Abstract

This paper will reveal a fundamental logical error of Special Relativity. It will be shown, strictly deducting from Special Relativity’s own concept, that lengths being measured with clocks do not differ between two systems in relative motion to each other. Furthermore, it is shown that Special Relativity de facto puts two different events into relation and concludes from this the loss of simultaneity.

1. Introduction

Most laymen, but also more and more experts who are confronted with the ideas of Special Relativity, have a faint feeling of lacking of logical reason about the basic structure of theory. In particular, the claim of invariance of the speed of light and equivalence of the relative uniformity of motion are under suspect. Although mathematics and the limited ability of abstraction make it difficult to fully understand the theory of special relativity, we will point out the fundamental fallacy right at the beginning. We will strictly follow Albert Einstein’s original paper and try to visualize the mathematical assumptions for clarity.
2. Clock Synchronisation

We first cite from “On the Electrodynamics of Moving Bodies”, Chapter A,1.: If there is a clock at point A in space, then an observer located at A can evaluate the time of events in the immediate vicinity of A by finding the positions of the hands of the clock that are simultaneous with these events. If there is another clock at point B that in all respects resembles the one at A, then the time of events in the immediate vicinity of B can be evaluated by an observer at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation. So far we have defined only an “A-time” and a “B-time,” but not a common “time” for A and B. The latter can now be determined by establishing by definition that the “time” required for light to travel from A to B is equal to the “time” it requires to travel from B to A. For suppose a ray of light leaves from A for B at “A-time” \( t_A \), is reflected from B toward A at “B-time” \( t_B \), and arrives back at A at “A-time” \( t'_A \). The two clocks are synchronous by definition if

\[ t_B = t_A = t'_A - t_B. \]

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily

Excerpt 1: Synchronization of two clocks

Now we visualize the principle of synchronization of two clocks in \( A \) and \( B \) in a stationary reference system, strictly based on the description given by Special Relativity. To be conform to the text we chose to denominate the system at rest with \( S' \). Light beam is orange, the rod is black, the involved clocks are highlighted in red:

![Fig. 1: Synchronization of two clocks](image)
Again we cite Special Relativity regarding synchronization of additional clocks:

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many points, and therefore that the following relations are generally valid:

1. If the clock at $B$ runs synchronously with the clock at $A$, the clock at $A$ runs synchronously with the clock at $B$.
2. If the clock at $A$ runs synchronously with the clock at $B$ as well as with the clock at $C$, then the clocks at $B$ and $C$ also run synchronously relative to each other.

Excerpt 2: Synchronisation of two more clocks

We now imagine to have two more clocks at $C$ and $D$ and we again synchronize such clocks with $B$ and therefore according to Special Relativity also with $A$. Positions of $C$ and $D$ appear random and we will show its purpose later:

Strictly according to Special Relativity we now have four synchronized clocks in $A$, $B$, $C$ and $D$. 
3. Length Measurement Methods

Special Relativity is proposing two different ways of length measurement. The first method is to simply have a scale and measure the length $l$ of a rod from within any moving reference system $S$:

**Excerpt 3: Measurement using scale (operation a.)**

Again the visualization for this, measurement of the rod length $l$ from inside the moving system $S$ (bluish).

The rod is blue, the measuring scale green:

$$v = \frac{1}{2}v$$

**Fig. 3: Measurement using scale (operation a.)**

The second method is measuring the length of the rod $\tau_{AB}$ by means of the position of the two ends of the rod at a given time $t$, to be measured with synchronized clocks from a stationary system $S'$:

**Excerpt 4: Measurement using clocks (operation b.)**

Using clocks at rest and synchronous in the rest system as outlined in section 2, the observer determines at which points of the rest system the beginning and end of the rod to be measured are located at some given time $t$. The distance between these two points, measured with the rod used before—but now at rest—is also a length that we can call the "length of the rod."
It has to be stressed that for now we still only deal with measuring of a rigid rod that is in relative motion, once being measured from within the moving system \( S \) (bluish), once from within the stationary system \( S' \) (grey). We do not deal with measurement of the length of any light beam. Also it must be emphasized that measurement of the moving rod from the stationary system \( S' \) takes place within one instant, as if the two clocks were stopwatches (measurement of the rod with a scale according to Special Relativity “at some given time \( t \), i.e. instant). The clocks being involved with the measurement are highlighted in red:

![Diagram](image)

Now Special Relativity announces that it will show the two ways of measurement giving different lengths for the rod. Also it is claiming that classic physics assume both measurement operations must be equal. We will prove the opposite, that according to classic physics it is not equal, but according to Special Relativity.

The \textit{length} determined using operation (b), which we shall call “the length of the (moving) rod in the rest system,” will be determined on the basis of our two principles, and we shall find that it differs from \( l \).

Current kinematics \textit{tacitly assumes} that the \textit{lengths determined by the above two operations are exactly equal} to each other, or, in other words, that at the time \( t \) a moving rigid body is totally replaceable, in geometric respects, by the \textit{same} body when it is \textit{at rest} in a particular position.
4. Non-Simultaneity within two reference systems moving relative to each other

Now Special Relativity introduces a light beam that is traveling along the moving rod towards a mirror from which it is being reflected back.

Further, we imagine the two ends (A and B) of the rod equipped with clocks that are synchronous with the clocks of the rest system, i.e., whose readings always correspond to the “time of the system at rest” at the locations the clocks happen to occupy; hence, these clocks are “synchronous in the rest system.”

We further imagine that each clock has an observer co-moving with it, and that these observers apply to the two clocks the criterion for the synchronous rate of two clocks formulated in section 1. Let a ray of light start out from A at time \( t_A \); it is reflected from B at time \( t_B \), and arrives back at A at time \( t'_A \). Taking into account the principle of the constancy of the velocity of light, we find that

\[
  t_B - t_A = \frac{r_{AB}}{V - u}.
\]

Excerpt 6: Measurement using clocks in moving system

The difference \( \Delta t_1 \) (\( t_B - t_A \)) and \( \Delta t_2 \) (\( t'_A - t_B \)) is computed strictly according to classic speed addition (rather than on the assumption of “the principle of the constancy of the velocity of light”), as shown by the following visualization.

The light beam (orange) is within the moving system \( S \) (bluish), the traveling length of the beam is being measured by classic speed addition, i.e. light travels from A to the right with speed \( V \) while the rod (blue) is traveling to the right with speed \( u \) inside the moving system. According to classic physics (and to Special Relativity) movement of the source is irrelevant.

The beam then hits the mirror on the moving rod in \( B \). Special Relativity has not mentioned at this how the measurement is done at the stationary frame at this same instant, and we show the stationary system with its measuring scale for clarification.
Fig. 5: Measurement using clocks in moving system
Now Special Relativity claims that $\Delta t_1 \ (t_B - t_A)$ in one direction is different from $\Delta t_2 \ (t_A' - t_B)$ in the opposite direction, hence the principle outlined in excerpt 1 and fig. 1 is violated and thus synchronicity of clocks is broken.

and

$$t_A' - t_B \approx \frac{r_{AB}}{V + v}.$$

where $r_{AB}$ denotes the length of the moving rod, measured in the rest system. Observers co-moving with the rod would thus find that the two clocks do not run synchronously, while observers in the system at rest would declare them to be running synchronously.

Thus we see that we cannot ascribe absolute meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system.

Excerpt 7: Violation of simultaneity

5. Critic

At first we have to have a closer look to the mathematic scheme, in order to see what is being herewith presented to us. As for excerpt 1 we have:

$$t_B - t_A = t_A' - t_B \quad (1)$$

And as per excerpt 6 and 7 we have:

$$t_B - t_A = \frac{-r_{AB}}{(V - v)}$$

and

$$t_A' - t_B = \frac{r_{AB}}{(V + v)} \quad (2)$$

Now it is suggesting itself to do a simple operation, we insert the two formulae from (2) into (1):

$$t_B - t_A = t_A' - t_B$$

$$\Rightarrow \frac{r_{AB}}{(V - v)} = \frac{r_{AB}}{(V + v)}$$

$$\Rightarrow (V - v) = (V + v)$$

The above could be true only if light speed were infinite. In any other case we obtain:
This already gives us a faint glimpse of the problem’s root.

We clarify now, how such could happen. At first, we have added the two additional clocks to the stationary system (thus $A'$, $B'$, $C'$, $D'$) that we have already synchronized according to fig. 2, and we have added them equally on both systems.

Secondly we get rid of the basic mistake that is to relate the traveling length of one light beam within the moving system against the measured length of a rigid rod within the stationary system, i.e. we get rid of the fact that Special Relativity does mix an instantaneous event (measuring a length $l$ at one given time $t$) with a period event (measuring the length of a light beam in one period $t_{B'-A}$), i.e. by no means the same events.

Instead, let us examine the question of how one and the same light beam within the moving system $S$ is actually measured in both systems ($S$ and $S'$) that are in relative motion to each other.

The first event is the emergence of the beam from the source in $A$. In both systems clocks in $A$ and $A'$ give equal and simultaneous readings. The second event is when the light beam hits the mirror on the rod in $B$ while the rod has moved with system $S'$, and again we get equal and simultaneous readings, but this time from clocks $B$ and $C'$ (keep in mind that all clocks are synchronized). At the third event the light beam hits back to the source in $A$ and we obtain equal and simultaneous readings this time from clocks in $A$ and $D'$. Obviously the distances that have been covered by the light beam are equal in both systems, represented by the orange beam in $S$ and the green scale in $S'$. We have highlighted in red such clocks (being all synchronized according to Special Relativity’s rule) that are involved with measurement in each reference system at the respective simultaneous events.
Fig. 6: Scheme with correctly measured light beam (with clocks) in two systems in relative motion

\[ v = \frac{1}{2} V \]
5. Conclusion

As we can easily see, we have now the following results:

1) All clocks are synchronized.

2) Clocks in $A$ and $A'$ are showing the same reading for the same event and instant and are on the very same locality. Likewise for clocks in $B$ and $C'$ and clocks in $A$ and $D'$ for their respective events, instances and localities.

3) Clock readings in both reference systems result in the same traveling length of the light beam.

4) Equal events and instants take place simultaneously and at the same locality.

5) Light speed is equal and constant in both reference systems.

6) Both reference systems are equivalent.

7) Simultaneous events are observed as simultaneous events in both reference systems.

We have shown that Special Relativity de facto puts two different events into relation, which leads to the claim that simultaneity does not exist within two systems in uniform relative motion. All further conclusions of Special Relativity must be put on the touchstone.

References and Acknowledgments: