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Interval Neutrosophic Tangent Similarity Measure Based MADM strategy and its Application to MADM Problems

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Abstract: In this paper, tangent similarity measure of interval valued neutrosophic sets is proposed and its properties are examined. The concept of interval valued neutrosophic set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. The concept of this tangent similarity measure is based on interval valued neutrosophic information. We present a multi-attribute decision making strategy based on the proposed similarity measure. Using this tangent similarity measure, an application, namely, selection of suitable sector for money investment of a government employee for a financial year is presented. Finally, a comparison of the proposed strategy with the existing strategies has been provided in order to exhibit the effectiveness and practicality of the proposed strategy.

Keywords: Neutrosophic set, interval valued neutrosophic set, tangent function, similarity measure, multi attribute decision making

1 Introduction

Decision making in every real field is a very challenging task for an individual. Decision making is done based on some attributes. In real life situations, attribute information involves indeterminacy, incompleteness and inconsistency. Indeterminacy plays an important role in real world decision-making problems. Neutrosophic set [1] is an important tool to deal with imprecise, indeterminate, and inconsistent data.

The concept of neutrosophic set generalizes the fuzzy set [2], intuitionistic fuzzy set [3]. Wang et al. [4] proposed interval valued neutrosophic sets in which the truth-membership, indeterminacy-membership, and false-membership were extended to interval valued numbers. Realizing the difficulty in applying the neutrosophic sets in realistic problems, Wang et al. [5] introduced the concept of single valued neutrosophic set, a subclass of neutrosophic set. Single valued neutrosophic set can be applied in real scientific and engineering fields. It offers us extra possibility to represent uncertainty, imprecise, incomplete, and inconsistent information.

During the last seven years neutrosophic sets and single valued have been studied and applied in different fields such as medical diagnosis [6, 7], decision making problems [8-12], social problems [13, 14], educational problem [15, 16], image processing [17, 18], conflict resolution [19] , etc.

The concept of similarity is very important for decision making problems. Some strategies [20, 21] have been proposed for measuring the degree of similarity between fuzzy sets. However, these strategies are not capable of dealing with the similarity measures involving indeterminacy, and inconsistency. In the literature, few studies have addressed similarity measures for neutrosophic sets, single-valued neutrosophic sets and interval valued neutrosophic sets [22-28].

Salama and Blowi [29] defined the correlation coefficient on the domain of neutrosophic sets, which is another kind of similarity measure. Broumi and Smarandache [30] extended the Hausdorff distance to neutrosophic sets. After that, a new series of similarity measures has been proposed for neutrosophic set using different approaches. Broumi and Smarandache [31] also proposed the correlation coefficient between interval valued neutrosphic sets. Majumdar and Smanta [32] studied several similarity measures of single valued neutrosophic sets (SVNS) based on distances, a matching function, memebership grades, and entropy measure for a SVNS.

Ye [33] proposed the distance-based similarity measure of SVNSs and applied it to the group decision making problems with single-valued neutrosophic information. Ye [34] also proposed three vector similarity measures for SNSs, an instance of SVNS and interval valued neutrosophic set, including the Jaccard, Dice, and cosine similarity and applied them to multi-attribute decision-making problems with simplified neutrosophic information. Recently, Ye [35] presented similarity measures on interval valued neutrosophic set based on Hamming distance and Euclidean distance and offered a numerical example of its use in decision making problems. Broumi and Smarandache [36] proposed a cosine similarity measure of interval valued neutrosophic sets.

Ye [37] further studied and found that there exsit some disadvantages of existing cosine similarity measures defined in vector space in some situations. Ye [37] mentioned that the defined function may produce absurd result in some real cases. In order to overcome theses disadvantages, Ye [37] proposed improved cosine similarity measures based on cosine function, including single-valued neutrosophic cosine similarity measures and interval valued neutrosophic cosine similarity measures. In his study, Ye [37] proposed medical diagnosis strategy based on the improved cosine similarity measures. Ye and Fu [38] further studied medical diagnosis problem namely, multi-period medical diagnosis using a single-valued neutrosophic similarity measure based on tangent function. Recently, Biswas et al. [39] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. In hybrid environment, Pramanik and Mondal [40] proposed cosine similarity measure of rough neutrosophic sets and provided its application in medical diagnosis. Pramanik and Mondal [41] also proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

Pramanik and Mondal [42] proposed weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. Pramanik and Mondal [43] also proposed tangent similarity measures between intuitionistic fuzzy sets and studied some of its properties and applied it for medical diagnosis. Mondal and Pramanik [44] also proposed tangent similarity measures between single-valued neutrosophic sets and studied some of its properties and applied in decision making.

Research gap: MADM strategy using similarity measure based on tangent function under interval neutrosophic environment is yet to appear.

Research questions:

- Is it possible to define a new similarity measure between interval neutrosophic sets using tangent function?
- Is it possible to develop a new MADM strategy based on the proposed similarity measure in interval neutrosophic environment?

Having motivated from the above researches on neutrosophic similarity measures, we have extended the concept of neutrosophic tangent similarity measure [44] to interval valued neutrosophic environment. We have defined a new similarity measure called "interval valued tangent similarity measure'' for interval valued neutrosophic sets. The properties of similarity are established. We establish a multi-attribute decision making strategy based on the interval valued tangent similarity measure. The proposed tangent similarity measure based MADM strategy is applied to money investment decision making problem.

The objectives of the paper:

- To define tangent similarity measures for interval valued neutrosophic set environment and prove its basic properties.
- To develop a multi-attribute decision making strategy based on proposed similarity measures.
- To present a numerical example for the effectiveness of the proposed strategy.

Rest of the paper is structured as follows. Section 2 presents neutrosophic preliminaries. In Section 3 we present tangent similarity measure for interval valued neutrosophic sets and prove some of its properties. Section 4 is devoted to presents multi attribute decision-making based on interval valued neutrosophic tangent similarity measure. Section 5 presents the application of the proposed multi attribute decision-making strategy to a problem, namely, money investment of an Indian government employee after a financial year. Section 6 conducts a comparative analysis of the approach to other existing strategies. Section 7 presents the contributions of the paper. Finally, Section 8 presents concluding remarks and scope for future research.

2 Neutrosophic preliminaries

2.1 Neutrosophic sets

Assume that X be an universe of discourse. Then the neutrosophic set [1] P can be presented of the form: $P = \{< x: T_P(x), I_P(x), F_P(x)>, x \in X\}$, where the functions T, I, F: $X\rightarrow\ ]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set P satisfying the following the condition

$$^-0 \leq \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \leq 3^+ .$$

For two netrosophic sets (NSs), $P_{NS} = \{<x: T_P(x), I_P(x), F_P(x) > | x \in X\}$ and $Q_{NS} = \{< x, T_Q(x), I_Q(x), F_Q(x) > | x \in X\}$ the two relations are defined as follows:

(1) $P_{NS} \subseteq Q_{NS}$ if and only if $T_P(x) \leq T_Q(x)$, $I_P(x) \geq I_Q(x)$, $F_P(x) \geq F_Q(x)$

(2) $P_{NS} = Q_{NS}$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$

2.2 Single valued neutrosophic sets (SVNS)

Assume that X be a space of points with generic element in X denoted by x. A SVNS [5] P in X is characterized by a truth-membership function $T_P(x)$, an indeterminacy-membership function $I_P(x)$, and a falsity membership function $F_P(x)$, for each point x in X, $T_P(x)$, $I_P(x)$, $F_P(x) \in [0, 1]$. When X is continuous, a SVNS P can be written as follows:

$$P = \int_x \frac{< T_P(x), I_P(x), F_P(x) >}{x} : x \in X$$

When X is discrete, a SVNS P can be written as follows:

$$P = \sum_{i=1}^{n} \frac{< T_P(x_i), I_P(x_i), F_P(x_i) >}{x_i} : x_i \in X$$

For two SVNSs , $P = \{<x: T_P(x), I_P(x), F_P(x)> \mid x \in X\}$ and $Q = \{<x, T_Q(x), I_Q(x), F_Q(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $P \subseteq Q$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, $F_P(x) \ge F_Q(x)$

(2) $P = Q$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$ for any $x \in X$

2.3 Interval valued neutrosophic sets (IVNS)

Assume that X be a space of points with generic element $x \in X$. An interval valued neutrosophic set [4] A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$, $F_A(x)$ are considered as interval form.

We have, $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for all $x \in X$.
Assume that

$A = \{<x, \langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle > \mid x \in X\}$ and

$B = \{<x, \langle [T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)] \rangle > \mid x \in X\}$ be two IVNS. Then the following relations are defined as follows:

- $A \subseteq B$ if and only if $T_A^L \le T_B^L, T_A^U \le T_B^U$; $I_A^L \ge I_B^L$, $I_A^U \ge I_B^U$; $F_A^L \ge F_B^L$, $F_A^U \ge F_B^U$
- $A = B$ if and only if $T_A^L = T_B^L, T_A^U = T_B^U$; $I_A^L = I_B^L$, $I_A^U = I_B^U$; $F_A^L = F_B^L$, $F_A^U = F_B^U$
  for all $x \in X$

3 Tangent similarity measures for interval valued neutrosophic sets

Definition 1: Assume that

$A = \langle [T_A^L(x_i), T_A^U(x_i)], [I_A^L(x_i), I_A^U(x_i)], [F_A^L(x_i), F_A^U(x_i)] \rangle$ and

$B = \langle [T_B^L(x_i), T_B^U(x_i)], [I_B^L(x_i), I_B^U(x_i)], [F_B^L(x_i), F_B^U(x_i)] \rangle$ be any two interval valued neutrosophic sets. Now, similarity measure based on tangent function between two interval valued neutrosophic sets is defined as follows:

$T_{IVNS}(A,B) =$

$$1 - \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{12}\left(\begin{array}{l} |T_A^\lambda(x_i) - T_B^\lambda(x_i)| + |I_A^\lambda(x_i) - I_B^\lambda(x_i)| \\ + |F_A^\lambda(x_i) - F_B^\lambda(x_i)| \end{array}\right)\right) \quad (1)$$

Here,

$T_A^\lambda(x_i) = \lambda T_A^L(x_i) + (1-\lambda) T_A^U(x_i)$,

$T_B^\lambda(x_i) = \lambda T_B^L(x_i) + (1-\lambda) T_B^U(x_i)$,

$I_A^\lambda(x_i) = \lambda I_A^L(x_i) + (1-\lambda) I_A^U(x_i)$,

$I_B^\lambda(x_i) = \lambda I_B^L(x_i) + (1-\lambda) I_B^U(x_i)$,

$F_A^\lambda(x_i) = \lambda F_A^L(x_i) + (1-\lambda) F_A^U(x_i)$,

$F_B^\lambda(x_i) = \lambda F_B^L(x_i) + (1-\lambda) F_B^U(x_i)$ and $0 \le \lambda \le 1$.

Theorem 1:

The defined tangent similarity measure $T_{IVNS}(A, B)$ between IVNS A and B satisfies the following properties:

1.1. $0 \le T_{IVNS}(A, B) \le 1$
1.2. $T_{IVNS}(A, B) = 1$ if and only if $A = B$
1.3. $T_{IVNS}(A, B) = T_{IVNS}(B, A)$
1.4. If C is a IVNS in X and $A \subset B \subset C$ then $T_{IVNS}(A, C) \le T_{IVNS}(A, B)$ and $T_{IVNS}(A, C) \le T_{IVNS}(B, C)$.

Proofs:

1.1. Tangent function is monotonic incresing in the interval $[0, \pi/4]$. It also lies in the interval [0, 1]. Therefore, $0 \le T_{IVNS}(A, B) \le 1$.

1.2. For any two IVNS A and B and $0 \le \lambda \le 1$,

$A = B$

$\Rightarrow T_A^\lambda(x_i) = T_B^\lambda(x_i)$, $I_A^\lambda(x_i) = F_B^\lambda(x_i)$, $F_A^\lambda(x_i) = F_B^\lambda(x_i)$

$\Rightarrow |T_A^\lambda(x_i) - T_B^\lambda(x_i)| = 0$, $|I_A^\lambda(x_i) - I_B^\lambda(x_i)| = 0$, $|F_A^\lambda(x_i) - F_B^\lambda(x_i)| = 0$

Therefore, $T_{IVNS}(A, B) = 1$.

Conversely,

$T_{IVNS}(A, B) = 1$

$\Rightarrow |T_A^\lambda(x_i) - T_B^\lambda(x_i)| = 0$, $|I_A^\lambda(x_i) - I_B^\lambda(x_i)| = 0$, $|F_A^\lambda(x_i) - F_B^\lambda(x_i)| = 0$

$\Rightarrow T_A^\lambda(x_i) = T_B^\lambda(x_i)$, $I_A^\lambda(x_i) = F_B^\lambda(x_i)$, $F_A^\lambda(x_i) = F_B^\lambda(x_i)$

Therefore $A = B$.

1.3. $T_{IVNS}(A,B)=$

$$1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|+\\ \left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$=$

$$1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_B^{\lambda}(x_i)-T_A^{\lambda}(x_i)\right|+\left|I_B^{\lambda}(x_i)-I_A^{\lambda}(x_i)\right|\\ +\left|F_B^{\lambda}(x_i)-F_A^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$= T_{IVNS}(B, A)$

1.4. If $A\subset B\subset C$

then $T_A^{\lambda}(x_i) \le T_B^{\lambda}(x_i) \le T_C^{\lambda}(x_i)$, $I_A^{\lambda}(x_i) \le I_B^{\lambda}(x_i) \le I_C^{\lambda}(x_i)$, $F_A^{\lambda}(x_i) \le F_B^{\lambda}(x_i) \le F_C^{\lambda}(x_i)$,

for $x\in X$. Now, we have the inequalities:

$\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$,

$\left|T_B^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$;

$\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$,

$\left|I_B^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$;

$\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$,

$\left|F_B^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$.

From eqn (1), we can say that $T_{IVNS}(A, C) \le T_{IVNS}(A, B)$ and $T_{IVNS}(A, C) \le T_{IVNS}(B, C)$.

Definition 2: Assume that

$A = \{<x, ([T_A^L(x), T_A^U(x)],[I_A^L(x), I_A^U(x)],[F_A^L(x), F_A^U(x)])> \mid x \in X\}$ and

$B = \{<x, ([T_B^L(x), T_B^U(x)],[I_B^L(x), I_B^U(x)],[F_B^L(x), F_B^U(x)])> \mid x \in X\}$ be any two interval valued neutrosophic sets.

Now, weighted similarity measure based on tangent function between two interval valued neutrosophic sets is defined as follows:

$T_{WIVNS}(A,B)=$

$$1-\sum_{i=1}^{n}w_i.\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\\ +\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right) \quad (2)$$

Here $\sum_{i=1}^{n}w_i=1$.

Theorem 2:

The weighted tangent similarity measure $T_{W\text{-}IVNS}(A,B)$ between IVNS A and B satisfies the following properties:

2.1. $0 \le T_{W\text{-}IVNS}(A, B) \le 1$

2.2. $T_{W\text{-}IVNS}(A, B) = 1$ if and only if $A = B$

2.3. $T_{W\text{-}IVNS}(A, B) = T_{W\text{-}IVNS}(B, A)$

2.4. If C is a IVNS in X and $A\subset B\subset C$ then $T_{W\text{-}IVNS}(A, C) \le T_{W\text{-}IVNS}(A, B)$ and $T_{W\text{-}IVNS}(A, C) \le T_{W\text{-}IVNS}(B, C)$.

Proofs:

2.1. Tangent function is monotonic incresing in the interval $[0, \pi/4]$. It also lies in the interval [0, 1] and $\sum_{i=1}^{n}w_i=1$. Therefore, $0\le T_{IVNS}(A, B)\le 1$.

2.2. For any two IVNS A and B,

$A = B$

$\Rightarrow T_A^{\lambda}(x_i)=T_B^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$

$\Rightarrow \left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|=0$, $\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|=0$, $\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|=0$

Therefore, $T_{W\text{-}IVNS}(A, B) = 1$ for $0\le\lambda\le 1$ and $\sum_{i=1}^{n}w_i=1$.

Conversely,

$T_{W\text{-}IVNS}(A, B) = 1$

$\Rightarrow \left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|=0$, $\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|=0$, $\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|=0$

$\Rightarrow T_A^{\lambda}(x_i)=T_B^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$

Therefore $A = B$.

2.3. $T_{W\text{-}IVNS}(A, B) =$

$$1-\sum_{i=1}^{n}w_i\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\\ +\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$=$

$$1-\sum_{i=1}^{n}w_i\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_B^{\lambda}(x_i)-T_A^{\lambda}(x_i)\right|+\left|I_B^{\lambda}(x_i)-I_A^{\lambda}(x_i)\right|\\ +\left|F_B^{\lambda}(x_i)-F_A^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$= T_{W\text{-}IVNS}(B, A)$

2.4. If $A\subset B\subset C$

then $T_A^{\lambda}(x_i) \le T_B^{\lambda}(x_i) \le T_C^{\lambda}(x_i)$, $I_A^{\lambda}(x_i) \le I_B^{\lambda}(x_i) \le I_C^{\lambda}(x_i)$, $F_A^{\lambda}(x_i) \le F_B^{\lambda}(x_i) \le F_C^{\lambda}(x_i)$, for $x\in X$ and $\sum_{i=1}^{n}w_i=1$. Now, we have the inequalities:

$\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$,

$\left|T_B^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$;

$\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$,

$\left|I_B^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$;

$\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$,

$\left|F_B^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$. From eqn (2), we can say that $T_{W\text{-}IVNS}(A, C) \le T_{W\text{-}IVNS}(A, B)$ and $T_{W\text{-}IVNS}(A, C) \le T_{W\text{-}IVNS}(B, C)$.

The following notations are adopted in the paper.

$P = \{P_1, P_2, ..., P_m\}$ $(m \geq 2)$ is the set of alternatives
$C = \{C_1, C_2, ..., C_n\}$ $(n \geq 2)$ is the set of attributes.
The decision maker provides the ranking of alternatives with respect to each attribute. The ranking presents the performances of alternatives P_i (i = 1, 2,..., m) based on the attributes C_j (j = 1, 2, ..., n). The values associated with the alternatives for multi- attributes decision making problem can be presented in the following decision matrix (see Table 1). The relation between alternatives and attributes in terms of IVNSs are given in the following decision matrix (see Table 1):

Table 1: The decision matrix

	C_1	C_2	...	C_n
P_1	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	$\left\langle [T_{12}^L, T_{12}^U], [I_{12}^L, I_{12}^U], [F_{12}^L, F_{12}^U] \right\rangle$	...	$\left\langle [T_{1n}^L, T_{1n}^U], [I_{1n}^L, I_{1n}^U], [F_{1n}^L, F_{1n}^U] \right\rangle$
P_2	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	...	$\left\langle [T_{2n}^L, T_{2n}^U], [I_{2n}^L, I_{2n}^U], [F_{2n}^L, F_{2n}^U] \right\rangle$
.	...	...	...	...
P_m	$\left\langle [T_{m1}^L, T_{m1}^U], [I_{m1}^L, I_{m1}^U], [F_{m1}^L, F_{m1}^U] \right\rangle$	$\left\langle [T_{m2}^L, T_{m2}^U], [I_{m2}^L, I_{m2}^U], [F_{m2}^L, F_{m2}^U] \right\rangle$	...	$\left\langle [T_{mn}^L, T_{mn}^U], [I_{mn}^L, I_{mn}^U], [F_{mn}^L, F_{mn}^U] \right\rangle$

Here, $\left\langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \right\rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) are interval valued neutrosophic sets. Multi attributes decision making procedure based on tangent similarity measure in interval valued neutrosophic environment is presented using the following steps.

Step 1: Determine the decision matrix in terms of SVNSs

Decision matrix in terms of SVNSs is constructed with the transformation $\Omega_{ij}^{\lambda} = \lambda \Omega_{ij}^{L} + (1-\lambda)\Omega_{ij}^{U}$,

where $\Omega = T, I, F$; i = 1, 2, ..., m; j = 1, 2, ..., n and $0 \leq \lambda \leq 1$.

Table 2: Decision matrix in terms of SVNSs

	C_1	C_2	...	C_n
P_1	$\left\langle T_{11}^{\lambda}, I_{11}^{\lambda}, F_{11}^{\lambda} \right\rangle$	$\left\langle T_{12}^{\lambda}, I_{12}^{\lambda}, F_{12}^{\lambda} \right\rangle$	...	$\left\langle T_{1n}^{\lambda}, I_{1n}^{\lambda}, F_{1n}^{\lambda} \right\rangle$
P_2	$\left\langle T_{21}^{\lambda}, I_{21}^{\lambda}, F_{21}^{\lambda} \right\rangle$	$\left\langle T_{22}^{\lambda}, I_{22}^{\lambda}, F_{22}^{\lambda} \right\rangle$	...	$\left\langle T_{2n}^{\lambda}, I_{2n}^{\lambda}, F_{2n}^{\lambda} \right\rangle$
…	…	…	…	…
…	…	…	…	…
P_m	$\left\langle T_{m1}^{\lambda}, I_{m1}^{\lambda}, F_{m1}^{\lambda} \right\rangle$	$\left\langle T_{m2}^{\lambda}, I_{m2}^{\lambda}, F_{m2}^{\lambda} \right\rangle$	...	$\left\langle T_{mn}^{\lambda}, I_{mn}^{\lambda}, F_{mn}^{\lambda} \right\rangle$

where $\Omega = T, I, F$; i = 1, 2, ..., m; j = 1, 2, ..., n and $0 \leq \lambda \leq 1$.

Step 2: Determine the benefit type attributes and cost type attributes

Generally, the attributes can be categorized into two types: benefit attributes and cost attributes. In the proposed decision making strategy, an ideal alternative can be identified by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes to determine the best value of each attribute among all alternatives. Therefore, we define an ideal alternative as follows:
$P^* = \{C_1^*, C_2^*, \ldots, C_m^*\}$,
Here the benefit attributes is

$$C_j^* = \left\lfloor \max_i T_{C_j}^{\lambda\ (P_i)}, \min_i I_{C_j}^{\lambda\ (P_i)}, \min_i F_{C_j}^{\lambda\ (P_i)} \right\rfloor \quad (3)$$

and the cost attributes is

$$C_j^* = \left[\min_i T_{C_j}^{\lambda\ (P_i)}, \max_i I_{C_j}^{\lambda\ (P_i)}, \max_i F_{C_j}^{\lambda\ (P_i)} \right] \quad (4)$$

Step 3: Calculate of the measure values between ideal alternatives and decision elements

Calculate tangent similarity measures (choosing various values of λ) between ideal alternatives and the decision elements of Table 2 using eqn.(1).

Step 4: Determine the weights of the attributes

The importance of all the attributes may or may not be same in decision making context. The decision maker may use normalized weights or differential weights for attributes based on his/her needs and practical decision making situation. If the attributes are assumed as extremely importance to the decision maker, then the weight of each attribute will be taken as $1/n$ where n is the number of attributes.

Step 5: Determination of the accumulated measure values

To aggregate the similarity measures corresponding to each alternative, we define accumulated measure function (AMF) as follows:

$$D_{AMF}^{i} = \sum_{j=1}^{n} w_j \cdot T_{IVNS}(P_{ij}, P^*) \quad (5)$$

Step 6: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of accumulated measure values. Highest value reflects the best alternative.

Step 7: End

5 Numerical example

Consider the illustrative example, which is very important for Indian government employees after a financial year to select suitable money Investment Company for more tax rebate and more return value after investment span. For a financial year, every government employee desires to invest a sum of money to reduce his/her annual income tax amount and to place the money in more secure investment company. This is the crucial time when most of the government employee gets confused too much and takes a decision which he/she starts to dislike later. Employees often confuse to decide which money Investment Company should choose. If the chosen Investment Company is improper, the employee may encounter a negative impact to his/her future economical condition. It is very important for any employee to choose carefully from various options available to him/her in which he/she is interested. So, it is necessary to utilize a suitable mathematical decision making strategy.

The feature of the proposed strategy is that it includes interval valued truth membership, interval valued indeterminate and interval valued falsity membership function simultaneously. Assume that, a government employee determines to invest a sum of money to a suitable investment sector, namely, Public provident fund (S_1), Postal Life insurance (S_2), Stock Market (S_3). The employee must invest his/her money with respect to the attributes, namely, Growth analysis (C_1), Risk analysis (C_2), Government norms and regulation (C_3). Our solution is to make decision to choose suitable money Investment Company. The values associated with the alternatives for multi- attributes decision-making problem can be presented in the following decision matrix:

Table 3: The decision matrix

	C_1	C_2	C_3
A_1	$\langle [0.6, 0.8], [0.2, 0.4], [0.3, 0.5] \rangle$	$\langle [0.2, 0.4], [0.4, 0.6], [0.3, 0.7] \rangle$	$\langle [0.4, 0.8], [0.5, 0.7], [0.4, 0.6] \rangle$
A_2	$\langle [0.4, 0.6], [0.3, 0.5], [0.5, 0.7] \rangle$	$\langle [0.1, 0.3], [0.2, 0.6], [0.2, 0.6] \rangle$	$\langle [0.5, 0.7], [0.4, 0.8], [0.3, 0.7] \rangle$
A_3	$\langle [0.6, 0.8], [0.2, 0.4], [0.4, 0.6] \rangle$	$\langle [0.3, 0.5], [0.1, 0.5], [0.3, 0.5] \rangle$	$\langle [0.3, 0.7], [0.3, 0.5], [0.1, 0.5] \rangle$

The decision making calculation is presented using the following steps:

Step 1: Determine the decision matrix in terms of SVNS

Each element of IVNS in Table 3 is transformed to an element of SVNS. This transformation is shown in Table 4.

Table 4: Relation between alternatives and attributes in terms of SVNSs

	C_1	C_2	C_3
A_1	$\langle 0.6\lambda + 0.8(1-\lambda), 0.2\lambda + 0.4(1-\lambda), 0.3\lambda + 0.5(1-\lambda) \rangle$	$\langle 0.2\lambda + 0.4(1-\lambda), 0.4\lambda + 0.6(1-\lambda), 0.3\lambda + 0.7(1-\lambda) \rangle$	$\langle 0.4\lambda + 0.8(1-\lambda), 0.5\lambda + 0.7(1-\lambda), 0.4\lambda + 0.6(1-\lambda) \rangle$
A_2	$\langle 0.4\lambda + 0.6(1-\lambda), 0.3\lambda + 0.5(1-\lambda), 0.5\lambda + 0.7(1-\lambda) \rangle$	$\langle 0.1\lambda + 0.3(1-\lambda), 0.2\lambda + 0.6(1-\lambda), 0.2\lambda + 0.6(1-\lambda) \rangle$	$\langle 0.5\lambda + 0.7(1-\lambda), 0.4\lambda + 0.8(1-\lambda), 0.3\lambda + 0.7(1-\lambda) \rangle$
A_3	$\langle 0.6\lambda + 0.8(1-\lambda), 0.2\lambda + 0.4(1-\lambda), 0.4\lambda + 0.6(1-\lambda) \rangle$	$\langle 0.3\lambda + 0.5(1-\lambda), 0.1\lambda + 0.5(1-\lambda), 0.3\lambda + 0.5(1-\lambda) \rangle$	$\langle 0.3\lambda + 0.7(1-\lambda), 0.3\lambda + 0.5(1-\lambda), 0.1\lambda + 0.5(1-\lambda) \rangle$

Step 2: Determine the benefit type attributes and cost type attributes

C_1, C_3 are treated as benefit type attributes and C_2 is treated as cost type attributes. Using Table 2, eqn.(3) and eqn.(4), we calculate ideal alternative solutions as follows (Table 5):

Table 5: Ideal alternative solutions

	λ	C_1	C_2	C_3
P^*	0.1	[0.78, 0.38, 0.48]	[0.28, 0.58, 0.66]	[0.76, 0.48, 0.46]
	0.2	[0.76, 0.36, 0.46]	[0.26, 0.56, 0.62]	[0.72, 0.46, 0.42]
	0.3	[0.74, 0.34, 0.44]	[0.24, 0.54, 0.58]	[0.68, 0.44, 0.38]
	0.4	[0.72, 0.32, 0.42]	[0.22, 0.52, 0.54]	[0.62, 0.42, 0.34]
	0.5	[0.70, 0.30, 0.40]	[0.20, 0.50, 0.50]	[0.60, 0.40, 0.30]
	0.6	[0.58, 0.28, 0.38]	[0.18, 0.48, 0.46]	[0.56, 0.38, 0.26]
	0.7	[0.66, 0.26, 0.36]	[0.16, 0.32, 0.42]	[0.56, 0.36, 0.22]
	0.8	[0.64, 0.24, 0.34]	[0.14, 0.44, 0.38]	[0.54, 0.34, 0.18]
	0.9	[0.62, 0.22, 0.32]	[0.12, 0.42, 0.34]	[0.52, 0.32, 0.14]

Step 3: Calculate the measure values between ideal alternatives and decision elements

Using eqn. (1), we calculate tangent similarity measures for different values of λ between ideal alternatives (Table 5) and the decision elements in Table 4 (see Table 6).

Step 4: Determine the weights of the attributes

We take each attribute weight as $w_i = 1/3$ (i = 1, 2, 3).

Step 5: Determine the accumulated measure values

Using eqn. 5, we calculate AMF values as follows (Table 7).

Table 7: Ranking results (with equal attributes weights)

Proposed strategy	λ	Measure values	Ranking orders
	0.1	0.9633; 0.8964; 0.9386	$S_1 \succ S_3 \succ S_2$
	0.2	0.9615; 0.8982; 0.9386	$S_1 \succ S_3 \succ S_2$
$T_{IVNS}(P, P^*)$	0.3	0.9598; 0.9000; 0. 9386	$S_1 \succ S_3 \succ S_2$
	0.4	0.9562; 0.9036; 0.9404	$S_1 \succ S_3 \succ S_2$
	0.5	0.9562; 0.9036, 0.9616	$S_3 \succ S_1 \succ S_2$
	0.6	0.9545; 0.9107; 0.9386	$S_1 \succ S_3 \succ S_2$
	0.7	0.9369;0.9070, 0.9475	$S_3 \succ S_1 \succ S_2$
	0.8	0.9456; 0.9036; 0.9333	$S_1 \succ S_3 \succ S_2$
	0.9	0.9420; 0.9036, 0.9316	$S_1 \succ S_3 \succ S_2$

Step 6: Ranking the alternatives

Ranking of the alternatives is prepared based on the descending order of accumulated measure values. When $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9,$ Public provident fund (S_1) is the best alternative to invest money (see Table 7). When $\lambda = 0.5, 0.7,$ Stock market (S_3) is the best alternative to invest money (see Table 7).

6 Comparative analysis

For the sake of validating the flexibility and feasibility of the proposed strategy, a comparative study is conducted. In order to do so, different existing strategies are used to solve the same decision-making problem with the interval valued neutrosophic information. Literature review reflects that Broumi and Smarandache [36] proposed cosine similarity measure of interval valued neutrosophic sets. Ye [35] proposed Similarity measures between interval neutrosophic sets and apply in multicriteria decision-making. Şahin [45] proposed cross-entropy measure on interval valued neutrosophic sets and presenter its applications in multicriteria decision making. Table 8 shows that the ranking results obtained from different strategy differ. Ranking results from proposed strategy with $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9$ are similar to the ranking result of cosine similarity measure [36] (Broumi and Smarandache, 2014). Ranking results obtained from proposed strategy with $\lambda = 0.5, 0.7$ are similar to the ranking results of Ye`s strategy (Ye, 2014d) and cross entropy strategy [45].

Table 8: The ranking results of different strategies

strategies	Ranking results
Proposed strategy with $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9$	$S_1 \succ S_3 \succ S_2$
Proposed strategy with $\lambda = 0.5, 0.7$	$S_3 \succ S_1 \succ S_2$
Cosine similarity measure (Broumi and Smarandache, [36]	$S_1 \succ S_2 \succ S_3$
Ye [35]	$S_3 \succ S_1 \succ S_2$
Cross entropy strategy [45]	$S_3 \succ S_1 \succ S_2$

7. Contributions of the paper

- We define tangent similarity measures for IVNS. We have also proved their basic properties.
- We developed a decision making strategy based on the proposed weighted tangent similarity measure.
- Steps and calculations of the proposed strategy are easy to use.
- We have solved a numerical example to show the applicability of the proposed strategy.

8. Conclusion

In this paper, we have defined tangent similarity measure and proved its properties in interval valued neutrosophic environment. We also also developed a novel multi attribute decision making strategy based on the proposed tangent similarity measure in interval valued neutrosophic environment. We have presented an application, namely, selection of best investment sector for an Indian government employee. We also presented a comparative analysis with the existing strategies in the literature.The concept presented in this paper can be applied in teacher selection, school choice, medical diagnosis, pattern rcognition, purchasing decision making, commodity recommendation in interval valued neutrosophic environment. It is worth of further study to formulate a multi attribute decision making strategy that considers the priority of attributes.

References

[1] F. Smarandache. A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth, 1998.

[2] L. A. Zadeh. Fuzzy Sets. Information and Control, 8 (1965), 338–353.

[3] K. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986), 87–96.

[4] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Interval neutrosophic sets and logic: theory and applications in computing, Hexis, Phoenix, Ariz, USA, 2005.

[5] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Single valued neutrosophic, sets. Multispace and Multi-structure, 4 (2010), 410–413.

[6] S. Ye, and J. Ye. Dice similarity measure between single valued neutrosophic multi-sets and its application in medical diagnosis, Neutrosophic Sets and Systems, 6 (2014), 49–54.

[7] J. Ye, and J. Fu. Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function, Computer Methods and Programs in Biomedicine, 123 (2016), 142–149.

[8] A. Q. Ansari, R. Biswas, and S. Aggarwal. Proposal for applicability of neutrosophic set theory in medical AI. International Journal of Computer Applications, 27(5)(2011), 5–11.

[9] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications, 27(3) (2016), 727–737.

[10] A. Kharal. A neutrosophic multi-criteria decision making method. New Mathematics and Natural Computation, Creighton University, USA, 2013.

[11] S. Pramanik, P. Biswas, and B. C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28(5) (2017), 1163–1176.

[12] J. Ye. Multiple criteria group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. Journal of Intelligent and Fuzzy Systems, 27 (2014), 2927–2935.

[13] S. Pramanik, and S. N. Chackrabarti. A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. International Journal of Innovative Research in Science Engineering and Technology, 2(11) (2013), 6387–6394.

[14] K. Mondal, and S. Pramanik. A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets and Systems, 5 (2014), 21–26.

[15] K. Mondal, and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28–34.

[16] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62–68.

[17] Y. Guo, and H. D. Cheng. New neutrosophic approach to image segmentation. Pattern Recognition, 42 (2009), 587–595.

[18] M. Zhang, L. Zhang, and H. D. Cheng. A neutrosophic approach to image segmentation based on watershed method. Signal Processing, 90(5) (2010), 1510–1517.

[19] S. Pramanik and T. K. Roy. Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. Neutrosophic Sets and Systems, 2 (2014), 82–101.

[20] L. K. Hyung, Y. S. Song, and K. M. Lee. Similarity measure between fuzzy sets and between elements. Fuzzy Sets and Systems, 62 (1994), 291–293.

[21] S. M. Chen, S. M. Yeh, and P. H. Hsiao. A comparison of similarity measures of fuzzy values. Fuzzy Sets and Systems, 72 (1995), 79–89.

[22] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-criteria decision-making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2 (2014), 102–110.

[23] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-criteria decision making with unknown weight information. Neutrosophic Sets and Systems, 3 (2014), 42–52.

[24] P. D. Liu, and L. Shi. The generalized hybrid weighted average operator based on interval valued neutrosophic hesitant set and its application to multiple criteria decision making. Neural Computing and Application, 26(2) (2015), 457–471.

[25] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, X. H. Chen. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International journal of systems science, 47(10), 2342–2358.

[26] R. Şahin, and M. Karabacak. A multi criteria decision making method based on inclusion measure for interval valued neutrosophic sets, International Journal of Engineering and Applied Sciences, 2(2) (2015), 13–15.

[27] J. Ye, and Q. Zhang. Single valued neutrosophic similarity measures for multiple criteria decision-making Neutrosophic Sets and System, 2 (2012), 48–54.

[28] J. Ye. Single-valued neutrosophic clustering algorithms based on similarity measures. Journal of Classification, 34(1) (2017), 148–162.

[29] A. A. Salama, and A. L. Blowi. Correlation of neutrosophic data. International Refereed Journal of Engineering and Science, 1(2) (2012), 39–43.

[30] S. Broumi, and F. Smarandache. Several similarity measures of neutrosophic sets. Neutrosophic Sets and Systems, 1(2013), 54–62.

[31] S. Broumi, and F. Smarandache. Correlation coefficient of interval valued neutrosophic set. Applied Mechanics and Materials, 436 (2013), 511–517.

[32] P. Majumder, and S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26 (2014), 1245–1252.

[33] J. Ye. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42(4) (2013), 386–394.

[34] J. Ye. Vector similarity measures of simplified neutrosophic sets and their application in multi-criteria decision making. International Journal of Fuzzy Systems, 16(2) (2014), 204–215.

[35] J. Ye. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. Journal of Intelligent & Fuzzy Systems, 26 (2014), 165–172.

[36] S. Broumi, and F. Smarandache. Cosine similarity measure of interval valued neutrosophic sets. Neutrosophic Sets and Systems, 5 (2014), 15–20.

[37] J. Ye. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine, 63(3) (2015), 171–179.

[38] S. Ye, J. Fu, and J. Ye. Medical diagnosis using distance-based similarity measures of single valued neutrosophic multi-sets. Neutrosophic Sets and Systems, 7 (2014), 47–52.

[39] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-criteria decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets and System, 8 (2015), 47–58.

[40] K. Mondal, and S. Pramanik S. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global Journal of Advanced Research, 2(1) (2015), 212–220.

[41] S. Pramanik, and K. Mondal. Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Journal of New Theory, 4 (2015), 464–471.

[42] S. Pramanik, and K. Mondal. Weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. International Journal of Innovative Research in Science, Engineering and Technology, 4(2) (2015), 158–164.

[43] K. Mondal, and S. Pramanik. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-criteria decision. Global Journal of Advanced Research, 2(2) (2015), 464–471.

[44] K. Mondal, and S. Pramanik. Neutrosophic tangent similarity measure and its application to multiple criteria decision making. Neutrosophic Sets and Systems, 9 (2015), 92–98.

[45] R. Șahin. Cross-entropy measure on interval neutrosophic sets and its applications in multicriteria decision making. Neural computing and application, 28(5) (2017), 1177–1187.

Table 6: Tangent similarity measure values

	$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.3$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9738	0.9160	1.0000	0.9738	0.9108	1.0000	0.9738	0.9055
S_2	0.8683	0.9686	0.8523	0.8683	0.9633	0.8630	0.8683	0.9581	0.8737
S_3	0.9738	0.8683	0.9738	0.9738	0.8683	0.9738	0.9738	0.8683	0.9738
	$\lambda = 0.4$			$\lambda = 0.5$			$\lambda = 0.6$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9738	0.8949	1.0000	0.9738	0.8949	1.0000	0.9738	0.8896
S_2	0.8683	0.9528	0.8896	0.8683	0.9476	0.8949	0.8949	0.9423	0.8949
S_3	0.9738	0.8683	0.9790	0.9738	0.9371	0.9738	0.9738	0.8683	0.9738
	$\lambda = 0.7$			$\lambda = 0.8$			$\lambda = 0.9$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9371	0.8737	1.0000	0.9738	0.8630	1.0000	0.9738	0.8523
S_2	0.8683	0.9738	0.8790	0.8683	0.9318	0.9108	0.8949	0.9266	0.9160
S_3	0.9738	0.9055	0.9633	0.9738	0.8683	0.9580	0.9738	0.8683	0.9528

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