Generalized Neutrosophic Closed Sets

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ABSTRACT

In this paper, the concept of generalized neutrosophic closed set is introduced. Further, generalized neutrosophic continuous mapping, generalized neutrosophic irresolute mapping, strongly neutrosophic continuous mapping, perfectly neutrosophic continuous mapping, strongly generalized neutrosophic continuous mapping and perfectly generalized neutrosophic continuous mapping are introduced. Several interesting properties and characterizations are also discussed.

KEYWORDS: Generalized neutrosophic closed sets; generalized neutrosophic continuity; strongly generalized neutrosophic continuity; generalized neutrosophic irresolute; strongly neutrosophic continuity; perfectly neutrosophic continuity.

1 INTRODUCTION AND PRELIMINARIES

The notion of fuzzy set has invaded almost all branches of mathematics since its introduction by Zadeh (1965). Fuzzy sets have applications in many fields such as information theory (Smets (1981)) and control theory (Sugeno (1985)). The notion of fuzzy topological space was introduced and developed by Chang (1968) and since then various notions in classical topology have been extended to fuzzy topological spaces. The idea of "intuitionistic fuzzy set" was first published by Krassimir Atanassov (1983) and developed further by him and his colleagues (Atanassov (1986, 1988); Atanassov and Stoeva (1983)). Intuitionistic fuzzy set is an extension of Zadeh's notion of fuzzy set which itself has extended the classical notion of a set. Later, this concept was generalized to "intuitionistic L - fuzzy sets" by Atanassov and Stoeva (1984). The concept of generalized intuitionistic fuzzy closed set was first introduced and investigated by Thakur and Chaturvedi (2006) and later independently by Dhavaseelan *et al.* (2010). After the introduction of the concepts of neutrosophy and neutrosophic set by Smarandache Smarandache (1999, 2000), the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces were introduced by Salama and Alblowi (2012).

In this paper, the concept of generalized neutrosophic closed set is introduced. Further, generalized neutrosophic continuous mapping, generalized neutrosophic irresolute mapping, strongly neutrosophic continuous mapping, perfectly neutrosophic continuous mapping, strongly generalized neutrosophic continuous mapping and perfectly generalized neutrosophic continuous mapping are introduced. Several interesting properties and characterizations are also discussed.

Definition 1.1. Let T, I and F be real standard or non standard subsets of $]0^-, 1^+[$, with
$sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T, I and F are neutrosophic components.

Definition 1.2. Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1.1. (1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.3. Let X be a nonempty set and the neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [Complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1-\gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.4. Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.5. $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

2 NEUTROSOPHIC TOPOLOGY

Definition 2.1. A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (NTS) and each neutrosophic set in T is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (NCS) in X.

Definition 2.2. Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Corollary 2.1. Let A, B and C be neutrosophic sets in X. Then the basic properties of inclusion and complementation:

(a) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$ and $A \cap C \subseteq B \cap D$,

(b) $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(c) $A \subseteq C$ and $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(d) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$,

(e) $\overline{A \cup B} = \overline{A} \cap \overline{B}$,

(f) $\overline{A \cap B} = \overline{A} \cup \overline{B}$,

(g) $A \subseteq B \Rightarrow \overline{B} \subseteq \overline{A}$,

(h) $\overline{\overline{(A)}} = A$,

(i) $\overline{1_N} = 0_N$,

(j) $\overline{0_N} = 1_N$.

Now we introduce the notions of image and preimage of neutrosophic sets. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition 2.3. (a) If $B = \{\langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle : y \in Y\}$ is a neutrosophic set in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(b) If $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ is a neutrosophic set in X, then the image of A under f, denoted by $f(A)$, is the neutrosophic set in Y defined by
$f(A) = \{\langle y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)\rangle : y \in Y\}$. where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \sigma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $1 - f(1 - \gamma_A)$.

Corollary 2.2. Let A , $A_i(i \in J)$ be neutrosophic sets in X, B, $B_i(i \in K)$ be neutrosophic sets in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(c) $A \subseteq f^{-1}(f(A))$ { If f is injective, then $A = f^{-1}(f(A))$ } ,

(d) $f(f^{-1}(B)) \subseteq B$ { If f is surjective, then $f(f^{-1}(B)) = B$ },

(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,

(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,

(g) $f(\bigcup A_i) = \bigcup f(A_i)$,

(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ { If f is injective, then $f(\bigcap A_i) = \bigcap f(A_i)\}$,

(i) $f^{-1}(1_N) = 1_N$,

(j) $f^{-1}(0_N) = 0_N$,

(k) $f(1_N) = 1_N$, if f is surjective,

(l) $f(0_N) = 0_N$,

(m) $\overline{f(A)} \subseteq f(\overline{A})$, if f is surjective,

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

3 GENERALIZED NEUTROSOPHIC CLOSED SETS AND GENERALIZED NEUTROSOPHIC CONTINUOUS FUNCTIONS

Definition 3.1. Let (X, T) be a neutrosophic topological space. A neutrosophic set A in (X, T) is said to be a generalized neutrosophic closed set if $Ncl\,(A) \subseteq G$ whenever $A \subseteq G$ and G is a neutrosophic open set. The complement of a generalized neutrosophic closed set is called a generalized neutrosophic open set.

Definition 3.2. Let (X, T) be a neutrosophic topological space and A be a neutrosophic set in X. Then the neutrosophic generalized closure and neutrosophic generalized interior of A are defined by,

(i) $NGcl(A) = \bigcap\{$G: G is a generalized neutrosophic closed set in X and $A \subseteq G\}$.

(ii) $NGint(A) = \bigcup\{$G: G is a generalized neutrosophic open set in X and $A \supseteq G\}$.

Proposition 3.1. Let (X, T) be any neutrosophic topological space and let A and B be neutrosophic sets in (X, T). Then the neutrosophic generalized closure operator satisfy the following properties:

(i) $A \subseteq NGcl(A)$.

(ii) $NGint(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow NGcl(A) \subseteq NGcl(B)$.

(iv) $A \subseteq B \Rightarrow NGint(A) \subseteq NGint(B)$.

(v) $NGcl(A \cup B) = NGcl(A) \cup NGcl(B)$.

(vi) $NGint(A \cap B) = NGint(A) \cap NGint(B)$.

(vii) $\overline{NGcl(A)} = NGint(\overline{A})$.

(viii) $\overline{NGint(A)} = NGcl(\overline{A})$.

Proof. (i) NGcl(A)= $\bigcap$\{G: G is a generalized neutrosophic closed set in X and $A \subseteq G$\}. Thus, $A \subseteq NGcl(A)$.

(ii) NGint(A)=$\bigcup$\{G: G is a generalized neutrosophic open set in X and $A \supseteq G$\}. Thus, $NGint(A) \subseteq A$.

(iii) NGcl(B)=$\bigcap$\{G: G is a generalized neutrosophic closed set in X and $B \subseteq G$\},
$\supseteq \bigcap\{G : G$ is a generalized neutrosophic closed set in X and $A \subseteq G\}$,
$\supseteq NGcl(A)$.
Thus, $NGcl(A) \subseteq NGcl(B)$.

(iv) NGint(B)=$\bigcup$\{G: G is a generalized neutrosophic open set in X and $B \supseteq G$\},
$\supseteq \bigcup$\{G: G is a generalized neutrosophic open set in X and $A \supseteq G$\},
$\supseteq NGint(A)$.
Thus, $NGint(A) \subseteq NGint(B)$.

(v) $NGcl(A \cup B) = \bigcap$\{G: G is a generalized neutrosophic closed set in X and $A \cup B \subseteq G$\},
($\bigcap$ \{G: G is a generalized neutrosophic closed set in X and $A \subseteq G$\}) $\cup$ ($\bigcap$\{G: G is a generalized neutrosophic closed set in X and $B \subseteq G$\}),
$= NGcl(A) \cup NGcl(B)$.
Thus, $NGcl(A \cup B)$=$NGcl(A) \cup NGcl(B)$.

(vi) $NGint(A \cap B)$= $\bigcup$\{G: G is a generalized neutrosophic open set in X and $A \cap B \supseteq G$\},
($\bigcup\{G : G$ is a generalized neutrosophic open set in X and $A \supseteq G\}) \cap$ ($\bigcup$\{G: G is a generalized neutrosophic open set in X and $B \supseteq G$\}),
$= NGint(A) \cap NGint(B)$.
Thus, $NGint(A \cap B) = NGint(A) \cap NGint(B)$.

(vii) $NGcl(A) = \bigcap$\{G: G is a generalized neutrosophic closed set in X and $A \subseteq G$\},
$\overline{NGcl(A)} = \cup\{\overline{G} : \overline{G}$ is a generalized neutrosophic open set in X and $\overline{A} \supseteq \overline{G}\}$,
$= NGint(\overline{A})$.
Thus, $\overline{NGcl(A)} = NGint(\overline{A})$.

(viii) NGint(A)=$\bigcup$\{G: G is a generalized neutrosophic open set in X and $A \supseteq G$\},
$\overline{NGint(A)} = \bigcap\{\overline{G} : \overline{G}$ is a generalized neutrosophic closed set in X and $\overline{A} \subseteq \overline{G}\}$,
$= NGcl(\overline{A})$. Thus,$\overline{NGint(A)} = NGcl(\overline{A})$.

□

Proposition 3.2. Let (X,T) be a neutrosophic topological space. If B is a generalized neutrosophic closed set and $B \subseteq A \subseteq Ncl(B)$, then A is a generalized neutrosophic closed set.

Proof. Let G be a neutrosophic open set in (X,T), such that $A \subseteq G$. Since $B \subseteq A$, $B \subseteq G$. Now, B is a generalized neutrosophic closed set and $Ncl(B) \subseteq G$. But $Ncl(A) \subseteq Ncl(B)$. Since $Ncl(A) \subseteq Ncl(B) \subseteq G$, $Ncl(A) \subseteq G$. Hence, A is a generalized neutrosophic closed set. $\square$

Proposition 3.3. Let (X,T) be a neutrosophic topological space. An neutrosophic set A is a generalized neutrosophic open set if and only if $B \subseteq Nint(A)$, whenever B is an neutrosophic closed set and $B \subseteq A$.

Proof. Let A be a generalized neutrosophic open set and B be a neutrosophic closed set such that $B \subseteq A$. Now, $B \subseteq A \Rightarrow \overline{A} \subseteq \overline{B}$ and since $\overline{A}$ is a generalized neutrosophic closed set, then $Ncl(\overline{A}) \subseteq \overline{B}$. This means that $B = \overline{(\overline{B})} \subseteq \overline{Ncl(\overline{A})}$. But $\overline{Ncl(\overline{A})} = Nint(A)$. Hence, $B \subseteq Nint(A)$.

Conversely, suppose that A is a neutrosophic set such that $B \subseteq Nint(A)$, whenever B is a neutrosophic closed set and $B \subseteq A$. Let $\overline{A} \subseteq B$ whenever B is a neutrosophic open set. Now, $\overline{A} \subseteq B \Rightarrow \overline{B} \subseteq A$. Hence by assumption, $\overline{B} \subseteq Nint(A)$. That is, $\overline{Nint(A)} \subseteq B$. But $\overline{Nint(A)} = Ncl(\overline{A})$. Hence, $Ncl(\overline{A}) \subseteq B$. This means that $\overline{A}$ is a generalized neutrosophic closed set. Therefore, A is a generalized neutrosophic open set. $\square$

Proposition 3.4. If $Nint(A) \subseteq B \subseteq A$ and if A is a generalized neutrosophic open set, then B is also a generalized neutrosophic open set.

Proof. Now, $\overline{A} \subseteq \overline{B} \subseteq \overline{Nint(A)} = Ncl(\overline{A})$. Since A is a generalized neutrosophic open set, then $\overline{A}$ is a generalized neutrosophic closed set. By Proposition 3.2, $\overline{B}$ is a generalized neutrosophic closed set. That is, B is a generalized neutrosophic open set. $\square$

Definition 3.3. Let (X,T) and (Y,S) be any two neutrosophic topological spaces.

(i) A map $f : (X,T) \to (Y,S)$ is said to be generalized neutrosophic continuous if the inverse image of every neutrosophic closed set in (Y,S) is a generalized neutrosophic closed set in (X,T).

Equivalently if the inverse image of every neutrosophic open set in (Y,S) is a generalized neutrosophic open set in (X,T).

(ii) A map $f : (X,T) \to (Y,S)$ is said to be generalized neutrosophic irresolute if the inverse image of every generalized neutrosophic closed set in (Y,S) is a generalized neutrosophic closed set in (X,T).

Equivalently if the inverse image of every generalized neutrosophic open set in (Y,S) is a generalized neutrosophic open set in (X,T).

(iii) A map $f : (X, T) \to (Y, S)$ is said to be strongly neutrosophic continuous if $f^{-1}(A)$ is both neutrosophic open and neutrosophic closed in (X, T) for each neutrosophic set A in (Y, S).

(iv) A map $f : (X, T) \to (Y, S)$ is said to be perfectly neutrosophic continuous if $f^{-1}(A)$ is both neutrosophic open and neutrosophic closed in (X, T) for each neutrosophic open set A in (Y, S).

(v) A map $f : (X, T) \to (Y, S)$ is said to be strongly generalized neutrosophic continuous if the inverse image of every generalized neutrosophic open set in (Y, S) is an neutrosophic open set in (X, T).

(vi) A map $f : (X, T) \to (Y, S)$ is said to be perfectly generalized neutrosophic continuous if the inverse image of every generalized neutrosophic open set in (Y, S) is both neutrosophic open and neutrosophic closed in (X, T).

Proposition 3.5. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a generalized neutrosophic continuous mapping. Then for every neutrosophic set A in X, $f(NGcl(A)) \subseteq Ncl(f(A))$.

Proof. Let A be an neutrosophic set in (X, T). Since $Ncl(f(A))$ is a neutrosophic closed set and f is a generalized neutrosophic continuous mapping, $f^{-1}(Ncl(f(A))$ is a generalized neutrosophic closed set and $f^{-1}(Ncl(f(A))) \supseteq A$. Now, $NGcl(A) \subseteq f^{-1}(Ncl(f(A)))$. Therefore, $f(NGcl(A)) \subseteq Ncl(f(A))$. □

Proposition 3.6. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a generalized neutrosophic continuous mapping. Then for every neutrosophic set A in Y, $NGcl(f^{-1}(A)) \subseteq f^{-1}(Ncl(A))$.

Proof. Let A be a neutrosophic set in (Y, S). Let $B = f^{-1}(A)$. Then, $f(B) = f(f^{-1}(A)) \subseteq A$. By Proposition 3.5., $f(NGcl(f^{-1}(A))) \subseteq Ncl(f(f^{-1}(A)))$. Thus, $NGcl(f^{-1}(A)) \subseteq f^{-1}(Ncl(A))$. □

Proposition 3.7. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If A is a generalized neutrosophic closed set in (X, T) and if $f : (X, T) \to (Y, S)$ is neutrosophic continuous and neutrosophic closed mapping then $f(A)$ is a generalized neutrosophic closed set in (Y, S).

Proof. Let G be a neutrosophic open set in (Y, S). If $f(A) \subseteq G$ then $A \subseteq f^{-1}(G)$ in (X, T). Since A is a generalized neutrosophic closed set and $f^{-1}(G)$ is a neutrosophic open set in (X, T), $Ncl(A) \subseteq f^{-1}(G)$. That is, $f(Ncl(A)) \subseteq G$. Now, by assumption, $f(Ncl(A))$ is a neutrosophic closed set in (Y, S) and $Ncl(f(A)) \subseteq Ncl(f(Ncl(A))) = f(Ncl(A)) \subseteq G$. Hence, $f(A)$ is a generalized neutrosophic closed set. □

Proposition 3.8. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a neutrosophic continuous mapping then it is a generalized neutrosophic continuous mapping.

Proof. Let A be a neutrosophic open set in (Y, S). Since f is a neutrosophic continuous mapping, $f^{-1}(A)$ is a neutrosophic open set in (X, T). Every neutrosophic open set is a generalized neutrosophic open set. Now, $f^{-1}(A)$ is a generalized neutrosophic open set in (X, T). Hence, f is a generalized neutrosophic continuous mapping.

The converse of Proposition 3.8., need not be true as shown in Example 3.1. □

Example 3.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A and B in X as follows:
$A = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.3})\rangle$, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.3})\rangle$. Then the families $T = \{0_N, 1_N, A\}$ and $S = \{0_N, 1_N, B\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as $f(a) = b, f(b) = a, f(c) = c$. **Then f is a generalized neutrosophic continuous mapping.** But, $f^{-1}(B)$ is not a neutrosophic open set in (X, T) for $B \in S$. **Hence, f is not a neutrosophic continuous mapping.**

Proposition 3.9. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a generalized neutrosophic irresolute mapping then it is a generalized neutrosophic continuous mapping.

Proof. Let A be a neutrosophic open set in (Y, S). Every neutrosophic open set is a generalized neutrosophic open set. Now, A is a generalized neutrosophic open set. Since f is a generalized neutrosophic irresolute mapping, $f^{-1}(A)$ is a generalized neutrosophic open set in (X, T). Thus, f is a generalized neutrosophic continuous mapping.

The converse of Proposition 3.9., need not be true as shown in Example 3.2. □

Example 3.2. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B and C in X as follows:
$A = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$, $B = \langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5})\rangle$ and $C = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$. Then the families $T = \{0_N, 1_N, A, B\}$ and $S = \{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as follows: $f(a) = c, f(b) = c, f(c) = c$. **Then f is a generalized neutrosophic continuous mapping.** But for a generalized neutrosophic open set $D = \langle x, (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$ in (X, S), $f^{-1}(D)$ is not a generalized neutrosophic open set in (X, T). **Thus, f is not a generalized neutrosophic irresolute mapping.**

Proposition 3.10. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a strongly generalized neutrosophic continuous mapping then f is a neutrosophic continuous mapping.

Proof. Let A be a neutrosophic open set in (Y,S). Every neutrosophic open set is a generalized neutrosophic open set. Now, A be a generalized neutrosophic open set in (Y,S). Since f is strongly generalized neutrosophic continuous, $f^{-1}(A)$ is a neutrosophic open set in (X,T). Hence, f is a neutrosophic continuous mapping.

The converse of Proposition 3.10., need not be true as shown in Example 3.3 $\square$

Example 3.3. Let $X=\{a,b,c\}$. Define the neutrosophic sets A, B and C in X as follows: $A=\langle x,(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.1})\rangle$, $B=\langle x,(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0})\rangle$ and $C=\langle x,(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.1},\frac{b}{0},\frac{c}{0.1})\rangle$. Then the families $T=\{0_N,1_N,A,B\}$ and $S=\{0_N,1_N,C\}$ are neutrosophic topologies on X. Thus, (X,T) and (X,S) are neutrosophic topological spaces. Define $f:(X,T)\to(X,S)$ as follows: $f(a)=a, f(b)=c, f(c)=b$. **Then f is a neutrosophic continuous mapping.**
Let $D=\langle x,(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.99}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.99}),(\frac{a}{0.05},\frac{b}{0},\frac{c}{0.01})\rangle$ be a generalized neutrosophic open set in (X,S). Now, $f^{-1}(D)$ is not a neutrosophic open set in (X,T). **Thus, f is not a Strongly generalized neutrosophic continuous mapping.**

Proposition 3.11. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If $f:(X,T)\to(Y,S)$ is a perfectly generalized neutrosophic continuous mapping then f is a strongly generalized neutrosophic continuous mapping.

Proof. Let A be a generalized neutrosophic open set in (Y,S). Since f is a perfectly generalized neutrosophic continuous mapping, $f^{-1}(A)$ is a neutrosophic open set in (X,T). Thus, f is a strongly generalized neutrosophic continuous mapping.

The converse of Proposition 3.11., need not be true as shown in Example 3.4. $\square$

Example 3.4. Let $X=\{a,b,c\}$. Define the neutrosophic sets A_n and B in X as follows: $A_n=\langle x,\mu_{A_n},\sigma_{A_n},\gamma_{A_n}:n=0,1,2,...\rangle$ where

$$\mu_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\leq\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\leq 1.\end{cases}\quad \sigma_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\leq\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\leq 1.\end{cases}\quad \text{and}$$

$$\gamma_{A_n}=\begin{cases}(\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \alpha=0;\\ (\frac{a}{\alpha},\frac{b}{\alpha},\frac{c}{\alpha}), & 0<\alpha\leq\frac{4n}{10n+1};\\ (\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \frac{4n}{10n+1}<\alpha\leq 1.\end{cases}\quad \text{and } B=\langle x,(\tfrac{a}{0.6},\tfrac{b}{0.6},\tfrac{c}{0.6}),(\tfrac{a}{0.6},\tfrac{b}{0.6},\tfrac{c}{0.5}),(\tfrac{a}{0.4},\tfrac{b}{0.4},\tfrac{c}{0.4})\rangle.$$

Then the families $T=\{0_N,1_N,A_n,n=0,1,2,...\}$ and $S=\{0_N,1_N,B\}$ are neutrosophic topologies on X. Thus, (X,T) and (X,S) are neutrosophic topological spaces. Define $f:(X,T)\to(X,S)$ as follows: $f(a)=c, f(b)=c, f(c)=c$. **Then f is a strongly generalized neutrosophic continuous mapping.**
Let $C=\langle x,(\frac{a}{0.7},\frac{b}{0.7},\frac{c}{0.7}),(\frac{a}{0.7},\frac{b}{0.7},\frac{c}{0.7}),(\frac{a}{0.3},\frac{b}{0.3},\frac{c}{0.3})\rangle$ be a generalized neutrosophic open set in (X,S). Now, $f^{-1}(D)$ is neutrosophic open and not neutrosophic closed in (X,T). **Hence, f is not a perfectly generalized neutrosophic continuous mapping.**

Proposition 3.12. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If $f:(X,T)\to(Y,S)$ is a strongly neutrosophic continuous mapping then f is a strongly generalized neutrosophic continuous mapping.

Proof. Let A be a generalized neutrosophic open set in (Y,S). Since f is a strongly neutrosophic continuous mapping, $f^{-1}(A)$ is neutrosophic open and neutrosophic closed in (X,T). Hence, f is a strongly generalized neutrosophic continuous mapping.

The converse of Proposition 3.12., need not be true as shown in Example 3.5. □

Example 3.5. Let $X=\{a,b,c\}$. Define the neutrosophic sets A_n and B in X as follows: $A_n=\langle x,\mu_{A_n},\sigma_{A_n},\gamma_{A_n}:n=0,1,2,...\rangle$ where

$$\mu_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\le\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\le 1.\end{cases};\ \sigma_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\le\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\le 1.\end{cases}\text{ and}$$

$$\gamma_{A_n}=\begin{cases}(\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \alpha=0;\\ (\frac{a}{\alpha},\frac{b}{\alpha},\frac{c}{\alpha}), & 0<\alpha\le\frac{4n}{10n+1};\\ (\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \frac{4n}{10n+1}<\alpha\le 1.\end{cases}\text{ and } B=\langle x,(\frac{a}{0.6},\frac{b}{0.6},\frac{c}{0.6}),(\frac{a}{0.6},\frac{b}{0.6},\frac{c}{0.6}),(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.4})\rangle.$$

Then the families $T=\{0_N,1_N,A_n,n=0,1,2,...\}$ and $S=\{0_N,1_N,B\}$ are neutrosophic topologies on X. Thus, (X,T) and (X,S) are neutrosophic topological spaces. Define $f:(X,T)\to(X,S)$ as follows: $f(a)=c,f(b)=c,f(c)=c$. **Then f is a strongly generalized neutrosophic continuous mapping.**

Let $D=\langle x,(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.9}),(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.1})\rangle$ be a neutrosophic set in (X,S). Then $f^{-1}(D)$ is a neutrosophic open set and but not a neutrosophic closed set in (X,T). **Hence, f is not a strongly neutrosophic continuous mapping.**

Proposition 3.13. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If $f:(X,T)\to(Y,S)$ is a strongly neutrosophic continuous mapping then f is a generalized neutrosophic irresolute mapping.

Proof. Let A be a generalized neutrosophic open set in (Y,S). Since f is a strongly neutrosophic continuous mapping, $f^{-1}(A)$ is neutrosophic open and neutrosophic closed in (X,T). Since every neutrosophic open set is a generalized neutrosophic open set, $f^{-1}(A)$ is a generalized neutrosophic open set in (X,T). Hence, f is a generalized neutrosophic irresolute mapping.

The converse of Proposition 3.13., need not be true as shown in Example 3.6. □

Example 3.6. Let $X=\{a,b,c\}$. Define the neutrosophic sets A_n and B in X as follows: $A_n=\langle x,\mu_{A_n},\sigma_{A_n},\gamma_{A_n}:n=0,1,2,...\rangle$ where

$$\mu_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\le\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\le 1.\end{cases};\ \sigma_{A_n}=\begin{cases}(\frac{a}{0},\frac{b}{0},\frac{c}{0}), & \alpha=0;\\ (\frac{a}{1-\alpha},\frac{b}{1-\alpha},\frac{c}{1-\alpha}), & 0<\alpha\le\frac{4n}{10n+1};\\ (\frac{a}{1},\frac{b}{1},\frac{c}{1}), & \frac{4n}{10n+1}<\alpha\le 1.\end{cases}\text{ and}$$

$\gamma_{A_n} = \begin{cases} (\frac{a}{1}, \frac{b}{1}, \frac{c}{1}), & \alpha = 0; \\ (\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}), & 0 < \alpha \leq \frac{4n}{10n+1}; \\ (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), & \frac{4n}{10n+1} < \alpha \leq 1. \end{cases}$ and $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$.Then the families $T = \{0_N, 1_N, A_n, n = 0, 1, 2, ...\}$ and $S = \{0_N, 1_N, B\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as follows: $f(a) = c, f(b) = c, f(c) = c$. **Then f is a generalized neutrosophic irresolute mapping**.

Let $D = \langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1})\rangle$ be a neutrosophic set in (X, S). Then $f^{-1}(D)$ is neutrosophic open and not neutrosophic closed in (X, T). **Hence, f is not a strongly neutrosophic continuous mapping.**

Proposition 3.14. Let (X, T),(Y, S) and (Z, R) be any three neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a generalized neutrosophic irresolute mapping and $g : (Y, S) \to (Z, R)$ be a generalized neutrosophic continuous mapping. Then $g \circ f$ is a generalized neutrosophic continuous mapping.

Proof. Let A be a neutrosophic open set in (Z, R). Since g is a generalized neutrosophic continuous mapping, $g^{-1}(A)$ is a generalized neutrosophic open set in (Y, S). Since f is a generalized neutrosophic irresolute mapping, $f^{-1}(g^{-1}(A))$ is a generalized neutrosophic open set in (X, T). Thus, $g \circ f$ is a generalized neutrosophic continuous mapping. □

Proposition 3.15. Let (X, T),(Y, S) and (Z, R) be any three neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a strongly generalized neutrosophic continuous mapping and $g : (Y, S) \to (Z, R)$ be a generalized neutrosophic continuous mapping. Then $g \circ f$ is a neutrosophic continuous mapping.

Proof. Let A be a neutrosophic closed set in (Z, R). Since g is a generalized neutrosophic continuous mapping. $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y, S). Since f is a strongly generalized neutrosophic continuous mapping, $f^{-1}(g^{-1}(A))$is a neutrosophic closed set in (X, T). Thus, $g \circ f$ is a neutrosophic continuous mapping. □

Definition 3.4. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. The graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x)), \forall x \in X$

Proposition 3.16. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. If the graph $g : X \to X \times Y$ of f is a strongly neutrosophic continuous mapping then f is also a strongly neutrosophic continuous mapping.

Proof. Let A be a neutrosophic set in (Y, S). By Definition 3.4., $f^{-1}(A) = 1_\sim \cap f^{-1}(A) = g^{-1}(1_\sim \times A)$. Since g is a strongly neutrosophic continuous mapping, $g^{-1}(1_\sim \times A)$ is both neutrosophic open and neutrosophic closed in (X, T). Now, $f^{-1}(A)$ is both neutrosophic open and neutrosophic closed in (X, T). Hence, f is a strongly neutrosophic continuous mapping. □

Proposition 3.17. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a mapping. If the graph $g : X \to X \times Y$ of f is a perfectly neutrosophic continuous mapping then f is also a perfectly neutrosophic continuous mapping.

Proof. Let A be a neutrosophic set in (Y, S). By Definition 3.4., $f^{-1}(A) = 1_\sim \cap f^{-1}(A) = g^{-1}(1_\sim \times A)$. Since g is a perfectly neutrosophic continuous mapping, $g^{-1}(1_\sim \times A)$ is both neutrosophic open and neutrosophic closed in (X, T). Hence, f is a perfectly neutrosophic continuous mapping. □

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