td>( S^4 )</td>
<td>8\pi^2/3 - ( S^4 ) is 4-dimensional</td>
</tr>
<tr>
<td>color</td>
<td>( CP^2 )</td>
<td>8\pi^2/3 - ( CP^2 ) is 4-dimensional</td>
</tr>
<tr>
<td>weak</td>
<td>( S^2 \times S^2 )</td>
<td>2 \times 4\pi - ( S^2 ) is a 2-dim boundary of 3-dim ball</td>
</tr>
</tbody>
</table>

\( 4\text{-dim } S^2 \times S^2 = \) topological boundary of 6-dim 2-polyball
Shilov Boundary of 6-dim 2-polyball = \( S^2 + S^2 = \)
\( = \) 2-dim surface frame of 4-dim \( S^2 \times S^2 \)

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<tr>
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<tbody>
<tr>
<td>e-mag</td>
<td>( T^4 )</td>
<td>4 \times 2\pi - ( S^1 ) is 1-dim boundary of 2-dim disk</td>
</tr>
</tbody>
</table>

\( 4\text{-dim } T^4 = S^1 \times S^1 \times S^1 \times S^1 = \) topological boundary of 8-dim 4-polydisk
Shilov Boundary of 8-dim 4-polydisk = \( S^1 + S^1 + S^1 + S^1 = \)
\( = \) 1-dim wire frame of 4-dim \( T^4 \)

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<td>color</td>
<td>( CP^2 )</td>
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</tbody>
</table>
Armand Wyler (1971 - C. R. Acad. Sc. Paris, t. 271, 186-188) showed how to use Green’s Functions = Kernel Functions of Classical Domain structures characterizing Sources = Leptons, Quarks, and Gauge Bosons, to calculate Particle Mass and **Force Strength** = \((1 / M_{\text{force}}^2) \left( \frac{\text{Vol}(M)}{\text{Vol}(Q)} \right)^{1 / m_{\text{force}}} \)

where \(M_{\text{force}}\) = characteristic mass (Planck for Gravity, Weak Bosons for Weak)

<table>
<thead>
<tr>
<th>Gauge Group</th>
<th>Force</th>
<th>Characteristic Energy Level</th>
<th>Geometric Strength</th>
<th>Full Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin(5)</td>
<td>gravity</td>
<td>approx 10^{19} GeV</td>
<td>1</td>
<td>GGm_{\text{proton}}^2 approx 5 \times 10^{-39}</td>
</tr>
<tr>
<td>SU(3)</td>
<td>color</td>
<td>approx 245 MeV</td>
<td>0.6286</td>
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</tr>
<tr>
<td>SU(2)</td>
<td>weak</td>
<td>approx 100 GeV</td>
<td>0.2535</td>
<td>GWm_{\text{proton}}^2 approx 1.05 \times 10^{-5}</td>
</tr>
<tr>
<td>U(1)</td>
<td>e-mag</td>
<td>approx 4 KeV</td>
<td>1/137.03608</td>
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</tbody>
</table>

Schwinger (1969 - see physics/0610054) said: “... operator field theory ... replace[s] the particle with ... properties ... distributed throughout ... small volumes of three-dimensional space ... particles ... must be created ... even though we vary a number of experimental parameters ... The properties of the particle ... remain the same ... We introduce a quantitative description of the particle source in terms of a source function ... we do not have to claim that we can make the source arbitrarily small ... the experimenter... must detect the particles ...[by]... collision that annihilates the particle ... the source ... can be ... an abstraction of an annihilation collision, with the source acting negatively, as a sink ... The basic things are ... the source functions ... describing the intermediate propagation of the particle ...”.
Schwinger Sources can be described by continuous manifold structures of Bounded Complex Domains and their Shilov Boundaries but E8 Physics at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines with $8 + 8 + 8 = 24$-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about $8 \times 10^{53}$.

The Monster Group is of order

\[
\begin{align*}
8080 & , 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000 \\
= & \quad 2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71
\end{align*}
\]

or about $8 \times 10^{53}$

This chart (from Wikipedia) shows the Monster M and other Sporadic Finite Groups

\[
\begin{align*}
\text{Co1} \times \text{Th} \times \text{He} \times \text{HN} / \text{HS} \text{ together have order about } 4 \times 9 \times 4 \times 10^{(18+16+9+7)} \\
= & \quad \text{about } 10^{52}
\end{align*}
\]

The order of Co1 is $2^{21.3} \cdot 9.5^{4.7} \cdot 2.11.13.23$ or about $4 \times 10^{18}$.

\[
\begin{align*}
\text{Aut(Leech Lattice)} & = \text{double cover of Co1}. \\
\text{The order of the double cover } 2.\text{Co1} & = 2^{22.3} \cdot 9.5^{4.7} \cdot 2.11.13.23 \text{ or about } 0.8 \times 10^{19}.
\end{align*}
\]

Taking into account the non-sporadic part of the Leech Lattice symmetry according to the ATLAS at brauer.maths.qmul.ac.uk/Atlas/v3/spor/M/ the Schwinger Source Kerr-Newman Cloud Symmetry is $2^{\times 1}(1+24).\text{Co1}$ of order $139511839126336328171520000 = 1.4 \times 10^{26}$
The components of the Monster Group describe the composition of Schwinger Sources:

Co1 gives the number of particles in the Schwinger Source Kerr-Newman Cloud emanating from a Valence particle in a Planck-scale cell of E8 Physics SpaceTime.

Th = Thompson Group. Wikipedia says “... Th ... acts on a vertex operator algebra over the field with 3 elements. This vertex operator algebra contains the E8 Lie algebra over \( \mathbb{F}_3 \), giving the embedding of Th into E8(3) ...”. Th gives the 3-fold E8 Triality structure relating 8-dim SpaceTime to First-Generation Fermion Particles and AntiParticles.

He = Held Group. Wikipedia says “... The smallest faithful complex representation has dimension 51; there are two such representations that are duals of each other. It centralizes an element of order 7 in the Monster group. ...”. He gives the 7-fold algebraically independent Octonion Imaginary E8 Integral Domains that make up 7 of the 8 components of Octonion Superposition E8 SpaceTime.

HN = Harada-Norton Group. Wikipedia says “... The prime 5 plays a special role ... it centralizes an element of order 5 in ... the Monster group ...”. HN / HS gives the 5-fold symmetry of 120-element Binary Icosahedral E8 McKay Group beyond the 24-element Binary Tetrahedral E6 McKay Group at which level the Shilov Boundaries of Bounded Complex Domains emerge to describe SpaceTime and Force Strengths and Particle Masses.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single Planck-scale entity because Tachyons create a cloud of particles/antiparticles. The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole. That Kerr-Newman cloud constitutes the E8 Physics model Schwinger Source.

The cloud structure comes from the 24-dim Leech lattice part of the Monster Group which is \( 2^{(1+24)} \) times the double cover of Co1, for a total order of about \( 10^{^\text{26}} \).

Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices. The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order, so the volume of the Kerr-Newman Cloud is on the order of \( 10^{^\text{27}} \) x Planck scale and the Kerr-Newman Cloud should contain about \( 10^{^\text{27}} \) particle/antiparticle pairs. Its size should be about \( 10^{(27/3)} \times 1.6 \times 10^{(-33)} \) cm = roughly \( 10^{(-24)} \) cm.

Each of those particle-antiparticle pairs should see (with Bohm Potential) the rest of our Universe in the perspective of \( 8 \times 10^{^\text{53}} \) Monster Symmetry so a single Schwinger Source acting as a Jewel of Indra’s Net should see / reflect...
10^27 x 8 x 10^53 = 8 x 10^80 Other Schwinger Source Jewels of Indra’s Net which is consistent with the number of Schwinger Sources in our Universe.

Andrew Gray in arXiv quant-ph/9712037 said: “... probabilities are ... assigned to entire fine-grained histories ... base[d] ... on the Feynman path integral formulation ...” so in E8 Physics the Indra’s Net of Schwinger Source Jewels would not have Bohm Quantum Potential interactions between two Jewels, rather the interactions would be between the two entire World-Line History Strings

![blockchain diagram](https://www.blockchaintechnologies.com/)

Each Node is a Schwinger Source that is connected by Bohm Quantum Potential to all other Schwinger Source Nodes in our Universe and governed by the “algorithms and rules” of the E8 Physics Lagrangian and AQFT “... A **blockchain** is a type of distributed ledger, comprised of unchangable, digitally recorded data in packages called blocks. These digitally recorded "blocks" of data is stored in a linear chain ... A distributed ledger is a consensus of replicated, shared, and synchronized digital data geographically spread across multiple sites, countries, and/or institutions ...” or, for E8 Physics Indra’s Net of Schwinger Source Jewels, spread across the entirety of our Universe.
African Origin of Indra’s Net

About 50,000 years ago (National Geographic Genographic) YAP and M174 went out of Africa to Sunda (which was dry land South of Ankgor Wat and SouthEast of India) and on to Japan and Tibet. After M174 left Africa they no longer had a sufficiently extensive social network to pass their culture (IFA, Real Clifford Algebras, E8) from one generation to the next, so they had to develop more formal ways to preserve culture:

in Sunda / Angkor Wat / India - Sanskrit language and the Rig Veda
in Japan - Shinto Futomani 128 poems of Amateru = half of 256 Odu of IFA
in Tibet - 64 elements of I Ching = quarter of 256 Odu of IFA

Therefore the Rig Veda is likely to be the oldest book on Earth.

David Frawley in a hindubooks riverheaven web site said:
“... The Rig Veda is composed of ten books (called mandalas in Sanskrit) ...
The first book is a collection of hymns from seers of different families ...
Each hymn is given to a certain deity (devata).
The main deities are Indra, Agni, Soma and Surya. ...”
According to The Constitution of the Universe by Maharishi Mahesh Yogi
“... the Constitution of the Universe ... is embodied in the very structure of the sounds of the Rik Ved, the most fundamental aspect of the Vedic literature ... the structure of the Ved provides its own commentary - a commentary which is contained in the sequential unfoldment of the Ved itself in its various stages of expression. The knowledge of the total Ved ... is contained in the first sukt of the Rik Ved, which is ...

<table>
<thead>
<tr>
<th>First 24 Syllables</th>
<th>First 24 Gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 First Richa Syllables + 24 First Richa Gaps = D4sm + D4gde (purple box)</td>
<td></td>
</tr>
</tbody>
</table>

24 First Richa Syllables + 24 First Richa Gaps = D4sm + D4gde (purple box)

8x8 = 64 Last-8 Syllables of Last 8 Lines = D8 / D4sm x D4gde (blue box)

8x8 = 64 First-8 Syllables of Last 8 Lines (green box) and

8x8 = 64 Middle-8 Syllables of Last 8 Lines (red box)
give 128 = E8 / D8 = Fermion Particles and AntiParticles

Therefore the Rig Veda encodes E8 with 240 Root Vectors = 24+24+64+64+64

Indra and Indra’s Net, which are described in the Rig Veda, which describes E8, have their source in Africa and IFA

According to Wikipedia:
“... Indra is praised as the highest god in 250 hymns of the Rigveda ... the earliest reference to a net belonging to Indra is in the Atharva Veda (c. 1000 BCE) ... "Indra's net" is the net of the Vedic deva Indra, whose net hangs over his palace on Mount Meru, the axis mundi of Buddhist and Hindu cosmology. In this metaphor, Indra’s
net has a multifaceted jewel at each vertex, and each jewel is reflected in all of the other jewels. ... 

Aspects of Indra as a deity are cognate to other ... thunder gods 

Chango is the most feared god in Santería ... Ọṣàngó is viewed as the most powerful ... orisha ... He casts a "thundersone" to earth, which creates thunder and lightning ... Chango ... had three wives ... Princess Oshun, Princess Oba, and Princess Oya ... Oshun is the deity of the river ... She is connected to destiny and divination ... The abèbè is the ritual object most associated with Oṣun. The abèbè is a fan in circular form ... with a mirror in the center ...”.

Chango and Indra both use Thunder, 
and Chango’s wife Oshun does Divination with a Mirror 
so 
Chango and Oshun are two of the African IFA Orishas 
who are precursors of Vedic Indra and Indra’s Net.

Japan, the next stop beyond Sunda of Human M174 migration Out of Africa, also has similar culture:

the sacred Yata no Kagami, or Eight-Handed Mirror -
analogous to Indra Net Jewel Reflections

the Sword Kusanagi no Tsurugi -
analogous to ThunderBolts

the curved Yasakani no Magatama Jewel -
analogous to Indra Jewels
The E8 model constructs the Lagrangian integral such that the mass $m$ emerges as the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass. 

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space \( \text{Spin}(10) / \text{Spin}(8) \times U(1) \) with Bounded Complex Domain D8 of type IV8 and Shilov Boundary Q8 = RP1 x S7 which has local symmetry of the Spin(8) gauge group from which the first generation spinor fermions are formed as \( +\text{half-spinor} \) and \( -\text{half-spinor} \) spaces.
For the Gauge Gravity and Standard Model Gauge Bosons the process of breaking Octonionic 8-dim SpaceTime down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime. There are 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces with Quaternionic structure consistent with 4-dim Physical SpaceTime:

\[ S_4 = \text{4-sphere} = \text{Spin}(5) / \text{Spin}(4) \text{ where Spin}(5) = \text{Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity} \]

\[ CP_2 = \text{complex projective 2-space} = \text{SU}(3) / \text{U}(2) \text{ with the SU(3) of the Color Force} \]

\[ S_2 \times S_2 = \text{SU}(2)/\text{U}(1) \times \text{SU}(2)/\text{U}(1) \text{ with two copies of the SU(2) of the Weak Force} \]

\[ S_1 \times S_1 \times S_1 \times S_1 = \text{U}(1) \times \text{U}(1) \times \text{U}(1) \times \text{U}(1) = 4 \text{ copies of the U(1) of the EM Photon (1 copy for each of the 4 covariant components of the Photon)} \]

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8). They "see" M4 Physical spacetime as the 4-sphere S4 so that their part of the Physical Lagrangian is

\[ \int_{S_4} \text{Gravity Gauge Boson Term} \]

an integral over SpaceTime S4.

The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons. However, due to Stabilization of Condensate SpaceTime by virtual Planck Mass Gravitational Black Holes, for Gravity, the effective force strength that we see in our experiments is not just composed of the S4 volume and the Spin(5) Schwinger Source volume, but is suppressed by the square of the Planck Mass. The unsuppressed Gravity force strength is the Geometric Part of the force strength.
The Standard Model SU(3) Color Force bosons live in a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8). They "see" M4 Physical spacetime as the complex projective plane CP2 so that their part of the Physical Lagrangian is

\[ \int \text{SU(3) Color Force Gauge Boson Term} \ \text{CP2} \ . \]
an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons. The Color Force Strength is given by the SpaceTime CP2 volume and the SU(3) Schwinger Source volume. Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2 so that their part of the Physical Lagrangian is

\[ \int \text{SU(2) Weak Force Gauge Boson Term} \ \text{S2xS2} \ . \]
an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons. However, due to the action of the Higgs mechanism, for the Weak Force, the effective force strength that we see in our experiments is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume, but is suppressed by the square of the Weak Boson masses. The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4 (T4 = 4-torus) so that their part of the Physical Lagrangian is

\[ \int \text{(U(1) Electromagnetism Gauge Boson Term} \ \text{T4} \ . \]
an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons are the Complex Bounded Domains and Shilov Boundaries for U(1) photons. The Electromagnetic Force Strength is given by the SpaceTime T4 volume and the U(1) Schwinger Source volume.
Force Strength and Boson Mass Calculation

The Force Strength is made up of two parts:

- the relevant spacetime manifold of gauge group global action
- the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian $G_{GSM}$ gauge boson term is:

- the integral over spacetime as seen by gauge boson acting globally
- of the gauge force term of the gauge boson acting locally

for the gauge bosons of each of the four forces:

- $U(1)$ for electromagnetism
- $SU(2)$ for weak force
- $SU(3)$ for color force
- $Spin(5)$ - compact version of antiDeSitter $Spin(2,3)$ subgroup of Conformal $Spin(2,4)$ for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,

for each gauge force the gauge boson force term contains the force strength, which in Feynman's picture is the amplitude to emit a gauge boson, and can also be thought of as the probability = square of amplitude, in an explicit (like $g |F|^2$) or an implicit (incorporated into the $|F|^2$) form. Either way, the conventional picture is that the force strength $g$ is an ad hoc inclusion.

The E8 model does not put in force strength $g$ ad hoc, but constructs the integral such that the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

- the $U(1)$ photon sees 4-dim spacetime as $T^4 = S^1 \times S^1 \times S^1 \times S^1$
- the $SU(2)$ weak boson sees 4-dim spacetime as $S^2 \times S^2$
- the $SU(3)$ weak boson sees 4-dim spacetime as $CP^2$
- the $Spin(5)$ of gravity sees 4-dim spacetime as $S^4$

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson. The nontrivial Shilov boundaries are:

- for $SU(2)$ Shilov = $RP^1 \times S^2$
- for $SU(3)$ Shilov = $S^5$
- for $Spin(5)$ Shilov = $RP^1 \times S^4$
The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

\[
\begin{align*}
S1 & \text{ for } U(1) \\
S2 & = SU(2)/U(1) = Spin(3)/Spin(2) \text{ for } SU(2) \\
CP2 & = SU(3)/SU(2)xU(1) \text{ for } SU(3) \\
S4 & = Spin(5)/Spin(4) \text{ for } Spin(5)
\end{align*}
\]

Each gauge group is the local symmetry of a symmetric space

\[
\begin{align*}
U(1) & \text{ for itself} \\
SU(2) & \text{ for } Spin(5) / SU(2)xU(1) \\
SU(3) & \text{ for } SU(4) / SU(3)xU(1) \\
Spin(5) & \text{ for } Spin(7) / Spin(5)xU(1)
\end{align*}
\]

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

\[
\begin{align*}
SU(2) & \text{ for } Spin(5) / SU(2)xU(1) \text{ corresponds to } IV3 \\
SU(3) & \text{ for } SU(4) / SU(3)xU(1) \text{ corresponds to } B^6 \text{ (ball)} \\
Spin(5) & \text{ for } Spin(7) / Spin(5)xU(1) \text{ corresponds to } IV5
\end{align*}
\]

The nontrivial bounded complex domains have Shilov boundaries

\[
\begin{align*}
SU(2) & \text{ for } Spin(5) / SU(2)xU(1) \text{ corresponds to } IV3 \text{ Shilov } = RP^1xS^2 \\
SU(3) & \text{ for } SU(4) / SU(3)xU(1) \text{ corresponds to } B^6 \text{ (ball) Shilov } = S^5 \\
Spin(5) & \text{ for } Spin(7) / Spin(5)xU(1) \text{ corresponds to } IV5 \text{ Shilov } = RP^1xS^4
\end{align*}
\]

Very roughly, think of the force strength as

integral over global symmetry space of physical (ie Shilov Boundary) volume =

= strength of the force.

That is:

the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson means
that the highest force strength or probability should be 1
the gravity Volume product is normalized to be 1, and so (approximately):

\[
\begin{align*}
\text{Volume product for gravity} & = 1 \\
\text{Volume product for color} & = 2/3 \\
\text{Volume product for weak} & = 1/4 \\
\text{Volume product for electromagnetism} & = 1/137
\end{align*}
\]
There are two further main components of a force strength:
1 - for massive gauge bosons, a suppression by a factor of $1 / M^2$
2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:
Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes, so that the geometric strength of gravity should be reduced by $1/M_p^2$

The weak force is carried by weak bosons, so that the geometric strength of the weak force should be reduced by $1/M_W^2$
That gives the result (approximate):

- gravity strength = $G$ (Newton's G)
- color strength = $2/3$
- weak strength = $G_F$ (Fermi's weak force G)
- electromagnetism = $1/137$

Consider Renormalization Running for the Color Force: That gives the result:

- gravity strength = $G$ (Newton's G)
- color strength = $1/10$ at weak boson mass scale
- weak strength = $G_F$ (Fermi's weak force G)
- electromagnetism = $1/137$

The use of compact volumes is itself a calculational device, because it would be more nearly correct, instead of the integral over the compact global symmetry space of the compact physical (i.e. Shilov Boundary) volume=strength of the force to use the integral over the hyperbolic spacetime global symmetry space of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios, and the compact volumes (finite and easy to look up in the book by Hua) have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices, and that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.
Here are more detailed force strength calculations:

The force strength of a given force is

$$\alpha_{\text{force}} = \left(\frac{1}{M_{\text{force}}^2}\right) \left(\frac{\text{Vol}(\text{MIS}_{\text{force}})}{\text{Vol}(\text{Qforce}) / \text{Vol}(\text{Dforce})^{1/m_{\text{force}}}}\right)$$

where:

\(\alpha_{\text{force}}\) represents the force strength;

\(M_{\text{force}}\) represents the effective mass;

\(\text{MIS}_{\text{force}}\) represents the relevant part of the target Internal Symmetry Space;

\(\text{Vol}(\text{MIS}_{\text{force}})\) stands for volume of \(\text{MIS}_{\text{force}}\) and is sometimes also denoted by \(\text{Vol}(M)\);

\(\text{Qforce}\) represents the link from the origin to the relevant target for the gauge boson;

\(\text{Vol}(\text{Qforce})\) stands for volume of \(\text{Qforce}\);

\(\text{Dforce}\) represents the complex bounded homogeneous domain of which \(\text{Qforce}\) is the Shilov boundary;

\(m_{\text{force}}\) is the dimensionality of \(\text{Qforce}\), which is

- 4 for Gravity and the Color force,
- 2 for the Weak force (which therefore is considered to have two copies of \(Q_{W}\) for SpaceTime),
- 1 for Electromagnetism (which therefore is considered to have four copies of \(Q_{E}\) for SpaceTime)

\(\text{Vol}(\text{Dforce})^{1/m_{\text{force}}}\) stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex).

The \(\text{Qforce}\), Hermitian symmetric space, and \(\text{Dforce}\) manifolds for the four forces are:

| \(\text{Spin}(5)\) | \(\text{Spin}(7) / \text{Spin}(5) \times \text{U}(1)\) | \(\text{IV}_5\) | 4 | \(\text{RP}^1 \times \text{S}^4\) |
| \(\text{SU}(3)\) | \(\text{SU}(4) / \text{SU}(3) \times \text{U}(1)\) | \(\text{B}^6(\text{ball})\) | 4 | \(\text{S}^5\) |
| \(\text{SU}(2)\) | \(\text{Spin}(5) / \text{SU}(2) \times \text{U}(1)\) | \(\text{IV}_3\) | 2 | \(\text{RP}^1 \times \text{S}^2\) |
| \(\text{U}(1)\) | - | - | 1 | - |
The geometric volumes needed for the calculations are mostly taken from the book "Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains" (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

<table>
<thead>
<tr>
<th>Force</th>
<th>M</th>
<th>Vol(M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity</td>
<td>$S^4$</td>
<td>$8\pi^2/3$ - $S^4$ is 4-dimensional</td>
</tr>
<tr>
<td>color</td>
<td>$CP^2$</td>
<td>$8\pi^2/3$ - $CP^2$ is 4-dimensional</td>
</tr>
<tr>
<td>weak</td>
<td>$S^2 \times S^2$</td>
<td>$2 \times 4\pi$ - $S^2$ is a 2-dim boundary of 3-dim ball</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-dim $S^2 \times S^2$ = topological boundary of 6-dim 2-polyball</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shilov Boundary of 6-dim 2-polyball = $S^2 + S^2 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 2$-dim surface frame of 4-dim $S^2 \times S^2$</td>
</tr>
<tr>
<td>e-mag</td>
<td>$T^4$</td>
<td>$4 \times 2\pi$ - $S^1$ is 1-dim boundary of 2-dim disk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-dim $T^4 = S^1 \times S^1 \times S^1 \times S^1$ = topological boundary of 8-dim 4-polydisk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shilov Boundary of 8-dim 4-polydisk = $S^1 + S^1 + S^1 + S^1 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 1$-dim wire frame of 4-dim $T^4$</td>
</tr>
</tbody>
</table>

Note (thanks to Carlos Castro for noticing this) also that the volume listed for $CP^2$ is unconventional, but physically justified by noting that $S^4$ and $CP^2$ can be seen as having the same physical volume, with the only difference being structure at infinity.

Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

<table>
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<td>$\pi^5/2^4 \times 5!$</td>
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<tr>
<td>color</td>
<td>$CP^2$</td>
<td>$8\pi^2/3$</td>
<td>$S^5$</td>
<td>$4\pi^3$</td>
</tr>
<tr>
<td>weak</td>
<td>$S^2 \times S^2$</td>
<td>$2 \times 4\pi$</td>
<td>$8\pi^1 \times S^2$</td>
<td>$4\pi^2$</td>
</tr>
<tr>
<td>e-mag</td>
<td>$T^4$</td>
<td>$4 \times 2\pi$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note (thanks to Carlos Castro for noticing this) that the volume listed for $S^5$ is for a squashed $S^5$, a Shilov boundary of the complex domain corresponding to the symmetric space $SU(4) / SU(3) \times U(1)$.
Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5) gravity approx \(10^{19} \text{ GeV}\) \(1\) \(GGmproton^2 \approx 5 \times 10^{-39}\)

SU(3) color approx 245 MeV \(0.6286\) \(0.6286\)

SU(2) weak approx 100 GeV \(0.2535\) \(GWmproton^2 \approx 1.05 \times 10^{-5}\)

U(1) e-mag approx 4 KeV \(1/137.03608\) \(1/137.03608\)

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels. The effect is particularly pronounced with the color force. The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

<table>
<thead>
<tr>
<th>Energy Level</th>
<th>Color Force Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>245 MeV</td>
<td>0.6286</td>
</tr>
<tr>
<td>5.3 GeV</td>
<td>0.166</td>
</tr>
<tr>
<td>34 GeV</td>
<td>0.121</td>
</tr>
<tr>
<td>91 GeV</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV
**Higgs, W+, W-, Z0:**

As with forces strengths, the calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.

In the Cl(1,25) E8 model, the value of the fundamental mass scale vacuum expectation value \( v = \langle \Phi \rangle \) of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, \( W^+, W^-, \) and \( Z^0 \), whose tree-level masses will then be shown by ratio calculations to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively, and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and \( v \) is given by the Ginzburg-Landau term from the Mayer Mechanism as

\[
\frac{1}{4} \text{Tr} \left( [\Phi H, \Phi] - \Phi \right)^2
\]

or, in the notation of quant-ph/9806009 by Guang-jiong Ni

\[
\frac{1}{4!} \lambda \Phi^4 - \frac{1}{2} \sigma \Phi^2
\]

where the Higgs mass \( M_H = \sqrt{2 \sigma} \).

Ni says: "... the invariant meaning of the constant \( \lambda \) in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of \( \lambda \) is nothing but the ratio of two mass scales:

\[
\lambda = 3 \left( \frac{M_H}{\Phi} \right)^2
\]

which remains unchanged irrespective of the order ...".

Since \( \langle \Phi \rangle^2 = v^2 \), if \( v = 252.514 \text{ GeV} \) and \( \lambda = 1 \) for a single-mass-state Higgs,

\[
1 = \sqrt{3} \frac{M_H}{v} \quad \text{so that} \quad M_H = 252.514 / \sqrt{3} = 145.789 \text{ GeV}
\]

However, for 3-mass-state Higgs as Nambu - Jona-Lasinio Tquark condensate

\[
\lambda = \left( \cos \left( \frac{\pi}{6} \right) \right)^2 = 0.866^2 \quad \text{we have}
\]

\[
M_H^2 / v^2 = \left( \cos \left( \frac{\pi}{6} \right) \right)^2 / 3
\]

In E8 Physics, the fundamental mass scale vacuum expectation value \( v \) of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and \( v \) is set to be 252.514 GeV and we have

\[
M_H = v \cos \left( \frac{\pi}{6} \right) / \sqrt{1 / 3} = 126.257 \text{ GeV}
\]

This is the value of the Low Mass State of the Higgs observed by the LHC. Middle and High Mass States come from a Higgs-Tquark Condensate System. The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.
A Non-Condensate Higgs is represented by a Higgs at a point in M4 that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass with \( \lambda = 1 = 1^2 \) and Higgs mass \( M_H = v / \sqrt{3} = 145.789 \text{ GeV} \)

\[
\begin{array}{c|c|c}
\text{Higgs} & \text{Higgs in CP2 Internal Symmetry Space} & \text{Higgs in M4 spacetime} \\
\hline
& & \\
mass = 145 & Non-Condensate Higgs Mass = 145 & \\
& & \\
\text{Higgs} & \text{Higgs in M4 spacetime} & \\
\end{array}
\]

However, in E8 Physics, the Higgs has structure of a Tquark condensate

\[
\begin{array}{c|c|c}
\text{mass} = 145 & \text{Effective Higgs in CP2 Internal Symmetry Space} & \\
\hline
T \quad \text{---} \quad \text{Tbar} & \text{Higgs Effective Mass} = & 145 \times \cos(\pi/6) = 145 \times 0.866 = 126 \\
\text{\quad /} & & \\
\text{\quad /} & & \\
\text{\quad /} & & \\
Higgs & & \\
\end{array}
\]

In which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle composed of 2 right triangles (one from the CP2 origin to the T and to the M4 Higgs and another from the CP2 origin to the Tbar and to the M4 Higgs).

In the T-quark condensate picture

\[
\lambda = 1^2 = \lambda(T) + \lambda(H) = (\sin(\pi/6))^2 + (\cos(\pi/6))^2
\]

\[
\lambda(H) = (\cos(\pi/6))^2
\]

Therefore the Effective Higgs mass observed by LHC is:

\[
\text{Higgs Mass} = 145.789 \times \cos(\pi/6) = 126.257 \text{ GeV}.
\]
To get \textbf{W-boson masses}, denote the 3 SU(2) high-energy weak bosons (massless at energies higher than the electroweak unification) by \( W^+, W^-, \) and \( W^0 \), corresponding to the massive physical weak bosons \( W^+, W^-, \) and \( Z^0 \).

The triplet \( \{ W^+, W^-, W^0 \} \) couples directly with the \( T^- \)-\( \overline{T} \) quark-antiquark pair, so that the total mass of the triplet \( \{ W^+, W^-, W^0 \} \) at the electroweak unification is equal to the total mass of a \( T^- \)-\( \overline{T} \) pair, 259.031 GeV.

The triplet \( \{ W^+, W^-, Z^0 \} \) couples directly with the Higgs scalar, which carries the Higgs mechanism by which the \( W^0 \) becomes the physical \( Z^0 \), so that the total mass of the triplet \( \{ W^+, W^-, Z^0 \} \) is equal to the vacuum expectation value \( v \) of the Higgs scalar field, \( v = 252.514 \) GeV.

What are individual masses of members of the triplet \( \{ W^+, W^-, Z^0 \} \)?

First, look at the triplet \( \{ W^+, W^-, W^0 \} \) which can be represented by the 3-sphere \( S^3 \). The Hopf fibration of \( S^3 \) as
\[
S^1 \rightarrow S^3 \rightarrow S^2
\]
gives a decomposition of the \( W \) bosons into the neutral \( W^0 \) corresponding to \( S^1 \) and the charged pair \( W^+ \) and \( W^- \) corresponding to \( S^2 \).

The mass ratio of the sum of the masses of \( W^+ \) and \( W^- \) to the mass of \( W^0 \) should be the volume ratio of the \( S^2 \) in \( S^3 \) to the \( S^1 \) in \( S^3 \). The unit sphere \( S^3 \) in \( \mathbb{R}^4 \) is normalized by \( 1 / 2 \). The unit sphere \( S^2 \) in \( \mathbb{R}^3 \) is normalized by \( 1 / \sqrt{3} \). The unit sphere \( S^1 \) in \( \mathbb{R}^2 \) is normalized by \( 1 / \sqrt{2} \). The ratio of the sum of the \( W^+ \) and \( W^- \) masses to the \( W^0 \) mass should then be
\[
\frac{2}{\sqrt{3}} \frac{V(S^2)}{V(S^1)} = 1.632993
\]

Since the total mass of the triplet \( \{ W^+, W^-, W^0 \} \) is 259.031 GeV, the total mass of a \( T^- \)-\( \overline{T} \) pair, and the charged weak bosons have equal mass, we have
\[
M_{W^+} = M_{W^-} = 80.326 \text{ GeV} \quad \text{and} \quad M_{W^0} = 98.379 \text{ GeV}.
\]

The charged \( W^+/\overline{W}^0 \) neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the tree-level absence of right-handed neutrino particles requires that
the charged \( W^+/\overline{W}^0 \) SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged \( W^+/\overline{W}^0 \) SU(2) weak bosons act only on left-handed fermion particles of all types.
The neutral $W^0$ weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral $W^0$ weak bosons are related to the charged $W^{+/−}$ weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral $W^0$ must be equal to the left-handed (entire) component of the charged $W^{+/−}$.

Since the mass of the $W^0$ is greater than the mass of the $W^{+/−}$, there remains for the $W^0$ a component acting on both types of fermions.

Therefore the full $W^0$ neutral weak boson interaction is proportional to 
\[
\left( \frac{M_{W^+/−}}{M_W^0} \right)^2 \text{ acting on left-handed fermions}
\]
and 
\[
\left( 1 - \frac{M_{W^+/−}}{M_W^0} \right) \text{ acting on both types of fermions.}
\]

If \( 1 - \frac{M_{W^+/−}}{M_W^0} \) is defined to be \( \sin(\theta_w)^2 \) and denoted by \( K \), and if the strength of the $W^{+/−}$ charged weak force (and of the custodial SU(2) symmetry) is denoted by \( T \), then the $W^0$ neutral weak interaction can be written as $W^0L = T + K$ and $W^0LR = K$.

Since the $W^0$ acts as $W^0L$ with respect to the parity violating SU(2) weak force and as $W^0LR$ with respect to the parity conserving U(1) electromagnetic force, the $W^0$ mass $M_W^0$ has two components:
- the parity violating SU(2) part $m_{W^0L}$ that is equal to $M_{W^+/−}$
- and the parity conserving part $m_{W^0LR}$ that acts like a heavy photon.

As $M_W^0 = 98.379$ GeV = $M_{W^0L} + M_{W^0LR}$, and as $M_{W^0L} = M_{W^+/−} = 80.326$ GeV, we have $M_{W^0LR} = 18.053$ GeV.

Denote by $\alphaE = e^2$ the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength $\alphaE = e^2 = 1 / 137.03608$ was calculated above using the volume $V(S^1)$ of an $S^1$ in $R^2$, normalized by $1 / \sqrt{2}$.

The $\alphaE$ force is part of the SU(2) weak force whose strength $\alphaW = w^2$ was calculated above using the volume $V(S^2 \subset R^3)$, normalized by $1 / \sqrt{3}$.

Also, the electromagnetic force strength $\alphaE = e^2$ was calculated above using a 4-dimensional spacetime with global structure of the 4-torus $T^4$ made up of four $S^1$ 1-spheres,
while the SU(2) weak force strength $\alphaW = w^2$ was calculated above using two 2-spheres $S^2 \times S^2$,
each of which contains one 1-sphere of the $\alphaE$ force.
Therefore
\[ \alpha_E = \alpha_E \left( \frac{\sqrt{2}}{\sqrt{3}} \right) \left( \frac{2}{4} \right) = \alpha_E / \sqrt{6}, \]
\[ e = e / (\text{4th root of } 6) = e / 1.565, \]
and the mass \( m_{W0LR} \) must be reduced to an effective value
\[ m_{W0LR_{eff}} = m_{W0LR} / 1.565 = 18.053/1.565 = 11.536 \text{ GeV} \]
for the \( \alpha_E \) force to act like an electromagnetic force in the E8 model:
\[ e M_{W0LR} = e (1/5.65) M_{W0LR} = e M_{Z0}, \]
where the physical effective neutral weak boson is denoted by \( Z_0 \).

Therefore, the correct Cl(1,25) E8 model values for weak boson masses and the Weinberg angle \( \theta_w \) are:
\[ M_{W+} = M_{W-} = 80.326 \text{ GeV}; \]
\[ M_{Z0} = 80.326 + 11.536 = 91.862 \text{ GeV}; \]
\[ \sin^2(\theta_w) = 1 - (M_{W+/-} / M_{Z0})^2 = 1 - (6452.2663 / 8438.6270) = 0.235. \]

Radiative corrections are not taken into account here, and may change these tree-level values somewhat.
Fermion Mass Calculations

In E8 Physics, the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8). Due to Triality, Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime.

Take the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:
the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is taken in the same way ( unlike what happens for the force strength calculation ), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.

E8 Physics Fermions correspond to Schwinger Sources, so the quark mass in E8 Physics is a constituent mass.

Fermion masses are calculated as a product of four factors:

\[ V(Q\text{fermion}) \times N(\text{Graviton}) \times N(\text{octonion}) \times \text{Sym} \]

\( V(Q\text{fermion}) \) is the volume of the part of the half-spinor fermion particle manifold \( S^7 \times \text{RP}^1 \) related to the fermion particle by photon, weak boson, or gluon interactions.

\( N(\text{Graviton}) \) is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2). 2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

\( N(\text{octonion}) \) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

\( \text{Sym} \) is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.
3 Generation Fermion Combinatorics

First Generation (8)

( geometric representation of Octonions is from arXiv 1010.2979 )

<table>
<thead>
<tr>
<th>electron</th>
<th>red up quark</th>
<th>green up quark</th>
<th>blue up quark</th>
<th>red down quark</th>
<th>green down quark</th>
<th>blue down quark</th>
<th>neutrino</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Diagram of First Generation Fermions](image)

Second Generation (64)

Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element) .
Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

Blue Strange Quark (3)
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark (17)
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).

( Red and Green Strange and Charm Quarks follow similar rules )
Third Generation (512)

Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)
Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
   and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative and must be Blue.

( Red and Green Beauty and Truth Quarks follow similar rules )
The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless at tree level, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i. By gluon interactions, i can be taken into j and k, the blue and green down quarks. By also using weak boson interactions, it can also be taken into I, J, and K, the red, blue, and green up quarks. Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos. Therefore the red down quark (similarly, any down quark) is related to all parts of S<sup>7</sup> x RP<sup>1</sup>, the compact manifold corresponding to { 1, i, j, k, E, I, J, K } and therefore a down quark should have a spinor manifold volume factor V(Qdown quark) of the volume of S<sup>7</sup> x RP<sup>1</sup>. The ratio of the down quark spinor manifold volume factor to the electron spinor manifold volume factor is

\[
\frac{V(Q\text{down quark})}{V(Q\text{electron})} = \frac{V(S^7 \times RP^1)}{1} = \frac{\pi^5}{3}.
\]

Since the first generation graviton factor is 6,

\[
\frac{md}{me} = 6 \times \frac{V(S^7 \times RP^1)}{1} = 2 \pi^5 = 612.03937
\]

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

\[
u = md.
\]

Antiparticles have the same mass as the corresponding particles. Since the model only gives ratios of masses, the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV, and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

\[
\Sigma f_1 = 1.877 \text{ GeV}
\]
As the proton mass is taken to be the sum of the constituent masses of its constituent quarks
\[ m_{\text{proton}} = m_u + m_u + m_d = 938.25 \text{ MeV} \]
which is close to the experimental value of 938.27 MeV.

**The third generation** fermion particles correspond to triples of octonions. There are \(8^3 = 512\) such triples.

The triple \(\{1,1,1\}\) corresponds to the tau-neutrino.

The other 7 triples involving only 1 and \(E\) correspond to the tauon:

\[
\begin{align*}
\{E, E, E\} \\
\{E, E, 1\} \\
\{E, 1, E\} \\
\{1, E, E\} \\
\{1, 1, E\} \\
\{1, E, 1\} \\
\{E, 1, 1\}
\end{align*}
\]

The symmetry of the 7 tauon triples is the same as the symmetry of the first generation tree-level-massive fermions, 3 down quarks, the 3 up quarks, and the electron, so by the Sym factor the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV, the effective tauon mass should be renormalized from the energy level of 1 GeV at which the mass is 1.88 GeV to the energy level of 2 GeV. Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent, the effective tauon mass at 2 GeV would be about 1.78 GeV. The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.
The beauty quark corresponds to 21 triples. They are triples of the same form as the 7 tauon triples involving 1 and E, but for 1 and I, 1 and J, and 1 and K, which correspond to the red, green, and blue beauty quarks, respectively.

The seven red beauty quark triples correspond to the seven tauon triples, except that the beauty quark interacts with 6 Spin(0,5) gravitons while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times the third generation graviton factor $6/2 = 3$, so the red beauty quark mass is $m_b = 5.63111$ GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a constituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV. Therefore, the calculated beauty quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant $\alpha_s$ is known. The conventional value of $\alpha_s$ at about 5 GeV is about 0.22.

Using $\alpha_s (5 \text{ GeV}) = 0.22$, a pole mass of 5.0 GeV gives an MSbar 1-loop beauty quark mass of 4.6 GeV, and an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV, with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).
The theoretical model calculated Beauty Quark mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value of the color force strength constant alpha_s at about 5 GeV is about 0.166, while the conventional value of the color force strength constant alpha_s at about 5 GeV is about 0.216, and the theoretical model calculated value of the color force strength constant alpha_s at about 90 GeV is about 0.106, while the conventional value of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV), and a color force strength alpha_s at 5 GeV (0.166) such that 1 + alpha_s = 1.166 is about 4 percent lower than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Triples of the type \{ 1, I, J \}, \{ I, J, K \}, etc., do not correspond to the beauty quark, but to the truth quark. The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples, so the constituent mass of the red truth quark is 161 / 7 = 23 times the red beauty quark mass, and the red T-quark mass is

\[ m_T = 129.5155 \text{ GeV} \]

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in E8 Physics. The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994. The Low and High Mass States of the Truth Quark have, in my opinion, also been observed by Fermilab but the Fermilab and CERN establishments disagree.

All other masses than the electron mass (which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value \( v = 252.514 \) GeV), including the Higgs scalar mass and Truth quark mass, are calculated (not assumed) masses in E8 Physics. These results when added up give a total mass of third generation fermion particles:

\[ \Sigma f_3 = 1,629 \text{ GeV} \]
**The second generation** fermion particles correspond to pairs of octonions. There are $8^2 = 64$ such pairs.

The pair \( \{ 1,1 \} \) corresponds to the mu-neutrino.

The pairs \( \{ 1, E \}, \{ E, 1 \}, \) and \( \{ E, E \} \) correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles:
- The pair \( \{ E, E \} \) should correspond to the E electron.
- The other two muon pairs have a symmetry group \( S_2 \), which is 1/3 the size of the color symmetry group \( S_3 \) which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of
- the \( \{ E, E \} \) electron mass and
- the \( \{ 1, E \}, \{ E, 1 \} \) symmetry mass, which is 1/3 of the up or down quark mass.

Therefore, \( \text{mmu} = 104.76 \text{ MeV} \).

According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV which may be consistent with radiative corrections for the calculated tree-level \( \text{mmu} = 104.76 \text{ MeV} \) as Bailin and Love, in "Introduction to Gauge Field Theory", IOP (rev ed 1993), say: "... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to \( \text{mmu}^5 \) the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.
The red, blue and green strange quark each corresponds to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs \{ 1, i \}, \{ i, 1 \}, \{ i, i \} because i is the red down quark. Its mass should be the sum of two parts: the \{ i, i \} red down quark mass, 312.75 MeV, and the product of the symmetry part of the muon mass, 104.25 MeV, times the graviton factor.

Unlike the first generation situation, massive second and third generation leptons can be taken, by both of the colorless gravitons that may carry electric charge, into massive particles.

Therefore the graviton factor for the second and third generations is 6/2 = 3.

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and the red strange quark constituent mass is \( m_s = 312.75 \text{ MeV} + 312.75 \text{ MeV} = 625.5 \text{ MeV} \).

The blue strange quarks correspond to the three pairs involving j, the green strange quarks correspond to the three pairs involving k, and their masses are similarly determined to also be 625.5 MeV. The charm quark corresponds to the remaining 64 - 1 - 3 - 9 = 51 pairs.

Therefore, the mass of the red charm quark should be the sum of two parts: the \{ i, i \}, red up quark mass, 312.75 MeV; and the product of the symmetry part of the strange quark mass, 312.75 MeV, and the charm to strange octonion number factor 51 / 9, which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is \( m_c = 312.75 \text{ MeV} + 1,772.25 \text{ MeV} = 2.085 \text{ GeV} \).

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a constituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.
The pole mass can be converted to an MSbar mass if the color force strength constant \( \alpha_s \) is known. The conventional value of \( \alpha_s \) at about 2 GeV is about 0.39, which is somewhat lower than the theoretical model value. Using \( \alpha_s \) (2 GeV) = 0.39, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermion particles:

\[
\Sigma_{f2} = 32.9 \text{ GeV}
\]
The Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

\[ Smf_1 = 7.508 \text{ GeV}, \]

and the similar sums for second-generation and third-generation fermions, denoted by

\[ Smf_2 = 32.94504 \text{ GeV} \text{ and } Smf_3 = 1,629.2675 \text{ GeV}. \]

The resulting KM matrix is:

\[
\begin{array}{ccc}
     & \text{s} & \text{b} \\
\text{u} & 0.975 & 0.222 \quad 0.00249 \quad -0.00388i \\
\text{c} & -0.222 \quad -0.000161i & 0.974 \quad -0.0000365i \quad 0.0423 \\
\text{t} & 0.00698 \quad -0.00378i & -0.0418 \quad -0.00086i \quad 0.999 \\
\end{array}
\]
Below the energy level of ElectroWeak Symmetry Breaking the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current $W^\pm$ interactions couple to the ... quarks with couplings given by ...

\[
\begin{align*}
V_{ud} & \quad V_{us} & \quad V_{ub} \\
V_{cd} & \quad V_{cs} & \quad V_{cb} \\
V_{td} & \quad V_{ts} & \quad V_{tb}
\end{align*}
\]

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix. It can be parameterized by three mixing angles and the CP-violating KM phase ...

The most commonly used unitarity triangle arises from

\[V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,\]

by dividing each side by the best-known one, $V_{cd}V_{cb}^*$

\[\rho + i\eta = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)\]

is phase-convention-independent ...

\[\sin 2\beta = 0.673 \pm 0.023 \quad \alpha = 89.0 \pm 4.4 \quad -4.2 \text{ degrees} \quad \gamma = 73 \pm 22 \quad -25 \text{ degrees} \]

The sum of the three angles of the unitarity triangle, $\alpha + \beta + \gamma = (183 \pm 22 \quad -25)$ degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, $J$, which is a phase-convention-independent measure of CP violation, defined by

\[\text{Im } V_{ij} V_{kl} V_{il}^* V_{kj}^* = J \text{ SUM}(m,n) \varepsilon_{ikm} \varepsilon_{jln}\]
The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015  
0.2253 ± 0.0007  
0.97345 +0.00015 −0.00016

0.2252 ± 0.0007  
0.97345 +0.00015 −0.00016  
0.0410 +0.0011 −0.0007

0.00862 +0.00026 −0.00020  
0.0403 +0.0011 −0.0007  
0.999152 +0.000030 −0.000045

and the Jarlskog invariant is $J = (2.91 +0.19-0.11) \times 10^{-5}$. ..."
Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed that in the Massless Realm the mixing matrix might be democratic. In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...

the mass matrix ... MD ... of the type ... 1/3 x m x

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

\[
\begin{array}{ccc}
1/\sqrt{2} & -1/\sqrt{2} & 0 \\
1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\
1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\
\end{array}
\]

as A MD At =

\[
\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & m \\
\end{array}
\]

"...

Up in the Massless Realm you might just say that there is no mass matrix, just a democratic mixing matrix of the form 1/3 x

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking then you might as a first approximation use m = 1 so that all the mass first goes to the third generation as

\[
\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{array}
\]

which is physically like the Higgs being a T-Tbar quark condensate.
Consider a 3-dim Euclidean space of generations:

The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

If mass only goes to one other generation
that can be represented by a red line extending to a second dimension
forming a small blue-red-black triangle

that can be extended by reflection to form six small triangles making up a large triangle

Each of the six component triangles has 30-60-90 angle structure:
If mass goes on further to all three generations that can be represented by a green line extending to a third dimension.

If you move the blue line from the top vertex to join the green vertex you get a small blue-red-green-gray-gray-gray tetrahedron that can be extended by reflection to form 24 small tetrahedra making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds to the $12+12 = 24$ elements of the Binary Tetrahedral Group.
The basic blue-red-green triangle of the basic small tetrahedron has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module" with lengths

\[ V1.V2 = (1/2) \, EL = \text{Half of the regular Tetrahedron's edge length.} \]
\[ V1.V3 = (1/\sqrt{3}) \, EL = 0.577350269 \, EL \]
\[ V1.V4 = 3/(2\sqrt{6}) \, EL = 0.612372436 \, EL \]
\[ V2.V3 = 1/(2\sqrt{3}) \, EL = 0.288675135 \, EL \]
\[ V2.V4 = 1/(2\sqrt{2}) \, EL = 0.353553391 \, EL \]
\[ V3.V4 = 1/(2\sqrt{6}) \, EL = 0.204124145 \, EL \]

the Unitarity Triangle angles are:

\[ \beta = V3.V1.V4 = \arccos\left(\frac{2\sqrt{2}}{3}\right) = 19.471220634 \, \text{degrees so sin} \, 2\beta = 0.6285 \]
\[ \alpha = V1.V3.V4 = 90 \, \text{degrees} \]
\[ \gamma = V1.V4.V3 = \arcsin\left(\frac{2\sqrt{2}}{3}\right) = 70.528779366 \, \text{degrees} \]

which is substantially consistent with the 2010 Review of Particle Properties

\[ \sin 2\beta = 0.673 \pm 0.023 \, \text{so} \, \beta = 21.1495 \, \text{degrees} \]
\[ \alpha = 89.0 \pm 4.4 \, \text{degrees} \]
\[ \gamma = 73 \pm 22 \, \text{degrees} \]

and so also consistent with the Standard Model expectation.
The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):

In the Cl(1,25) E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by $Smf_1 = 7.508$ GeV,

and the similar sums for second-generation and third-generation fermions, denoted by $Smf_2 = 32.94504$ GeV and $Smf_3 = 1,629.2675$ GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.
The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle \( d_{13} = \gamma = 70.529 \) degrees

\[
\sin(\theta_{12}) = s_{12} = \frac{m_e+3m_d+3m_{\mu}}{\sqrt{(m_e^2+3m_d^2+3m_{\mu}^2)+ (m_{\mu}^2+3m_s^2+3m_c^2)}} = 0.222198
\]

\[
\sin(\theta_{13}) = s_{13} = \frac{m_e+3m_d+3m_{\mu}}{\sqrt{(m_e^2+3m_d^2+3m_{\mu}^2)+ (m_{\tau}^2+3m_b^2+3m_{\tau}^2)}} = 0.004608
\]

\[
\sin(\theta_{23}) = \frac{m_{\mu}+3m_s+3m_c}{\sqrt{(m_{\tau}^2+3m_b^2+3m_{\tau}^2)+ (m_{\mu}^2+3m_s^2+3m_c^2)}}
\]

\[
\sin(\theta_{23}) = s_{23} = \sin(\theta_{23}) \sqrt{\Sigma f_2 / \Sigma f_1} = 0.04234886
\]

The factor \( \sqrt{\Sigma f_2 / \Sigma f_1} \) appears in \( s_{23} \) because an \( s_{23} \) transition is to the second generation and not all the way to the first generation, so that the end product of an \( s_{23} \) transition has a greater available energy than \( s_{12} \) or \( s_{13} \) transitions by a factor of \( \Sigma f_2 / \Sigma f_1 \).

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an \( s_{23} \) transition has greater available energy than the \( s_{12} \) or \( s_{13} \) transitions by a factor of \( \Sigma f_2 / \Sigma f_1 \) the effective magnitude of the \( s_{23} \) terms in the KM entries is increased by the factor \( \sqrt{\Sigma f_2 / \Sigma f_1} \).

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\
0 & -\sin(\theta_{23}) & \cos(\theta_{23})
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos(\theta_{13}) & 0 & \sin(\theta_{13}) \exp(-i \ d_{13}) \\
0 & 1 & 0 \\
-\sin(\theta_{13}) \exp(i \ d_{13}) & 0 & \cos(\theta_{13})
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\
-\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

\[
\begin{array}{ccc}
d & s & b \\
u & 0.975 & 0.222 & 0.00249 -0.00388i \\
c & -0.222 -0.000161i & 0.974 -0.0000365i & 0.0423 \\
t & 0.00698 -0.00378i & -0.0418 -0.00086i & 0.999
\end{array}
\]

The matrix is labelled by either (u c t) input and (d s b) output, or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

\[
\begin{array}{ccc}
d & s & b \\
d' & 0.975 & 0.222 & 0.00249 -0.00388i \\
s' & -0.222 -0.000161i & 0.974 -0.0000365i & 0.0423 \\
b' & 0.00698 -0.00378i & -0.0418 -0.00086i & 0.999
\end{array}
\]

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model, the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...
The study of CP violation is, at last, experiment driven. ...
The CKM matrix provides a consistent picture of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely, the KM mechanism is the dominant source of CP violation in flavor changing processes. ...
The result is consistent with the SM predictions. ..."."
Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos: $\nu_e$ (electron neutrino); $\nu_m$ (muon neutrino); $\nu_t$ and three neutrino mass states: $\nu_1$; $\nu_2$; $\nu_3$ and the division of 8-dimensional spacetime into 4-dimensional physical Minkowski spacetime plus 4-dimensional CP2 internal symmetry space.

The heaviest mass state $\nu_3$ corresponds to a neutrino whose propagation begins and ends in CP2 internal symmetry space, lying entirely therein. According to the Cl(1,25) E8 model the mass of $\nu_3$ is zero at tree-level but it picks up a first-order correction propagating entirely through internal symmetry space by merging with an electron through the weak and electromagnetic forces, effectively acting not merely as a point but as a point plus an electron loop at beginning and ending points.

So the first-order corrected mass of $\nu_3$ is given by

$$M_{\nu_3} \times \left(1/\sqrt{2}\right) = M_e \times GW(m_{\text{proton}}^2) \times \alpha_E$$

where the factor $(1/\sqrt{2})$ comes from the $U_{\nu 3}$ component of the neutrino mixing matrix so that

$$M_{\nu_3} = \sqrt{2} \times M_e \times GW(m_{\text{proton}}^2) \times \alpha_E = 1.4 \times 5 \times 10^{-5} \times 1.05 \times 10^{-5} \times (1/137) \text{ eV} = 7.35 / 137 = 5.4 \times 10^{-2} \text{ eV}.$$ 

The neutrino-plus-electron loop can be anchored by weak force action through any of the 6 first-generation quarks at each of the beginning and ending points, and that the anchor quark at the beginning point can be different from the anchor quark at the ending point, so that there are $6 \times 6 = 36$ different possible anchorings.
The intermediate mass state \( \nu_2 \) corresponds to a neutrino whose propagation begins or ends in \( \text{CP}2 \) internal symmetry space and ends or begins in \( M4 \) physical Minkowski spacetime, thus having only one point (either beginning or ending) lying in \( \text{CP}2 \) internal symmetry space where it can act not merely as a point but as a point plus an electron loop.

According to the Cl(1,25) E8 model the mass of \( \nu_2 \) is zero at tree-level but it picks up a first-order correction at only one (but not both) of the beginning or ending points so that there are 6 different possible anchorings for \( \nu_2 \) first-order corrections, as opposed to the 36 different possible anchorings for \( \nu_3 \) first-order corrections, so that

the first-order corrected mass of \( \nu_2 \) is less than

the first-order corrected mass of \( \nu_3 \) by a factor of 6,

so

the first-order corrected mass of \( \nu_2 \) is
\[
M_{\nu_2} = \frac{M_{\nu_3}}{\text{Vol}(\text{CP}2)} = \frac{5.4 \times 10^{-2}}{6} = 9 \times 10^{-3} \text{eV}.
\]

The low mass state \( \nu_1 \) corresponds to a neutrino whose propagation begins and ends in physical Minkowski spacetime, thus having only one anchoring to \( \text{CP}2 \) internal symmetry space.

According to the Cl(1,25) E8 model the mass of \( \nu_1 \) is zero at tree-level but it has only 1 possible anchoring to \( \text{CP}2 \) as opposed to the 36 different possible anchorings for \( \nu_3 \) first-order corrections or the 6 different possible anchorings for \( \nu_2 \) first-order corrections so that

the first-order corrected mass of \( \nu_1 \) is less than

the first-order corrected mass of \( \nu_2 \) by a factor of 6,

so

the first-order corrected mass of \( \nu_1 \) is
\[
M_{\nu_1} = \frac{M_{\nu_2}}{\text{Vol}(\text{CP}2)} = \frac{9 \times 10^{-3}}{6} = 1.5 \times 10^{-3} \text{eV}.
\]
Therefore:

the mass-squared difference $D(M_{23}^2) = M_{\nu_3}^2 - M_{\nu_2}^2 =
= (2916 - 81) \times 10^{-6} \text{ eV}^2 =
= 2.8 \times 10^{-3} \text{ eV}^2$

and

the mass-squared difference $D(M_{12}^2) = M_{\nu_2}^2 - M_{\nu_1}^2 =
= (81 - 2) \times 10^{-6} \text{ eV}^2 =
= 7.9 \times 10^{-5} \text{ eV}^2$

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix $U$

\[
\begin{pmatrix}
\nu_1 & \nu_2 & \nu_3 \\
\nu_e & U_{e1} & U_{e2} & U_{e3} \\
\nu_m & U_{m1} & U_{m2} & U_{m3} \\
\nu_t & U_{t1} & U_{t2} & U_{t3}
\end{pmatrix}
\]

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

\[
U = \begin{pmatrix}
c_{12} & c_{13} & s_{13} \\
s_{12} c_{23} & c_{12} c_{23} & -s_{12} c_{23} s_{13} e^{i\delta} \\
c_{12} s_{23} & -s_{12} c_{23} & c_{12} s_{23} s_{13} e^{i\delta} \\
-s_{12} c_{23} & s_{12} c_{23} & c_{12} c_{23} s_{13} e^{i\delta}
\end{pmatrix}
\]

where $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$
The angles are

\[ \theta_{23} = \frac{\pi}{4} = 45 \text{ degrees} \]
because
nu_3 has equal components of nu_m and nu_t so
that \( U_{m3} = U_{t3} = \frac{1}{\sqrt{2}} \) or, in conventional
notation, mixing angle \( \theta_{23} = \frac{\pi}{4} \)
so that \( \cos(\theta_{23}) = 0.707 = \frac{\sqrt{2}}{2} = \sin(\theta_{23}) \)

\[ \theta_{13} = 9.594 \text{ degrees} = \arcsin(\frac{1}{6}) \]
and \( \cos(\theta_{13}) = 0.986 \)
because \( \sin(\theta_{13}) = \frac{1}{6} = 0.167 = |U_{e3}| = \text{fraction of nu}_3 \text{ that is nu}_e \)

\[ \theta_{12} = \frac{\pi}{6} = 30 \text{ degrees} \]
because
\( \sin(\theta_{12}) = 0.5 = \frac{1}{2} = U_{e2} \) = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that \( \cos(\theta_{12}) = 0.866 = \frac{\sqrt{3}}{2} \)

\[ d = 70.529 \text{ degrees is the Dirac CP violation phase} \]
ei(70.529) = \( \cos(70.529) + i \sin(70.529) = 0.333 + 0.943 \text{i} \)
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
\[ \beta = V_{31}V_{11}V_{41} = \arccos\left(\frac{2 \sqrt{2}}{3}\right) \cong 19.471 \text{ 220 634 degrees} \text{ so } \sin 2\beta = 0.6285 \]
\[ \alpha = V_{11}V_{21}V_{41} = 90 \text{ degrees} \]
\[ \gamma = V_{11}V_{21}V_{41} = \arcsin\left(\frac{2 \sqrt{2}}{3}\right) \cong 70.528 \text{ 779 366 degrees} \]

The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):
Then we have for the neutrino mixing matrix:

<table>
<thead>
<tr>
<th></th>
<th>ν₁</th>
<th>ν₂</th>
<th>ν₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>νₑ</td>
<td>0.866 x 0.986</td>
<td>0.50 x 0.986</td>
<td>0.167 x e⁻id</td>
</tr>
<tr>
<td>νₘ</td>
<td>-0.5 x 0.707</td>
<td>0.866 x 0.707</td>
<td>0.707 x 0.986</td>
</tr>
<tr>
<td></td>
<td>-0.866 x 0.707 x 0.167 x e⁻id</td>
<td>-0.5 x 0.707 x 0.167 x e⁻id</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
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<tbody>
<tr>
<td>νₑ</td>
<td>0.853</td>
<td>0.493</td>
<td>0.167 e⁻id</td>
</tr>
<tr>
<td>νₘ</td>
<td>-0.354</td>
<td>0.612</td>
<td>0.697</td>
</tr>
<tr>
<td></td>
<td>-0.034 - 0.096 i</td>
<td>-0.020 - 0.056 i</td>
<td></td>
</tr>
<tr>
<td>νₜ</td>
<td>0.354</td>
<td>-0.612</td>
<td>0.697</td>
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<td>-0.020 - 0.056 i</td>
<td></td>
</tr>
</tbody>
</table>

Since \( e^{i(70.529)} = \cos(70.529) + i \sin(70.529) = 0.333 + 0.943 \text{i} \)
and \( 0.333e^{-i(70.529)} = \cos(70.529) - i \sin(70.529) = 0.333 - 0.943 \text{i} \)

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<tr>
<td>νₜ</td>
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<td>0.632 - 0.056 i</td>
<td>0.697</td>
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</tbody>
</table>

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero \( \theta_{13} = 9.54 \text{ degrees} \).
Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about

\[(ms - md) (md/ms)^2 a(w) |V_{ds}| = 312 \times 0.25 \times 0.253 \times 0.22 \text{ Mev} = 4.3 \text{ Mev},\]

(where \(a(w) = 0.253\) is the geometric part of the weak force strength and \(|V_{ds}| = 0.22\) is the magnitude of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass \(Q_{md}\) of the down quark is

\[Q_{md} = 312.75 + 4.3 = 317.05 \text{ MeV}.\]

Similarly, the up quark Quantum color force mass increase is about

\[(mc - mu) (mu.mc)^2 a(w) |V_{uc}| = 1777 \times 0.022 \times 0.253 \times 0.22 \text{ Mev} = 2.2 \text{ Mev},\]

(where \(|V_{uc}| = 0.22\) is the magnitude of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass \(Q_{mu}\) of the up quark is

\[Q_{mu} = 312.75 + 2.2 = 314.95 \text{ MeV}.\]

Therefore, the Quantum color force Neutron-Proton mass difference is

\[m_N - m_P = Q_{md} - Q_{mu} = 317.05 \text{ Mev} - 314.95 \text{ Mev} = 2.1 \text{ Mev}.\]

Since the electromagnetic Neutron-Proton mass difference is roughly

\[m_N - m_P = -1 \text{ MeV}\]

the total theoretical Neutron-Proton mass difference is

\[m_N - m_P = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},\]

an estimate that is comparable to the experimental value of 1.3 Mev.
Pion as Sine-Gordon Breather

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole with constituent mass M 312 MeV.

The antiquark is also a Schwinger Source Kerr-Newman Black Hole, with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [ of the axisymmetric configuration ]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ...

... toroidal stage just after merger ...

At merger, the two pincers join to form a single ... toroidal black hole.
The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it ...] produce[s] first a peanut shaped black hole and finally a spherical black hole. ...

In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus. The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface \( r = r^+ \) is ... the event horizon ... and is a null surface ...

... On the surface \( r = r^+ \) .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...". 
A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather. The soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion, analogous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

\[ L = \frac{1}{B^2} \left( \frac{1}{2} (df)^2 + A (\cos(f) - 1) \right) \]

Coleman says: “... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B. The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]

\[ L / \hbar = \frac{1}{B^2 \hbar} \left( \frac{1}{2} (df)^2 + A (\cos(f) - 1) \right) \]

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishing \hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set \hbar equal to one. ...

... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

\[ f(x, t) = \frac{4}{B} \arctan(\frac{n \sin(w t)}{\cosh(n w x)}) \]

where [eq. 4.60] \( n = \sqrt{A - w^2} / w \) and \( w \) ranges from 0 to \( A \). This solution has a simple physical interpretation ... a soliton far to the left ... [and]... an antisoliton far to the right. As \( \sin(w t) \) increases, the soliton and antisoliton move farther apart from each other. When \( \sin(w t) \) passes through one, they turn around and begin to approach one another. As \( \sin(w t) \) comes down to zero ... the soliton and antisoliton are on top of each other ... when \( \sin(w t) \) becomes negative. .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

\[ E = 2M \sqrt{1 - (w^2 / A)} \]

where [eq. 4.65] \( M = 8\sqrt{A} / B^2 \) is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soliton-antisoliton pair. ...

...[ found that ]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [ eq. 4.82 ]
\[ E_N = 2 M \sin( B'^2 N / 16 ) \]
where N = 0, 1, 2 ... < 8 \pi / B'^2 , [ eq. 4.83 ]
B'^2 = B^2 / ( 1 - ( B^2 / 8 \pi ) ) and M is the soliton mass.
M is not given by Eq. ( 4.65 ), but is the soliton mass corrected by the DHN formula,
or, equivalently, by the first-order weak coupling expansion. ...
I have written the equation in this form .. to eliminate A,
and thus avoid worries about renormalization conventions.
Note that the DHN formula is identical to the Bohr-Sommerfeld formula,
except that B is replaced by B'. ...
Bohr and Sommerfeld's ... quantization formula says that if we have a one-parameter
family of periodic motions, labeled by the period, T,
then an energy eigenstate occurs whenever [ eq. 4.66 ]

\[ \int_{0}^{T} \left( dt \right) \left( p qdot = 2 \pi N, \right) \]
where N is an integer. ... Eq.( 4.66 ) is cruder than the WKB formula,
but it is much more general;
it is always the leading approximation for any dynamical system ...
Dashen et al speculate that Eq. ( 4.82 ) is exact. ...
the sine-Gordon equation is equivalent ... to the massive Thirring model.
This is surprising,
because the massive Thirring model is a canonical field theory
whose Hamiltonian is expressed in terms of fundamental Fermi fields only.
Even more surprising, when B'^2 = 4 \pi , that sine-Gordon equation is equivalent
to a free massive Dirac theory, in one spatial dimension. ...
Furthermore, we can identify the mass term in the Thirring model
with the sine-Gordon interaction, [ eq. 5.13 ]
\[ M = - ( A / B^2 ) N_m \cos( B f ) \]
.. to do this consistently ... we must say [ eq. 5.14 ]
\[ B^2 / ( 4 \pi ) = 1 / ( 1 + g / \pi ) \]
....[where]... g is a free parameter, the coupling constant [ for the Thirring model ]...
Note that if B'^2 = 4 \pi , g = 0 ,
and the sine-Gordon equation is the theory of a free massive Dirac field. ...
It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.
Certainly this could not happen in three dimensions,
where it would be forbidden by the spin-statistics theorem.
However, there is no spin-statistics theorem in one dimension,
for the excellent reason that there is no spin. ...
the lowest fermion-antifermion bound state of the massive Thirring model
is an obvious candidate for the fundamental meson of sine-Gordon theory. ...
equation ( 4.82 ) predicts that
all the doublet bound states disappear when B'^2 exceeds 4 \pi .
This is precisely the point where the Thirring model interaction switches from attractive to repulsive. ... these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... define identical physics. ... I have computed the predictions of ...[various]... approximation methods for the ration of the soliton mass to the meson mass for three values of $B^2$ : 4 pi (where the qualitative picture of the soliton as a lump totally breaks down), 2 pi, and pi . At 4 pi we know the exact answer ... I happen to know the exact answer for 2 pi, so I have included this in the table. ...

<table>
<thead>
<tr>
<th>Method</th>
<th>$B^2 = \pi$</th>
<th>$B^2 = 2\pi$</th>
<th>$B^2 = 4\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zeroth-order weak coupling</td>
<td>2.55</td>
<td>1.27</td>
<td>0.64</td>
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<tr>
<td>expansion eq2.13b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coherent-state variation</td>
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<td>1.27</td>
<td>0.64</td>
</tr>
<tr>
<td>First-order weak coupling</td>
<td>2.23</td>
<td>0.95</td>
<td>0.32</td>
</tr>
<tr>
<td>expansion eq4.64</td>
<td>2.56</td>
<td>1.31</td>
<td>0.71</td>
</tr>
<tr>
<td>Bohr-Sommerfeld eq4.82</td>
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<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>DHN formula eq4.82</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exact</td>
<td>?</td>
<td>1.00</td>
<td>0.50</td>
</tr>
</tbody>
</table>

...[eq. 2.13b ]

$$E = \frac{8 \sqrt{A}}{B^2}$$

...[ is the ]... energy of the lump ... of sine-Gordon theory ... frequently called 'soliton...' in the literature ... [ Zeroth-order is the classical case, or classical limit. ] ... ... Coherent-state variation always gives the same result as the ... Zeroth-order weak coupling expansion ... . The ... First-order weak-coupling expansion ... explicit formula ... is $\left( \frac{8}{B^2} \right) - \left( \frac{1}{\pi} \right)$. ...".

Using the Cl(1,25) E8 model constituent mass of the Up and Down quarks and antiquarks, about 312.75 MeV, as the soliton and antisoliton masses, and setting $B^2 = \pi$ and using the DHN formula, the mass of the charged pion is calculated to be $(312.75 / 2.25) \text{ MeV} = 139 \text{ MeV}$ which is close to the experimental value of about 139.57 MeV.

**Why is the value $B^2 = \pi$ the special value that gives the pion mass ?**

( or, using Coleman's eq. (5.14), the Thirring coupling constant $g = 3 \pi$ )

**Because $B^2 = \pi$ is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula.** In other words, 

**The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.**
Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles for a total of 64 particle-antiparticle pairs. Of the 64 particle-antiparticle pairs, 12 are bosonic pions.

A typical combination should have about 6 pions so it should have a mass of about $0.14 \times 6 \text{ GeV} = 0.84 \text{ GeV}$.

Just as the pion mass of $0.14$ GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as $0.1$ GeV, and if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of $0.66$ GeV.

Summing over all $2^{64}$ combinations, the total mass of a one-vertex universe should give a Planck mass roughly around $0.66 \times 2^{64} = 1.217 \times 10^{19}$ GeV.

The value for the Planck mass given in by the 1998 Particle Data Group is $1.221 \times 10^{19}$ GeV.
Conformal Gravity+Dark Energy and DE : DM : OM

MacDowell-Mansourii Gravity is described by Rabindra Mohapatra in section 14.6 of his book “Unification and Supersymmetry”:

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators $X_A$ of a group

$$[X_A, X_B] = f_{AB}^C X_C,$$

where $f_{AB}^C$ are structure constants of the group. We can then introduce a gauge field connection $h^A_n$ as follows:

$$h^A_n = h^A_n X_A.$$

Let us denote the parameter associated with $X_A$ by $c^n A$. The gauge transformations on the fields $h^A_n$ are given as follows:

$$\delta h^A_n = \partial_x c^n A + h^B_n c_{f CB} f^{CA} = (D_x c)^A.$$

We can then define a covariant curvature

$$R^A_{\mu} = \partial_\mu h^A_n - \partial_\nu h^A_{\mu} + h^B_n h^C_{f CB} f^{CA}.$$  (14.6.4)

Under a gauge transformation

$$\delta_{\text{gauge}} R^A_{\mu} = R^B_{\mu} c_{f CB} f^{CA}.$$  (14.6.5)

We can then write the general gauge invariant action as follows:

$$I = \int d^4x \, Q_{\text{gauge}} R^A_{\mu} R^B_{\mu}.$$  (14.6.6)

Let us now apply this formalism to conformal gravity. In this case

$$h^A_n = P_{nm} e^m_n + M_{nm} c^{nm} + K_{nm} f^m_n + D_{nm}.$$  (14.6.7)

The various $R_{\mu}$ are:

$$R_{\mu}(P) = \partial_\mu e^m_n - \partial_n e^m_\mu + \omega^m_{\mu n} - \omega^m_{n\mu} e^m_n - b_n e^m_\mu + b_\mu e^m_n,$$  (14.6.8)

$$R_{\mu}(M) = \partial_\mu c^{nm} - \partial_n c^{m\mu} - \omega^m_{\mu n} c^{nm} - \omega^m_{n\mu} c^{nm} - 4(e^m_n f^m_\mu - e^m_\mu f^m_n),$$  (14.6.9)

$$R_{\mu}(K) = \partial_\mu f^m_n - \partial_n f^m_\mu - b_n f^m_\mu + b_\mu f^m_n + \omega^m_{\mu n} f^m_n - \omega^m_{n\mu} f^m_n,$$  (14.6.10)

$$R_{\mu}(D) = \partial_\mu b^m_n - \partial_n b^m_\mu + 2c^m_{n\mu} f^m_n - 2e^m_\mu f^m_n.$$  (14.6.11)

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4 x \, e^{nm} e^{ mn} R_{\mu}(P) R_{\mu}(M).$$  (14.6.12)

We also impose the constraint that

$$R_{\mu}(P) = 0.$$  (14.6.13)
which expresses $\omega^m_\mu$ as a function of $(e, b)$. The reason for imposing this constraint has to do with the fact that $P_m$ transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein $e^m_\mu$. Under coordinate transformations
\[ \delta e^m_\mu = \delta \xi \xi^\mu e^m_\mu + \xi^\nu \partial_\nu e^m_\mu. \] (14.6.14)

Using eqn. (14.6.8) we can rewrite
\[ \delta e^m_\mu = \delta \rho (\xi^\alpha) e^m_\mu + \delta \omega^m_\nu e^\nu_\mu + \delta b^\nu e^m_\mu + \xi^\gamma R^m_{\nu\gamma}(P). \]

where
\[ \delta \omega^m_\nu e^\nu_\mu = \delta \rho \xi^\mu + \xi^\nu \omega^\nu_\mu + \xi^\nu b. \] (14.6.15)

If $R^m_{\nu\gamma}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \delta_\mu - i X^a \partial_\mu a^a$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{p}$ such that
\[ \delta \bar{p}(\xi) = \delta e^m_\mu (\xi^\nu) - \sum_{\lambda'} \delta_{\lambda'} (\xi^m h_{\lambda'}^\nu), \]

where $\lambda'$ goes over all gauge transformations excluding translation. The rule is
\[ \delta \bar{p}(\xi) \phi = \xi^m D^m_{\phi} \phi. \] (14.6.17)

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to $D^m_{\phi}$ and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to $f^m_\mu$ to get an expression for it, i.e.,
\[ \delta_{\phi} f^m_\mu = - \frac{1}{2} [e^m_\nu R_{\nu\mu} - \delta_{\phi} R], \]

where $f^m_\mu$ has been set to zero in $R$ written in the right-hand side.

This eliminates (from the theory the degrees of freedom) $\omega^m_\nu$ and $f^m_\mu$ and we are left with $e^m_\nu$ and $b_\mu$. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field $\phi$. It has conformal weight $\lambda = 1$. So we can write a covariant derivative for it, eqn. (14.6.17)
\[ D^\nu_{\phi} \phi = \delta^\nu_{\phi} \phi - \phi b_{\mu}. \] (14.6.19)

We note that the conformal charge of $\phi$ can be assumed to be zero since $K_m = x^2 \partial$ and is the dimension of inverse mass. In order to calculate $\Box \phi$ we
After the scale and conformal gauges have been fixed, the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739). (Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ... S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... , and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)
The minimal group required to produce Gravity, and therefore the group that is used in calculating Force Strengths, is the [anti] de Sitter group, as is described by Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):

"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ... Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab (n_ab = Minkowski metric).

What happened to the w^ab_u ?

The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u ... The w do not propagate ...

We start from the four-dimensional de-Sitter algebra ... so(3,2).

Technically this is the anti-de-Sitter algebra ...

We envision space-time as a four-dimensional manifold M.

At each point of M we have a copy of SO(3,2) (a fibre ...)

and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.

From these potentials h^A_mu we calculate the field-strengths (curvature components) [let @ denote partial derivative]

R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu

[where]... the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....

We now wish to write down the action S as an integral over the four-manifold M ...

S(Q) = INTEGRAL_M R^A ∧ R^B Q_AB

where Q_AB are constants ... to be chosen ... we require ...

... the invariance of S(Q) under local Lorentz transformations ...
... the invariance of S(Q) under space inversions ...
...[ AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE ]...

we shall see ...[that]... the action becomes invariant under all local [anti]de-Sitter transformations ...[and]... we recognize t he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...

Variation of the vierbein leads to the Einstein equations with cosmological term. Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.

So at this level full sp(4) invariance has been checked.

... Were it not for the assumed space-inversion invariance ...

we could have had a parity violating gravity. ...

Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.

Parity violation and torsion go hand-in-hand.

Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether the universe as a whole is in a space-inversion symmetric configuration. ...".
According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by
the de Sitter instead of the Poincare group, the P-symmetry of the weak
cosmological-constant limit and the Q-symmetry of the strong cosmological constant
limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[ is the space ]... whose geometry is gravitationally related to an infinite
cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and
whose group of motion is Q. Analogously to the Minkowski case, N is also a
homogeneous space, but now under the kinematical group Q, that is, N = Q/L
[ where L is the Lorentz Group of Rotations and Boosts ]. In other words, the
point-set of N is the point-set of the special conformal transformations.
Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as
base space and L as the typical fiber. The kinematical group Q, like the Poincare
group, has the Lorentz group L as the subgroup accounting for both the isotropy
and the equivalence of inertial frames in this space. However, the special
conformal transformations introduce a new kind of homogeneity. Instead of
ordinary translations, all the points of N are equivalent through special conformal
transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the
sense that their geometries are determined respectively by a vanishing and an
infinite cosmological constants. The same can be said of their kinematical group of
motions: P is associated to a vanishing cosmological constant and Q to an infinite
cosmological constant.
The dual transformation connecting these two geometries is the spacetime
inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group
P is transformed into the group Q, and the Minkowski space M becomes the conespace
N. The points at infinity of M are concentrated in the vertex of the conespace
N, and those on the light-cone of M becomes the infinity of N. It is
concepts of space isotropy and equivalence between inertial frames in the conespace
N are those of special relativity. The difference lies in the concept of
uniformity as it is the special conformal transformations, and not ordinary
translations, which act transitively on N. ..."
Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional $\text{Spin}(2,4) = \text{SU}(2,2)$ Conformal Group, which is made up of:

- 3 Rotations
- 3 Boosts
- 4 Translations
- 4 Special Conformal transformations
- 1 Dilatation

The Cosmological Constant / Dark Energy comes from the 10 Rotation, Boost, and Special Conformal generators of the Conformal Group $\text{Spin}(2,4) = \text{SU}(2,2)$, so the fractional part of our Universe of the Cosmological Constant should be about $10 / 15 = 67\%$ for tree level.

Black Holes, including Dark Matter Primordial Black Holes, are curvature singularities in our 4-dimensional physical spacetime, and since Einstein-Hilbert curvature comes from the 4 Translations of the 15-dimensional Conformal Group $\text{Spin}(2,4) = \text{SU}(2,2)$ through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate), the fractional part of our Universe of Dark Matter Primordial Black Holes should be about $4 / 15 = 27\%$ at tree level.

Since Ordinary Matter gets mass from the Higgs mechanism which is related to the 1 Scale Dilatation of the 15-dimensional Conformal Group $\text{Spin}(2,4) = \text{SU}(2,2)$, the fractional part of our universe of Ordinary Matter should be about $1 / 15 = 6\%$ at tree level.

However, as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates, so that the differences in evolution must be taken into account from the initial End of Inflation to the Present Time.

Without taking into account any evolutionary changes with time, our Flat Expanding Universe should have roughly:

- 67\% Cosmological Constant
- 27\% Dark Matter - possibly primordial stable Planck mass black holes
- 6\% Ordinary Matter
As Dennis Marks pointed out to me, since density $\rho$ is proportional to $(1+z)^3(1+w)$ for red-shift factor $z$ and a constant equation of state $w$:

- $w = -1$ for $\Lambda$ and the average overall density of $\Lambda$ Dark Energy remains constant with time and the expansion of our Universe;
- and
- $w = 0$ for nonrelativistic matter so that the overall average density of Ordinary Matter declines as $1 / R^3$ as our Universe expands;
- and
- $w = 0$ for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as $1 / R^3$ as our Universe expands; so that the ratio of their overall average densities must vary with time, or scale factor $R$ of our Universe, as it expands.

Therefore, the above calculated ratio $0.67 : 0.27 : 0.06$ is valid only for a particular time, or scale factor, of our Universe.

When is that time? Further, what is the value of the ratio now?

Since WMAP observes Ordinary Matter at 4% NOW, the time when Ordinary Matter was 6% would be at redshift $z$ such that

\[
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , \text{ or } (1+z)^3 = 1.5 , \text{ or } 1+z = 1.145 , \text{ or } z = 0.145.
\]

To translate redshift into time, in billions of years before present, or Gy BP, use this chart

from a [www.supernova.lbl.gov](http://www.supernova.lbl.gov) file SNAPoverview.pdf to see that the time when Ordinary Matter was 6% would have been a bit over 2 billion years ago, or 2 Gy BP.
In the diagram, there are four Special Times in the history of our Universe: the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion (beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point = Beginning of Accelerating Expansion (purple vertical line at about z = 0.587 and about 7 Gy BP).

According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".

According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".

According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7 provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".
3 - the Last Intersection of the Accelerating Expansion of our Universe of Linear Expansion (green line) with the Third Intersection (at red vertical line at \( z = 0.145 \) and about 2 Gy BP), which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe2O3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.
Those four Special Times define four Special Epochs:
The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe ( see gr-qc/0007006 ).
The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.
The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.
The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant \( \Lambda \) is more prominent than it would be under the "standard norm" conditions of Linear Expansion. Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy \( \Lambda \) and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

\( \Lambda \) and CDM would be effectively created during expansion in their natural ratio \( 67 : 27 = 2.48 = 5 / 2 \), each having proportionate fraction \( 5 / 7 \) and \( 2 / 7 \), respectively; CDM Black Hole decay would be ignored; and pre-existing CDM Black Hole density would decline by the same \( 1 / R^3 \) factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.
The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the Λ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

$$\text{CDM Black Hole factor} = 0.18 + 0.11 \times \frac{2}{7} = 0.18 + 0.03 = 0.21$$

for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

$$0.75 : 0.21 : 0.04$$

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

2013 Planck Data ( arxiv 1303.5062 ) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

**E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05**

Planck: DE : DM : OM = 69 : 26 : 05


Since uncertainties are substantial, I think that there is reasonable consistency.
World-Line String Bohm Quantum Theory

A physically realistic Lattice Bosonic String Theory with Strings = World-Lines and Monster Group Symmetry
containing gravity and the Standard Model
can be constructed consistently with the E8 physics model
248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8
= (28 + 28 + 64) + (64 + 64)

Paths in C8 / WE8 correspond to World-Lines of Observers acting as Bosonic Strings.
Andrew Gray in arXiv quant-ph/9712037 said:
“... probabilities are ... assigned to entire fine-grained histories ...
base[d] ... on the Feynman path integral formulation ...
The formulation is fully relativistic and applicable to multi-particle systems.
It ... makes the same experimental predictions as quantum field theory ...”.
Luis E. Ibanez and Angel M. Uranga in “String Theory and Particle Physics” said:
“... String theory proposes ... small one-dimensional extended objects, strings,
of typical size Ls = 1/ Ms, with Ms known as the string scale ...
As a string evolves in time, it sweeps out a two-dimensional surface in spacetime,
known as the worldsheet, which is the analog of the ... worldline of a point particle ...
for the bosonic string theory ... the classical string action is the total area spanned by
the worldsheet ... This is the ... Nambu–Goto action ...”.

In my unconventional view

the red line and the green line are different strings/worldlines/histories and
the world-sheet is the minimal surface connecting them,
carrying the Bohm Potential,
as Standard Model gauge bosons carry Force Potential between Point Particles.
The t world-sheet coordinate is for Time of the string-world-line history.
The sigma world-sheet coordinate is for Bohm Potential Gauge Boson at a given Time.

( images adapted from “String Theory and Particle Physics” by Ibanez and Uranga )
Further, Ibanez and Uranga also said:
“... The string groundstate corresponds to a 26d spacetime tachyonic scalar field $T(x)$. This tachyon ... is ... unstable
...
The massless two-index tensor splits into irreducible representations of SO(24) ...
Its trace corresponds to a scalar field, the dilaton $\phi$, whose vev fixes the string interaction coupling constant $g_s$
...
the antisymmetric part is the 26d 2-form field BMN
...
The symmetric traceless part is the 26d graviton GMN ...”.

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analogous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The antisymmetric SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

Joe Polchinski in “String Theory, Volume 1, An Introduction to the Bosonic String” said:
“... we find at $m^2 = - 4/\alpha'$ the tachyon,
and at $m^2 = 0$ the 24 x 24 states of the graviton, dilaton, and antisymmetric tensor ...

Must the 24x24 symmetric matrices be interpreted as the graviton ? - !!! NO !!!

The 24x24 Real Symmetric Matrices form the Jordan Algebra J(24,R).

Jordan algebras correspond to the matrix algebra of quantum mechanical states, that is, from a particle physics point of view, the configuration of particles in spacetime upon which the gauge groups act.

24-Real-dim space has a natural Octonionic structure of 3-Octonionic-dim space.

The corresponding Jordan Algebra is J(3,O) = 3x3 Hermitian Octonion matrices.

Their 26-dim traceless part J(3,O)o describes the 26-dim of Bosonic String Theory and the algebra of its Quantum States, so that
the 24x24 traceless symmetric spin-2 particle is the Quantum Bohmion.
Joseph Polchinski, in his books String Theory vols. I and II (Cambridge 1998), says: "... the closed unoriented bosonic string theory has the maximal 26-dimensional Poincare invariance ... It is possible to have a consistent theory with the dilaton, the string-graviton, the tachyon, whose negative mass-squared means that the no-string 'vacuum' is actually unstable ... ".

The dilaton of E8 Physics sets the Planck scale as the scale for the 16 dimensions that are orbifolded fermion particles and anti-particles and the 4 dimensions of the CP2 Internal Symmetry Space of M4xCP2 spacetime. The remaining 26 - 16 - 4 = 6 dimensions are the Conformal Physical Spacetime with Spin(2,4) = SU(2,2) symmetry that produces M4 Physical Spacetime.

E8 Physics 26D String Theory Spacetime
10D = 6D Conformal Spacetime + 4D Compact CP2 Internal Symmetry Space
with CP2 = SU(3) / SU(2)xU(1) as unique Compactification
which specifies Gauge Groups of the Standard Model.

If Strings = World Lines and World Lines are past and future histories of particles, then spin-2 string entities carry Bohm Quantum Potential with Sarfatti Back-Reaction related to Cramer Transaction Quantum Theory.

Roger Penrose in "Road to Reality" (Knopf 2004) says: "... quantum mechanics ... alternates between unitary evolution U ... and state reduction R ... quantum state reduction ... is ... objective ... OR ... it is always a gravitational phenomenon ... [A] conscious event ... would be ... orchestrated OR ... of ... large-scale quantum coherence ... of ... microtubules ... ".

String-Gravity produces Sarfatti-Bohm Quantum Potential with Back-Reaction.
It is distinct from the MacDowell-Mansouri Gravity of stars and planets.
The tachyon produces the instability of a truly empty vacuum state with no strings.
It is natural, because if our Universe were ever to be in a state with no strings, then tachyons would create strings = World Lines thus filling our Universe with the particles and World-Lines = strings that we see. Something like this is necessary for particle creation in the Inflationary Era of non-unitary Octonionic processes.
Our construction of a 26D String Theory consistent with E8 Physics uses a structure that is not well-known, so I will mention it here before we start:

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octonions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices. An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices (viXra 1301.0150v2).
It can act as an effectively independent lattice as part of the basis subsets {1E8,EE8} or {1E8,iE8,jE8,kE8}.
With that in mind, here is the construction:

Step 1:
Consider the 26 Dimensions of Bosonic String Theory as the 26-dimensional traceless part $J_3(O)_{0}^{a}O^{+}O^{v}O^{*-a-b}$

(\text{where Ov, O+, and O- are in Octonion space with basis \{1,i,j,k,E,I,J,K\} and a and b are real numbers with basis \{1\}})

of the 27-dimensional Jordan algebra $J_3(O)$ of 3x3 Hermitian Octonion matrices.

Step 2:
Take a D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by \{i,j,k\} in the 8-dimensional Octonionic Ov space.

Step 3:
Compactify the 4-dimensional co-associative subspace spanned by \{E,I,J,K\} in the Octonionic Ov space as a $\text{CP}^2 = \text{SU}(3)/\text{U}(2)$, with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.
Add this subspace to D3, to get D7.

Step 4:
Orbifold the 1-dimensional Real subspace spanned by \{1\} in the Octonionic Ov space by the discrete multiplicative group $Z_2 = \{-1,+1\}$, with its fixed points \{-1,+1\} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by \{1\} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.
Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.
Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.
Step 5:
To get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group
\[ \text{Oct16} = \{+/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K\} \]
to reduce O+ to 16 singular points \{-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K\}.

Let the 8 O+ singular points \{-1,-i,-j,-k,-E,-I,-J,-K\} correspond to the fundamental fermion particles
{neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark}
located on the past D7 layer of D8.

Let the 8 O+ singular points \{+1,+i,+j,+k,+E,+I,+J,+K\} correspond to the fundamental fermion particles
{neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark}
located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:
8 world-brane scalars for Octonionic O- space;
1 world-brane scalars for real a space; and
1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:
To get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group
\[ \text{Oct16} = \{+/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K\} \]
to reduce O- to 16 singular points \{-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K\}.

Let the 8 O- singular points \{-1,-i,-j,-k,-E,-I,-J,-K\} correspond to the fundamental fermion anti-particles
{anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark}
located on the past D7 layer of D8.

Let the 8 O- singular points \{+1,+i,+j,+k,+E,+I,+J,+K\} correspond to the fundamental fermion anti-particles
{anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark}
located on the future D7 layer of D8.

The 8 components of 8 fundamental first-generation fermion anti-particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:
1 world-brane scalars for real a space; and
1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:
Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.
Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:
Fundamentally, physics is described on HyperDiamond Lattice structures.
There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octonions. denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices. An 8th 8-dim lattice 1E8 with 240 first-shell vertices related to the E8 adjoint part of E8 is related to the 7 octonion imaginary lattices. Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices. ( see viXra 1301.0150 )

Step 9:
Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

a gauge boson emanating from D8 from its 1E8 and EE8 lattices is a U(2) ElectroWeak boson thus accounting for the photon and W+, W- and Z0 bosons.

a gauge boson emanating from D8 from its IE8, JE8, and KE8 lattices is a U(3) Color Gluon boson thus accounting for the 8 Color Force Gluon bosons.

The 4+8 = 12 bosons of the Standard Model Electroweak and Color forces correspond to 12 of the 28 dimensions of 28-dim Spin(8) that corresponds to one of the 28 of the 120-dim adjoint D8 parts of E8.

a gauge boson emanating from D8 from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) boson for conformal U(2,2) = Spin(2,4)xU(1) MacDowell-Mansouri gravity plus conformal structures consistent with the Higgs mechanism and with observed Dark Energy, Dark Matter, and Ordinary matter.

The 16-dim U(2,2) is a subgroup of 28-dim Spin(2,6) that corresponds to the other 28 of the 120-dim adjoint D8 part of E8.
Step 10:  
Since Polchinski says

"... there will also be $r^2$ massless scalars from the components normal to the D-brane. ... the collective coordinates ... $X^u$ ... for the embedding of $n$ D-branes in spacetime are now enlarged to $n \times n$ matrices.

This 'noncommutative geometry' ...[may be]... an important hint about the nature of spacetime. ...",

make the following assignment:

The $8 \times 8$ matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E8 lattices of the D8 brane to the eight E8 lattices of the next D8 brane in the stack.

The $8 \times 8 = 64$ correspond to the 64 of the 120 adjoint D8 part of E8.

We have now accounted for all the scalars and have shown that the model has the physics content of the realistic E8 Physics model with Lagrangian structure based on $E_8 = (28 + 28 + 64) + (64 + 64)$ and AQFT structure based on $Cl(1,25)$ with real Clifford Algebra periodicity and generalized Hyperfinite II1 von Neumann factor algebra.
26D String Theory structure can also be formulated directly in the Root Vector picture using redundancy in the E8 description of Quantum States:

- Fermion components carry 8-dim Spacetime information
  - so E8 / D8 = 8x8 + 8x8 can be reduced to 8+8
- Spacetime position and momentum are redundant
  - so D8 / D4 x D4 = 8x8 can be reduced to 8
- Gauge Bosons and Ghosts are redundant
  - so D4 x D4 = 28+28 can be reduced to 28 = 16 for Gravity + 12 for Standard Model

Elimination of Redundancy gives 8+8 + 8 + 28 = 52-dim F4 with 48 Root Vectors forming a 24-cell plus its dual

52-dim F4 has 26-dim smallest non-trivial representation which has structure of

\[ J(3,O) o = \text{traceless part of 27-dim exceptional Jordan Algebra } J(3,O) \]

and is the minimal structure containing the basic information of E8 Physics.

so

E8 Physics Quantum Theory can be formulated in terms of 26-dim J(3,O) o.

The Cl(1,25) E8 AQFT inherits structure from the Cl(1,25) E8 Local Lagrangian

\[ \int \text{Gauge Gravity + Standard Model + Fermion Particle-AntiParticle} \]

8-dim SpaceTime

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes

\[ 8v = 8 \text{-dim SpaceTime Dimensions} + 8s+ = 8 \text{ Fermion Particle Types} + 8s- = 8 \text{ Fermion AntiParticle Types} \]

combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

\[
\begin{array}{ccc}
  a & 8s+ & 8v \\
  8s+* & b & 8s- \\
  8v* & 8s-* & -a-b \\
\end{array}
\]

which has total dimension \(8v + 8s+ + 8s- + 2 = 26\) and is the space of a 26D String Theory with Strings seen as World-Lines.

24 = \(8v + 8s+ + 8s-\) of the 26 dimensions of 26D String Theory correspond to \(24x8 = 192\) of the 240 E8 Root Vectors by representing the \(8v + 8s+ + 8s-\) as superpositions of their respective 8 components.
8v SpaceTime is represented by D8 branes. A D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8.

A single Snapshot of SpaceTime is represented by a D8 brane at each point of which is placed Fermion Particles or AntiParticles represented by 8+8 = 16 orbifolded dimensions of the 26 dimensions of 26D String Theory.
It is necessary to patch together SpaceTime Snapshots to form a Global Structure describing a Many-Worlds Global Algebraic Quantum Field Theory (AQFT) whose structure is described by Deutsch in "The Fabric of Reality" (Penguin 1997 pp. 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ...
Suppose ... we toss a coin ... Each point in the diagram represents one snapshot ... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. ...
in some regions of the multiverse, and in some places in space, the snapshots of some physical objects do fall, for a period, into chains, each of whose members determines all the others to a good approximation ...".  
**The Many-Worlds Snapshots are structured as a 26-dim Lorentz Leech Lattice of 26D String Theory parameterized by the a and b of J(3,0)o as indicated in this 64-element subset of Snapshots**

![Diagram showing the 64-element subset of Snapshots]

The 240 - 192 = 48 = 24+24 Root Vector Vertices of E8 that do not represent the 8-dim D8 brane or the 8+8 = 16 dim of Orbifolds for Fermions do represent the **Gauge Bosons (and their Ghosts) of E8 Physics:**

- Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give **U(2,2) Conformal Gravity**
- Gauge Bosons from EE8 part of a D8 give **U(2) Electroweak Force**
- Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give **SU(3) Color Force**
Each Deutsch chain of determination represents a World-Line of Particles / AntiParticles corresponding to a String of 26D String Theory such as the red line in this 64-element subset of Snapshots.

26D String Theory is the Theory of Interactions of Strings = World-Lines.

Interactions of World-Lines can describe Quantum Theory according to Andrew Gray (arXiv quant-ph/9712037): "... probabilities are ... assigned to entire fine-grained histories ... based on the Feynman path integral formulation ... The formulation is fully relativistic and applicable to multi-particle systems. It ... makes the same experimental predictions as quantum field theory ...".

Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986) "... For the ... closed ... bosonic string [26D String Theory] ... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ...

SO(24) ... little group of a ...[26-dim]... massless particle ... and ...
a ... massless ... spin two state ...".

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analogous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin 2 state = Bohmion = Carrier of the Bohm Force of the Bohm Quantum Potential.
Roderick Sutherland (arXiv 1509.02442) gave a Lagrangian for the Bohm Potential saying: “... This paper focuses on interpretations of QM in which the underlying reality is taken to consist of particles have definite trajectories at all times ... An example ... is the Bohm model ... This paper ... provide[s]... a Lagrangian ...[for]... the unfolding events ... ... describing more than one particle while maintaining a relativistic description requires the introduction of final boundary conditions as well as initial, thereby entailing retrocausality ...

In addition ... the Lagrangian approach pursued here to describe particle trajectories also entails the natural inclusion of an accompanying field to influence the particle’s motion away from classical mechanics and reproduce the correct quantum predictions. In so doing, it is ... providing a physical explanation for why quantum phenomena exist at all ... the particle is seen to be the source of a field which alters the particle’s trajectory via self-interaction ...

The Dirac case ... each particle in an entangled many-particle state will be described by an individual Lagrangian density ... of the form:

\[
\mathcal{L} = \text{Re} \left[ \frac{1}{\langle f | \bar{f} \rangle} \left( -i \bar{\psi}_f \gamma^0 \partial_0 \psi_f + m \bar{\psi}_f \psi_f \right) \right] + \sigma_0 \rho_0 \left| u_\alpha u^\alpha \right|^{1/2} + \sigma_0 u_\alpha j^\alpha
\]

... the ...[first]... term ...[is]... the ... Lagrangian densities for the PSI field alone ...
... \sigma_0 \rho_0 \text{ is the rest density distribution of the particle through space } ... j \text{ is the current density } ...
... \rho_0 \text{ and } u \text{ are the rest density and 4-velocity of the probability flow } ...”.

Jack Sarfatti extended the Sutherland Lagrangian to include Back-Reaction entanglement.

where a, b and VM4 form Cl(2,4) vectors and VCP2 forms CP2 and S+ and S- form OP2 so that
26D = 16D orbifolded fermions + 10D and 10D = 6D Conformal Space + 4D CP2 ISS (ISS = Internal Symmetry Space and 6D Conformal contains 4D M4 of Kaluza-Klein M4xCP2)
saying (linkedin.com Pulse 13 January 2016): “... the reason entanglement cannot be used as a direct messaging channel between subsystems of an entangled complex quantum system, is the lack of direct back-reaction of the classical particles and classical local gauge fields on their shared entangled Bohmian quantum information pilot wave ... Roderick. I. Sutherland ... using Lagrangian field theory, shows how to make the original 1952 Bohm pilot-wave theory completely relativistic,
and how to avoid the need for configuration space for many-particle entanglement. The trick is that final boundary conditions on the action as well as initial boundary conditions influence what happens in the present. The general theory is "post-quantum" ... and it is non-statistical ...
There is complete two-way action-reaction between quantum pilot waves and the classical particles and classical local gauge fields ... orthodox statistical quantum theory, with no-signaling ...[is derived]... in two steps, first arbitrarily set the back-reaction (of particles and classical gauge field on their pilot waves) to zero. This is analogous to setting the curvature equal to zero in general relativity, or more precisely in setting G to zero. Second, integrate out the final boundary information, thereby adding the statistical Born rule to the mix. ...
the mathematical condition for zero post-quantum back-reaction of particles and classical fields (aka "beables" J.S. Bell's term) is exactly de Broglie's guidance constraint. That is, in the simplest case, the classical particle velocity is proportional to the gradient of the phase of the quantum pilot wave. It is for this reason, that the independent existence of the classical beables can be ignored in most quantum calculations.
However, orthodox quantum theory assumes that the quantum system is thermodynamically closed between strong von Neumann projection measurements that obey the Born probability rule.
The new post-quantum theory in the equations of Sutherland, prior to taking the limit of orthodox quantum theory, should apply to pumped open dissipative structures. Living matter is the prime example. This is a clue that should not be ignored. ...

Jack Sarfatti (email 31 January 2016) said: “... Sabine [Hossenfelder]'s argument ...
"... two types of fundamental laws ... appear in contemporary theories.
One type is deterministic, which means that the past entirely predicts the future. There is no free will in such a fundamental law because there is no freedom. The other type of law we know appears in quantum mechanics and has an indeterministic component which is random. This randomness cannot be influenced by anything, and in particular it cannot be influenced by you, whatever you think “you” are. There is no free will in such a fundamental law because there is no “will" - there is just some randomness sprinkled over the determinism.
In neither case do you have free will in any meaningful way.”
... However ...[ There is a Third Way ]...
post-quantum theory with action-reaction between quantum information pilot wave and its be-able is compatible with free will. ...”
The Creation-Annihilation Operator structure of the Bohm Quantum Potential of 26D String Theory is given by the

Maximal Contraction of $E_8 = \text{semidirect product } A_7 \times h_92$
where $h_92 = 92+1+92 = 185$-dim Heisenberg algebra and $A_7 = 63$-dim $SL(8)$

The Maximal $E_8$ Contraction $A_7 \times h_92$ can be written as a 5-Graded Lie Algebra

$$28 + 64 + (SL(8,R) + 1) + 64 + 28$$

Central Even Grade 0 = $SL(8,R) + 1$

The 1 is a scalar and $SL(8,R) = \text{Spin}(8) + \text{Traceless Symmetric} \ 8\times8 \ \text{Matrices}$, so $SL(8,R)$ represents a local 8-dim $\text{SpaceTime}$ in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64
Each = 64 = $8\times8$ = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28
Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The $8\times8$ matrices linking one $D_8$ to the next $D_8$ of a World-Line String give $A_7 \times R = U(8)$ representing Position x Momentum
The Algebraic Quantum Field Theory (AQFT) structure of the Bohm Quantum Potential of 26D String Theory is given by the E8 Physics Local Lagrangian

\[ \int \text{Gauge Gravity} + \text{Standard Model} + \text{Fermion Particle-AntiParticle} \]

8-dim SpaceTime

and by 8-Periodicity of Real Clifford Algebras, as the Completion of the Union of all Tensor Products of the form

\[ \text{Cl}(1,25) \times \ldots (N \text{ times tensor product}) \ldots \times \text{Cl}(1,25) \]

which is analogous to Fock Space Hyperfinite II1 von Neumann factor algebra that is based on 2-Periodicity of Complex Clifford Algebras.

For \( N = 2^8 = 256 \) the copies of \( \text{Cl}(1,25) \) are on the 256 vertices of the 8-dim HyperCube

For \( N = 2^{16} = 65,536 = 4^8 \) the copies of \( \text{Cl}(1,25) \) fill in the 8-dim HyperCube as described by William Gilbert’s web page: “… The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. …”.

The vertices of the Hilbert curve are at the centers of the \( 2^8 \) sub-8-Hyper Cubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube.

As \( N \) grows, the copies of \( \text{Cl}(1,25) \) continue to fill the 8-dim HyperCube of E8 Space Time using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-Hyper Cubes.

If edges of sub-Hyper Cubes, equal to the distance between adjacent copies of \( \text{Cl}(1,25) \), remain constantly at the Planck Length, then the full 8-dim HyperCube of our Universe expands as \( N \) grows to \( 2^{16} \) and beyond similarly to the way shown by this 3-HyperCube example for \( N = 2^3, 4^3, 8^3 \) from William Gilbert’s web page:
The Union of all $\text{Cl}(1,25)$ tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe.

26D String Theory Structure is

Green, Schwartz, and Witten, in "Superstring Theory" vol. 1, describe 26D String Theory saying ".... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ... $\text{SO}(24)$ ... little group of a ...[26-dim]... massless particle ... and ... a ... massless ... spin two state ..."."
Tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions by filling their Schwinger Source regions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analogous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

**The massless spin 2 state = Bohmion = Carrier of the Bohm Force of the Bohm Quantum Potential.**

**Similarity of the spin 2 Bohmion to the spin 2 Graviton accounts for the Bohmion’s ability to support Penrose Consciousness with Superposition Separation Energy Difference \(G \frac{m^2}{a}\) where, for a Human Brain, \(m = \) mass of electron and \(a = 1\) nanometer in Tubulin Dimer

“...Bohm's Quantum Potential can be viewed as an internal energy of a quantum system...”

according to Dennis, de Gosson, and Hiley (arXiv 1412.5133) and

Bohm Quantum Potential inherits Sarfatti Back-Reaction from its spin-2 structure similar to General Relativity

Peter R. Holland says in "The Quantum Theory of Motion" (Cambridge 1993):

"... the total force... from the quantum potential... does not... fall off with distance... because... the quantum potential... depends on the form of...[the quantum state]... rather than... its... magnitude...".

**Penrose-Hameroff-type Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms.**

The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its \(10^18\) to \(10^19\) Tubulin Dimers described by a large Real Clifford Algebra. Paola Zizzi in gr-qc/0007006 describes the Octonionic Inflation Era of Our Universe as a Quantum Consciousness Superpositon of States ending with Self-Decoherence after 64 doublings of Octonionic Inflation, at which time Our Universe is "... a superposed state of quantum...[ qubits ].

the self-reduction of the superposed quantum state is... reached at the end of inflation...[at]... the decoherence time...[ Tdecoh = \(10^9\) Tplanck = \(10^{\sim 34}\) sec ]...

and corresponds to a superposed state of...[ \(10^{19} = 2^{64}\) qubits ]....

64 doublings to \(2^{64}\) qubits corresponds to the Clifford algebra

\[
\text{Cl}(64) = \text{Cl}(8x8) = \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8) \times \text{Cl}(8)
\]

By the periodicity-8 theorem of Real Clifford algebras, \(\text{Cl}(64)\) is the smallest Real Clifford algebra for which we can reflexively identify each component \(\text{Cl}(8)\) with a basis vector in the \(\text{Cl}(8)\) vector space.

This reflexive identification causes our universe to decohere at \(N = 2^{64} = 10^{19}\). Octonionic Quantum Processes are Not Unitary and so can produce Fermions.

(see Stephen Adler's book "Quaternionic Quantum Mechanics..." at pages 50-52 and 561).
At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about \((1/2) \cdot 16^{64} = (1/2) \cdot (2^4)^{64} = 2^{255} = 6 \times 10^{76}\) Fermions. At the End of Inflation Our Universe had Temperature / Energy \(10^{27} K = 10^{14} GeV\) so each of the \(10^{77}\) Fermions had energy of \(10^{14} GeV\) and collisions among them would for each of the \(10^{77}\) Fermions produce jets containing about \(10^{12}\) particles of energy 100 GeV or so so that the total number created by Inflation was about \(10^{89}\).

The End of Inflation time was at about \(10^{(-34)}\) sec = \(2^{64}\) Tplanck and the size of our Universe was then about \(10^{(-24)}\) cm which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud. The \(2^{64}\) qubits created by Inflation is roughly \(10^{19}\) which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.

Therefore

\[
\text{the Human Brain Quantum Consciousness has evolved in Our Universe to be roughly equivalent to the Maximum Consciousness of Our Inflationary Era Universe.}
\]

Further,
Each cell of E8 Classical Lagrangian Spacetime corresponds to 65,536-dim Cl(16) which contains 248-dim E8 = 120-dim D8 bivectors + 128-dim D8 half-spinors.

Human Brain Microtubules 40 microns long have 65,536 Tubulin Dimers

and so can have Bohm Quantum Resonance with Cl(16) Spacetime cells

so that at any and all Times the State of Consciousness of a Human is in exact resonant correspondence with a subset of the cells of E8 Classical Lagrangian Spacetime.

Therefore E8 Classical Lagrangian Spacetime NJL Condensate is effectively the Spirit World in which the Human States of Consciousness = Souls exist. After the death of the Human Physical Body the Spirit World interactions with its Soul are no longer constrained by Physical World interactions with its Body so that the Spirit World can harmonize the individual Soul with the collective Universal Soul.
Giza and Wakanda

At some time in the future (if humans stay around long enough) humans will invent time machines. Time travel of physical Humans might not be necessary. It might be sufficient for Future Humans to transmit Ideas to Past Humans by Resonant Quantum Consciousness Connection (see viXra 1805.0297 page 122).

When (and where) in their past would they be most likely to go? National Geographic Genographic Y-DNA project says that when humans left their original home in central Africa by going North up the Nile River to the Mediterranean Sea was about 36,000 years ago.

At that time the Giza Plateau would have been a relatively isolated (spatially) outpost of humanity so Time-Travelers-From-Future would choose Giza 36,000 years ago-from-now and they would build the two large Pyramids and the Sphinx with structure that encodes the fundamental laws of E8 Clifford Algebra Physics so that humans could eventually figure out the code and build Time Machines. Therefore

Giza of 36,000 years ago would be like Wakanda (invented by Stan Lee and Jack Kirby in 1966) except that it would not only be isolated in space but also hidden in time