Logical Errors of Special Relativity

by
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Abstract

This paper uncovers a fundamental logical error in Special Relativity (SR). By rigorously adhering to SR's own principles, it will be demonstrated that lengths measured using clocks do not differ between two systems in relative motion. Furthermore, it is shown that SR implicitly relates two distinct events, leading to the erroneous conclusion of a loss of simultaneity.

1. Introduction

Many laypeople, and even an increasing number of experts, sense a lack of logical consistency in the foundational structure of Special Relativity. Particularly, the invariance of the speed of light is often met with skepticism. While the mathematical complexity and the abstract nature of General Relativity may obscure its full understanding, this paper identifies a fundamental logical fallacy within SR. By following Einstein's original 1905 paper, we aim to clarify SR's assumptions through visualizations of its mathematical framework.

2. Clock Synchronisation

We begin by examining clock synchronization as described in Einstein's "On the Electrodynamics of Moving Bodies" (1905), Chapter A,1:

> If there is a clock at point A in space, then an observer located at A can evaluate the time of events in the immediate vicinity of A by finding the positions of the hands of the clock that are simultaneous with these events. If there is another clock at point B that in all respects resembles the one at A, then the time of events in the immediate vicinity of B can be evaluated by an observer at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation. So far we have defined only an "A-time" and a "B-time," but not a common "time" for A and B. The latter can now be determined by establishing *by definition* that the "time" required for light to travel from A to B is equal to the "time" it requires to travel from B to A. For, suppose a ray of light leaves from A for B at "A-time" t_A, is reflected from B toward A at "B-time" t_B, and arrives back at A at "A-time" t'_A. The two clocks are synchronous by definition if
>
> $$t_B - t_A = t'_A - t_B.$$
>
> We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily

Excerpt 1: Synchronization of two clocks

To visualize the principle of synchronizing two clocks located at points A and B in a stationary reference frame, we adopt SR's terminology and denote this reference frame as K. The light beam is represented in orange, the rigid rod in black, and the clocks involved are highlighted in red:

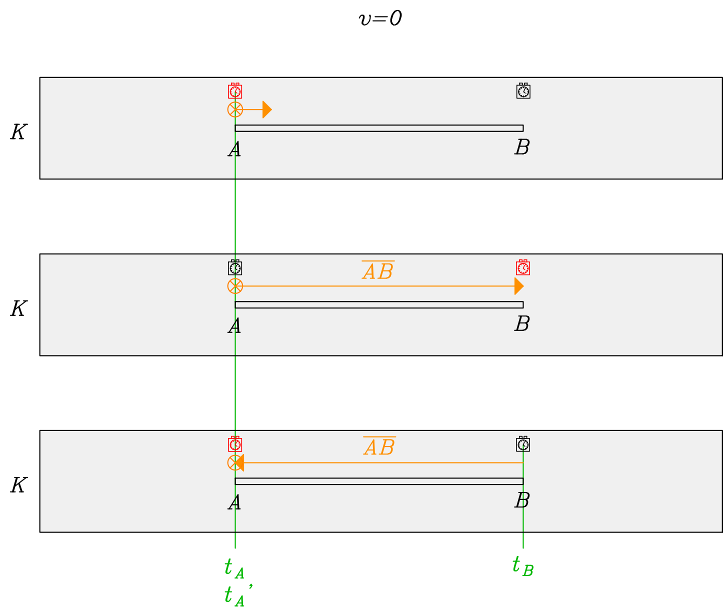
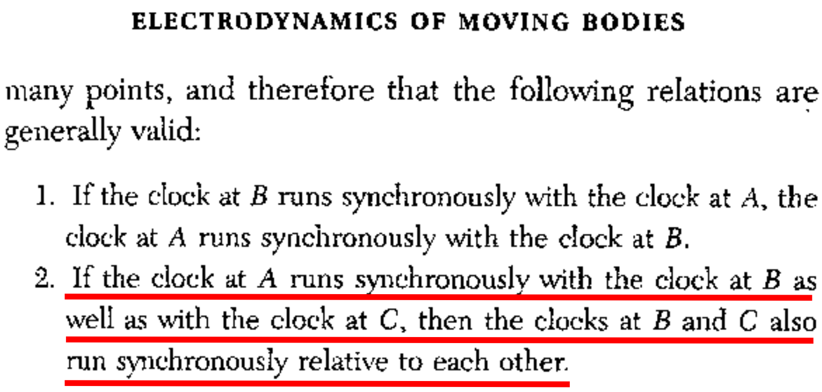


Fig. 1: Synchronization of two clocks

SR also describes the synchronization of additional clocks. Citing again:



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many points, and therefore that the following relations are generally valid:

1. If the clock at *B* runs synchronously with the clock at *A*, the clock at *A* runs synchronously with the clock at *B*.
2. If the clock at *A* runs synchronously with the clock at *B* as well as with the clock at *C*, then the clocks at *B* and *C* also run synchronously relative to each other.

Excerpt 2: Synchronisation of two more clocks

We now add two additional clocks at points C and D and synchronize them with clock B, thereby ensuring synchronization as per SR with clock A as well. Although the positions of C and D appear arbitrary, their relevance will become clear later:

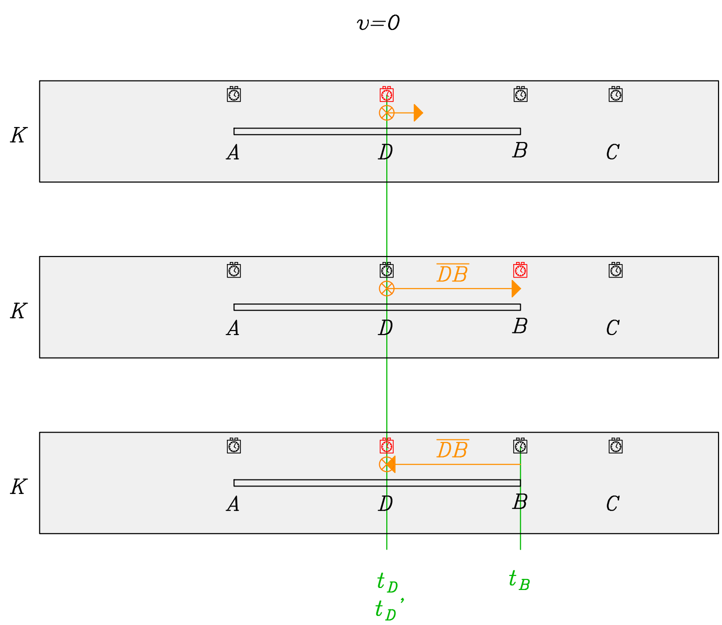


Fig. 2: Synchronisation of two more clocks

Thus, strictly adhering to SR, we now have four synchronized clocks at points A, B, C and D.

3. Length Measurement Methods

SR proposes two methods for measuring lengths. The first method involves using a physical scale to measure the length l of a rigid rod within the moving reference frame k:

> Take a rigid rod at rest; let its length, measured by a measuring rod that is also at rest, be l. Now imagine the axis of the rod placed along the X-axis of the rest coordinate system, and the rod then set into uniform parallel translational motion (with velocity v) along the X-axis in the direction of increasing x. We now inquire about the length of the *moving* rod, which we imagine to be ascertained by the following two operations:
>
> a. The observer moves together with the aforementioned measuring rod and the rigid rod to be measured, and measures the length of the rod by laying out the measuring rod in the same way as if the rod to be measured, the observer, and the measuring rod were all at rest.

Excerpt 3: Measurement using scale(operation a.)

This method is visualized as follows, where the moving rod is blue, and the scale is green:

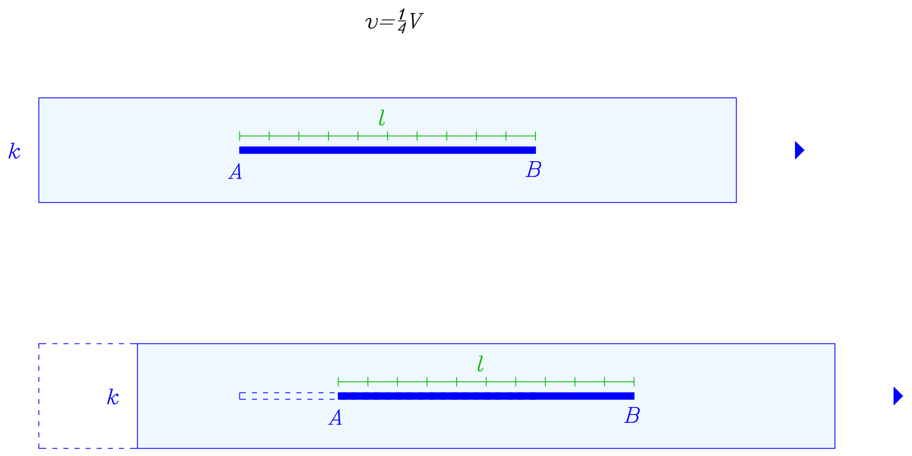


Fig. 3: Measurement using scale (operation a.)

The second method measures the length r_{AB} of the rod using synchronized clocks at the rod's endpoints. This is done from the stationary system K at a given time t:

> b. Using clocks at rest and synchronous in the rest system as outlined in section 1, the observer determines at which points of the rest system the beginning and end of the rod to be measured are located at some given time t. The distance between these two points, measured with the rod used before—but now at rest—is also a length that we can call the "length of the rod."

Excerpt 4: Measurement using clocks (operation b.)

It is important to emphasize that, for now, we are exclusively focused on measuring the length of a rigid rod in relative motion. This measurement is considered first from the perspective of the moving system k (represented in blue) and then from the stationary system K (represented in grey). At this stage, we are not addressing the measurement of the length of any light beam. Additionally, it must be stressed that the measurement of the moving rod from the stationary system K occurs within a single instant - essentially as though the two clocks were functioning as stopwatches. According to the principles of Special Relativity, this corresponds to measuring the rod using a scale "at a specific moment in time, t" (i.e., instantaneously). The clocks involved in this measurement are highlighted in red for clarity.

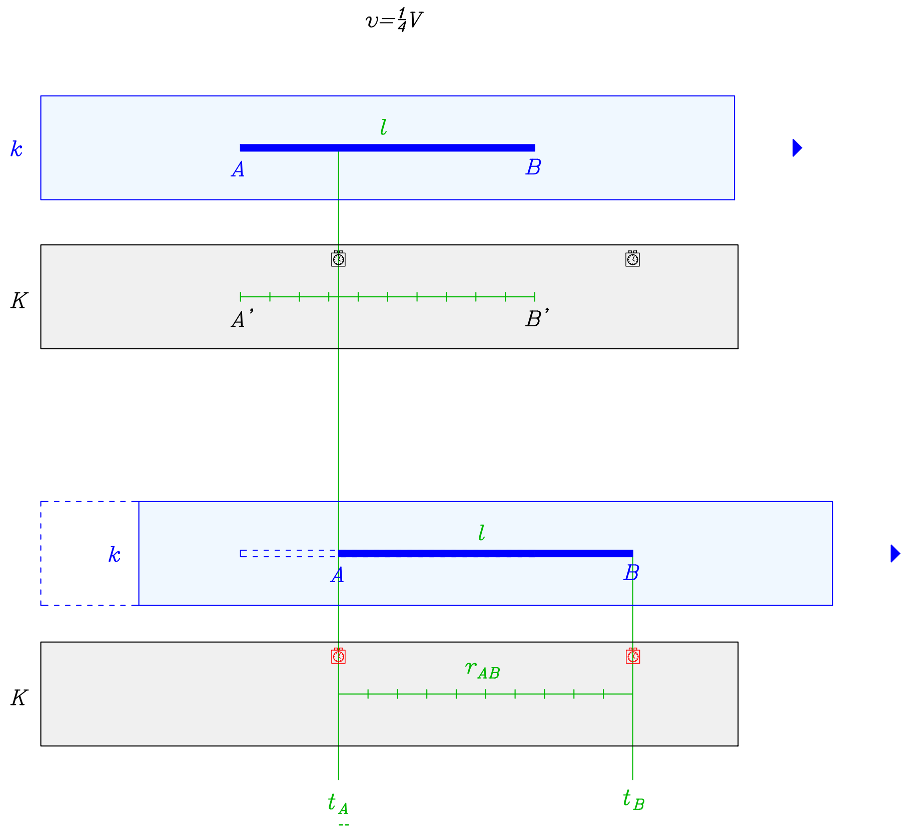
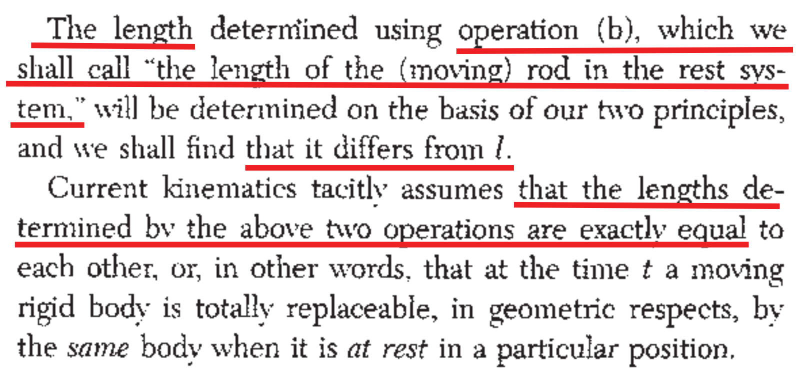


Fig. 4: Measurement using clocks in stationary system (operation b.)

SR claims that the two methods yield different measurements for the same rod and asserts that classical physics assumes equality between them. However, we argue that classical physics acknowledges the difference, whereas SR's claims equality.


The length determined using operation (b), which we shall call "the length of the (moving) rod in the rest system," will be determined on the basis of our two principles, and we shall find that it differs from *l*.

Current kinematics tacitly assumes that the lengths determined by the above two operations are exactly equal to each other, or, in other words, that at the time *t* a moving rigid body is totally replaceable, in geometric respects, by the *same* body when it is *at rest* in a particular position.

Excerpt 5: Differing lengths in moving system

4. Non- Simultaneity within two reference systems moving relative to each other

Now SR introduces a light beam traveling along a moving rod toward a mirror, where it is reflected back:

> Further, we imagine the two ends (A and B) of the rod equipped with clocks that are synchronous with the clocks of the rest system, i.e., whose readings always correspond to the "time of the system at rest" at the locations the clocks happen to occupy; hence, these clocks are "synchronous in the rest system."
>
> We further imagine that each clock has an observer co-moving with it, and that these observers apply to the two clocks the criterion for the synchronous rate of two clocks formulated in section 1. Let a ray of light start out from A at time[2] t_A; it is reflected from B at time t_B, and arrives back at A at time t'_A. Taking into account the principle of the constancy of the velocity of light, we find that
>
> $$t_B - t_A = \frac{r_{AB}}{V - v}$$

Excerpt 6: Measurement using clocks in moving system

The time intervals $\Delta t_1\ (t_B - t_A)$ and $\Delta t_2\ (t_A' - t_B)$ are computed strictly according to classic speed addition (rather than on the assumption of "the principle of the constancy of the velocity of light"), as shown by the following visualization.

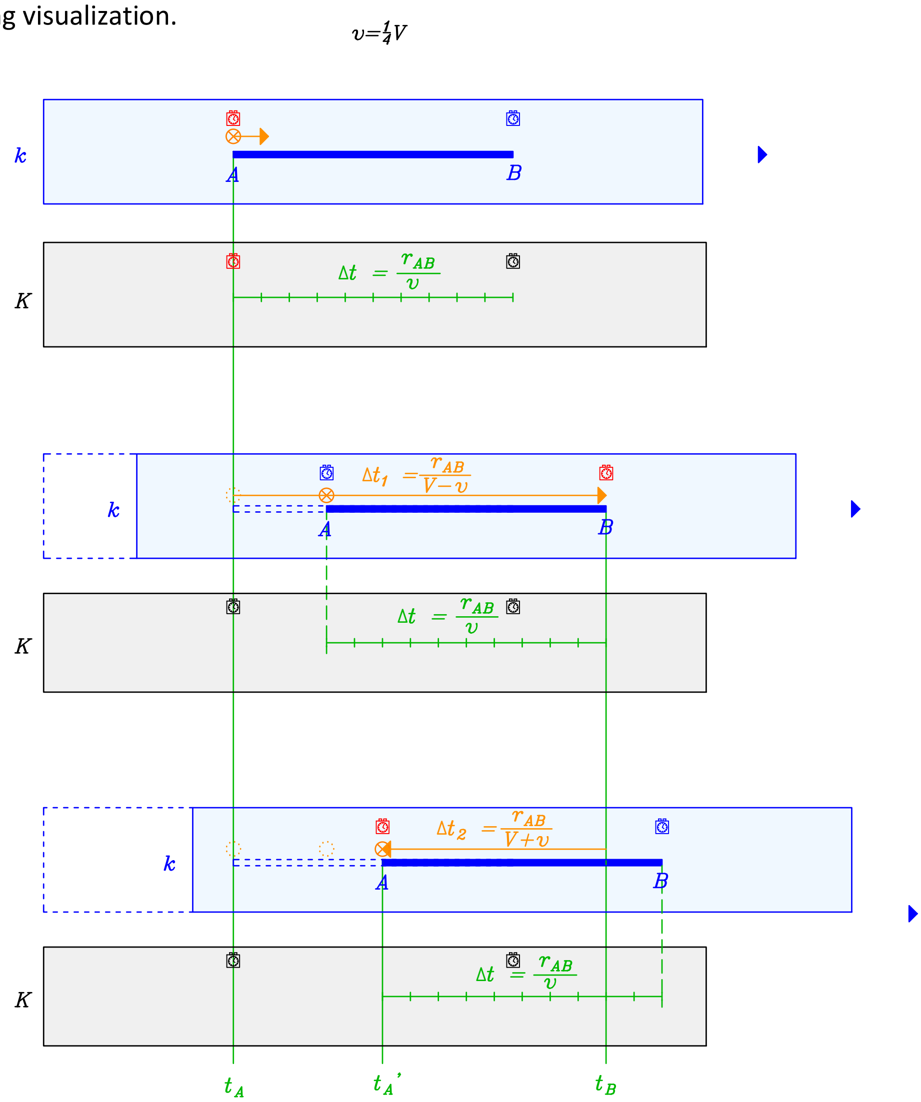


Fig. 5: Measurement using clocks in moving system

The light beam (shown in orange) is located in the moving system k (depicted in blue). Its traveling distance is measured using classical velocity addition: the beam travels from point A to the right at speed V, while the rod (in blue) moves to the right at speed v stationary with the moving system. According to both classical physics and Special Relativity, the motion of the source is irrelevant to the behavior of the light beam.

The beam eventually strikes the mirror located on the moving rod at point B. At this stage, Special Relativity has not yet specified how the measurement is performed in the stationary frame at this same instant. For clarity, we illustrated the stationary system K along with its measuring scale.

According to SR, the time intervals $\Delta t_1 (t_B - t_A)$ (outward) and $\Delta t_2 (t_A' - t_B)$ (return) differ due to relative motion. This contradicts the synchronization principle outlined earlier, effectively breaking simultaneity according to SR.

and

$$t'_A - t_B = \frac{r_{AB}}{V + v},$$

where r_{AB} denotes the length of the moving rod, measured in the rest system. Observers co-moving with the rod would thus find that the two clocks do not run synchronously, while observers in the system at rest would declare them to be running synchronously.

Thus we see that we cannot ascribe *absolute* meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system.

Excerpt 7: Violation of simultaneity

5. Critic

We now analyze the mathematical framework to identify the underlying issue. For Excerpt 1, we have:

$$t_B - t_A = t_A' - t_B \qquad (1)$$

From Excerpts 6 and 7, we also have:

$$t_B - t_A = \frac{r_{AB}}{(V - v)}$$

and

$$t_A' - t_B = \frac{r_{AB}}{(V + v)} \qquad (2)$$

Now it is suggesting itself to do a simple operation, we insert the two formulae from (2) into (1):

$$t_B - t_A = t_A' - t_B$$

$$\Rightarrow \frac{r_{AB}}{(V - v)} = \frac{r_{AB}}{(V + v)}$$

$$\Rightarrow (V - v) = (V + v)$$

The above could be true only if light speed were infinite. In any other case we obtain:

$$\boxed{v = -v}$$

This already gives us a glimpse into the root of the problem. Let us now clarify how this situation arises.

First, we have introduced additional clocks into the stationary system (denoted as A', B', C', D'), synchronized according to Figure 2, and placed these clocks symmetrically in both systems.

Second, we address a fundamental error: comparing <u>the traveling distance of a light beam</u> within the moving system with <u>the measured length of a rigid rod</u> in the stationary system. This mistake arises because Special Relativity mixes two fundamentally different types of events: an instantaneous event (measuring a length l at a specific time t) and a periodic event (measuring the distance traveled by a light beam over a time period $t_B - t_A$). These are categorically different events and cannot be directly equated.

Instead, we focus on the question of how the same light beam, originating within the moving system k, is measured in both the moving system k and the stationary system K, which are in relative motion.

First Event:
The light beam emerges from the source at point A. In both systems, the clocks at A and A' show identical, simultaneous time readings.

Second Event:
The light beam strikes the mirror on the rod at point B. By this time, the rod has moved with system k. Again, synchronized clocks at B (in k) and C' (in K) show equal and simultaneous readings (keep in mind: all clocks were synchronized).

Third Event:
The light beam returns to the source at A. At this point, synchronized clocks at A (in k) and D' (in K) show equal and simultaneous readings.

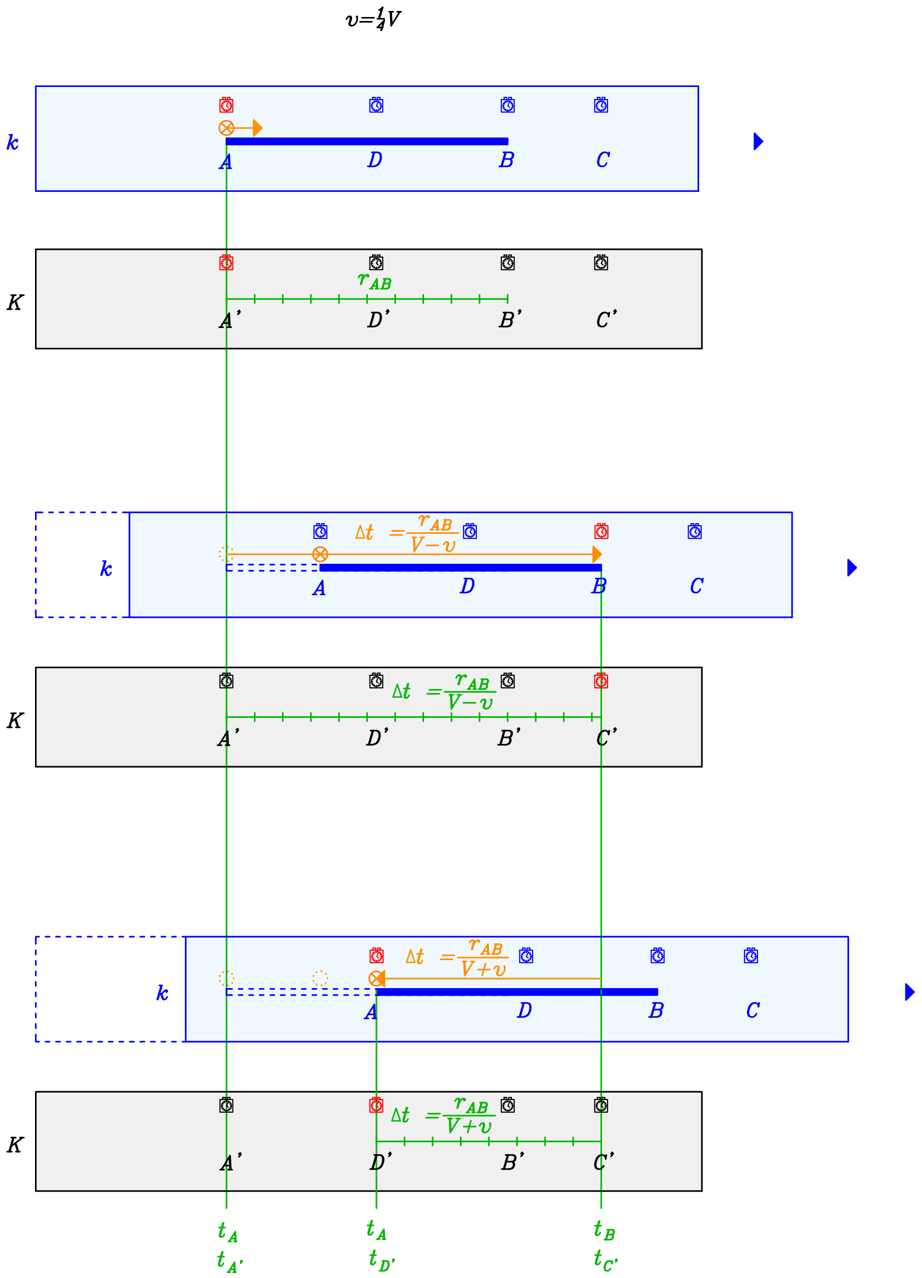


Fig. 6: Scheme with correctly measured light beam (with clocks) in two systems in relative motion

Notably, the distances covered by the light beam are identical when measured in both systems: the orange beam represents the measurement in k, while the green scale represents the measurement in K. For clarity, we have highlighted in red the clocks involved in the measurements, all of which are synchronized according to Special Relativity's rules, noting the corresponding simultaneous events in each reference system.

5. Conclusion

The following conclusions can be drawn:

a) All clocks are synchronized.
b) Identical events occur simultaneously in both systems at identical locations.
c) Clock readings in both reference frames yield identical light beam lengths.
d) The speed of light remains constant and equal in both frames.
e) Both reference systems are equivalent.

SR’s claim of non-simultaneity arises from relating distinct events, not from the nature of simultaneity itself. These findings challenge the foundational assertions of SR, suggesting the need for a critical reassessment of its conclusions.

References and Acknowledgments:

[1] Einstein, Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik und Chemie. 17, 1905, S. 891–921, 1905.