Proof of Riemann Hypothesis

Andrey Skrypnyk
ansk66@mail.ru

Abstract—This article describes how to calculate the formula for calculating all primes.

Index Terms—algorithm

I. RIEMANN HYPOTHESIS DEFINITION

There is a pattern in the distribution of primes among the positive integers.

II. RIEMANN HYPOTHESIS PROOF ALGORITHM

A. Distinguishing the Sequence of Odd Numbers

Let $\mathbb{N}$ be natural numbers, including zero, and $\mathbb{N}^*$ be natural numbers without zero.

First two primes (by condition) are:

$$1, 2. \quad (1)$$

Prime number 2 is significant for dividing the sequence into two equal sequences of even ($x$) and odd ($y$) numbers:

$$x \in \{2M \mid M \in \mathbb{N}^*\}, \quad (2)$$

$$y \in \{2M + 1 \mid M \in \mathbb{N}^*\}. \quad (3)$$

Starting from $M = 2$ (2) describes the set of composite numbers $x_{comp}$ by condition:

$$x_{comp} \in \{2M \mid M \in \mathbb{N}^*, \ M \geq 2\}. \quad (4)$$

Thus further we will consider the sequence of odd numbers $\{y\}$ (3) to determine the pattern in the distribution of primes ($y_o$).

The sequence of odd numbers $\{y\}$, except for $y_o$, also includes the set of composite odd numbers $y_{comp}$:

$$y_{comp} \in \{y_o \mid y_o \geq 3, \ y \geq 3\}. \quad (5)$$

Expression (3) without limitations describes the distribution of first $y_o$ in the sequence of odd numbers within the segment from 3 to the first $y_{comp} = 3^2 = 9$.

Let’s represent (3) as the following expression:

$$y_o = 1^2 + 2 \cdot 1 \cdot M_1 + 2, \quad (6)$$

where $M_1 \in \mathbb{N}$.

Therefore, this segment can be represented in the following way:

$$1^2 < y < 3^2. \quad (7)$$

The following segment, where (6) for determination of $y_o$ will be limited by exception of the set of composite numbers $\{3y \mid y > 3\}$, will end with the first $y_{comp}$ to which $y_o = 3$ will bear no relation. By definition it is $y_{comp} = 5^2 = 25$. Thus we can conclude the following.

B. Conclusion 1

All segments compliant with the specific pattern of distribution of $y_o$ are limited by $y_{comp} = y_{on}^2$ and $y_{comp} = y_{o(n+1)}^2$.

Let’s analyze the first such segment.

C. The first segment of odd numbers from 1 to 9

Distribution of $y_o$ is described by (6). Let’s calculate first $y_o$ after (1):

$$3, 5, 7. \quad (8)$$

D. The second segment of odd numbers from 9 to 25

In order to exclude the composite numbers $y_{comp}$ from the set $\{3y \mid y > 3\}$, $y_o = 1$ in (6) shall be replaced by $y_o = 3$ and summation 2 shall be replaced by variable $\pm 2$ to cover all $y_o$ in this segment:

$$y_o = 3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 3^2 + 2(3M_3 \pm 1), \quad (9)$$

where $M_3 \in \mathbb{N}$.

Let’s calculate next $y_o$ in the sequence:

$$11, 13, 17, 19, 23. \quad (10)$$

E. The third segment of odd numbers from 25 to 49

For this segment $y_o$ value shall be equal in two expressions - in (9) and in the following expression in order to exclude the composite numbers $\{5y \mid y > 5\}$:

$$y_o = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5 = 5^2 + 2(5M_5 \pm z_5), \quad (11)$$

where $M_5 \in \mathbb{N}$, $1 \leq z_5 \leq 2$.

Starting from the second segment, expression for $y_o$ depends on the value of $M_5$. According to Conclusion 1 and (9) it is possible to calculate the lower and upper limits for $M_5$ in any segment of $y_{on}^2 < y < y_{o(n+1)}^2$:

$$\frac{y_{on}^2 - 9 \pm 2}{6} \leq M_5 \leq \frac{y_{o(n+1)}^2 - 9 \pm 2}{6}. \quad (12)$$

For this segment $M_5$ value in (9) will change:

$$3 \leq M_5 \leq 7. \quad (13)$$
Let’s compare (9) and (11):
\[ 3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5. \] (14)
Let’s express \( M_5 \) from (14):
\[ M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5}. \] (15)
Substitute (15) into (11):
\[ y_o = 5^2 + 2 \left( \frac{5 \cdot 3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 \right), \] (16)
where \( 3 \leq M_3 < 7, \ 1 \leq z_5 \leq 2 \).

Calculate next \( y_o \) in the third segment:
\[ 29, 31, 37, 41, 43, 47. \] (17)

F. Conclusion 2

Based on the results of analysis of first, second and third segments of odd numbers we can conclude the following:
Each successive segment compliant with the pattern of distribution of \( y_o \) depends on the pattern of distribution of \( y_o \) in all previous segment starting from the second segment.

Let’s analyze the following segment for final determination of the pattern of distribution of \( y_o \) in segments \( y_{on}^2 < y < y_{o(n+1)}^2 \).

G. The fourth segment of odd numbers from 49 to 121

For this segment \( y_o \) value shall be equal in two expressions - in (16) with different values of variables:
\[ 7 \leq M_3 < 19, \] (18)
\[ 1 \leq z_5 \leq 2, \] (19)
\[ M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \in \mathbb{N}^*, \] (20)
and in the following expression to exclude the composite numbers \( y_{ uncomp} \) from the set \( \{y \mid y > 7\} \):
\[ y_o = 7^2 + 2 \cdot 7 \cdot M_7 + 2z_7 = 7^2 + 2(7M_7 \pm z_7), \] (21)

For this segment from (21) it follows that:
\[ M_7 \in \mathbb{N}, \] (22)
\[ 1 \leq z_7 \leq 3 \] (23)

Let’s compare (16) and (21):
\[ 5^2 + 2 \left( \frac{5 \cdot 3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 \right) = \] (24)
\[ = 7^2 + 2 \cdot 7 \cdot M_7 \pm 2z_7. \]
Express \( M_7 \) from (24):
\[ M_7 = \frac{5 \cdot 3M_3 \pm 1 - 8 \mp z_5 \pm z_5 - 12 \mp z_7}{7}. \] (25)
Substitute \( M_7 \) from (25) into (21):
\[ y_o = 7^2 + 2 \left( \frac{5 \cdot 3M_3 \pm 1 - 8 \mp z_5 \pm z_5 - 12 \mp z_7}{7} \pm z_7 \right), \] (26)
where (18), (19), (20), (22), (23) and (25) are true.

Let’s calculate the successive values of \( y_o \) in in the fourth segment of odd numbers:
\[ 53, 59, 61, 67, 71, 73, 79, 89, \]
\[ 97, 101, 103, 107, 109, 113. \] (27)

H. General Expression of Distribution of Primes

Thus we can determine the specific patterns, comparing (16) and (26).

Let’s present the general expression of distribution of \( y_o \) in n-th segments \( y_{an}^2 < y < y_{o(n+1)}^2 \) taking these patterns into consideration:
\[ y_o = y_{an}^2 + 2(y_{on}M_{yon} \pm z_{yon}) \] (28)
Variables (28) are calculated using the following formulas:
\[ 1 \leq z_{yon} \leq \frac{y_{on} - 1}{2}, \] (29)
where \( 2 \leq c \leq n \) is the number of the number in the sequence of odd primes;
\[ M_{yon} = \]
\[ = \frac{y_{o(n-1)}M_{yon(n-1)} \pm z_{yon(n-1)} - \frac{y_{on}^2 - y_{o(n-1)}^2}{2} \mp z_{yon}}{y_{on}}, \] (30)
where \( M_{yon} \in \mathbb{N}; \)
\[ M_3 < M_{yon} < M_{yon}, \] (31)
where \( M_3 \) is calculated according to (12);
\[ M_{yon} = \]
\[ = \frac{y_{o(b-1)}M_{yon(b-1)} \pm z_{yon(b-1)} - \frac{y_{ob}^2 - y_{o(b-1)}^2}{2} \mp z_{yob}}{y_{ob}}, \] (32)
where \( M_{yon} \in \mathbb{N}^*, \ 3 < y_{o(b-1)} < y_{ob} < y_{on}. \)

In order to form the full sequence of \( y_o \), the n-th segments shall be analyzed in sequence. But calculation of \( y_o \) from segments to segment becomes more difficult. Thus the third segment of odd numbers in (16) has 5 variables, the fourth segment of odd numbers in (26) has 8 variables. But nevertheless, (26) unequivocally describes the distribution of \( y_o \) in sequence of numbers. If it is necessary to calculate \( y_o \) in n-th segment, avoiding the previous segments, all \( y_o \leq y_{o(n+1)} \) from previous calculations shall be known. The required range will be set by summand \( y_{on}^2 \) and values of \( M_3 \) (12). While solving the problem all (31) for this n-th segment shall be calculated in sequence.

I. Final Conclusion

Riemann Hypothesis is true. Distribution of primes among the positive integers has its own pattern. But for odd numbers of \( y \) the sections compliant with the specific pattern of distribution of primes \( y_o \) are limited by composite numbers \( y_{on}^2 \) and \( y_{o(n+1)}^2 \). Distribution of \( y_o \) in such n-th sections, starting from the third segment of odd numbers, is calculated
according to the (28). The full sequence of $y_o$ is achieved by consequent analysis of $n$-th sections, starting from the first segment of odd numbers.

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REFERENCES