On The Proving Method of Fermat's Last Theorem
Haofeng Zhang
Beijing, China

Abstract: In this paper the author gives an elementary mathematics method to solve Fermat's Last Theorem (FLT), in which let this equation become an one unknown number equation, in order to solve this equation the author invented a method called “Order reducing method for equations”, where the second order root compares to one order root, and with some necessary techniques the author successfully proved when \(x^{(n-1)}+y^{(n-1)}-z^{(n-1)} \leq x^{(n-2)}+y^{(n-2)}-z^{(n-2)}\) there are no positive solutions for this equation, and also proves with the increasing of \(x\) there are still no positive integer solutions for this equation when \(x^{(n-1)}+y^{(n-1)}-z^{(n-1)} \leq x^{(n-2)}+y^{(n-2)}-z^{(n-2)}\) is not satisfied.

1. Some Relevant Theorems

There are some theorems for proving or need to be known. All symbols in this paper represent positive integers unless they are stated to be not.

Theorem 1.1. In the equation of
\[
\begin{align*}
\left| x^n + y^n = z^n \\
gcd(x, y, z) = 1 \\
n > 2
\end{align*}
\] 
\[ (1-1) \]
x, y, z meet
\(x \neq y\),
\(x + y > z\),
and if
\(x > y\)
then
\(z > x > y\).

Proof: Let
\(x = y\),
we have
\[ 2x^n = z^n \]
and
\[ \sqrt[3]{2}x = z \]
where \(\sqrt[3]{2}\) is not an integer and \(x, z\) are all positive integers, so \(x \neq y\).

Since
\[ (x + y)^n = x^n + C_n^1 x^{n-1} y + \ldots + C_n^{n-1} x y^{n-1} + y^n > z^n, \]
so we get
\[ x + y > z. \]

Since
\[ x^n + y^n = z^n, \]
so we have
\[ z^n > x^n, z^n > y^n \]
and get
\[ z > x > y \]
when
\[ x > y. \]

**Theorem 1.2.** In the equation of (1-1), \( x, y, z \) meet

\[ \gcd(x, y) = \gcd(y, z) = \gcd(x, z) = 1. \]

**Proof:** Since \( x^n + y^n = z^n \), if \( \gcd(x, y) > 1 \) then we have \( \left(x^n + y^n\right) \times \left[\gcd(x, y)\right]^n = z^n \)
which causes \( \gcd(x, y, z) > 1 \) since the left side contains the factor of \( \left[\gcd(x, y)\right]^n \) then the right side must also contain this factor but contradicts against (1-1) in which \( \gcd(x, y, z) = 1 \), so we have \( \gcd(x, y) = 1 \). Using the same way we have \( \gcd(x, z) = \gcd(y, z) = 1 \).

**Theorem 1.3.** If there is no positive integer solution for
\[ x^p + y^p = z^p \]
when \( p > 2 \) is a prime number then there is also no positive integer solution for
\[ \left(x^p\right)^k + \left(y^p\right)^k = \left(z^p\right)^k. \]

**Proof:** Since \( x^p + y^p = z^p \) has no positive integer solution, so there still no positive integer solution for
\[ \left(x^p\right)^k + \left(y^p\right)^k = \left(z^p\right)^k \]
which means there is also no positive integer solution for
\[ \left(x^p\right)^k + \left(y^p\right)^k = \left(z^p\right)^k. \]
So we only need to prove there is no positive integer solution for equation (1-1) when \( n \) is a prime number.

**Theorem 1.4.** There are no positive integer solutions for equation (1-1) when \( x \) or \( y \) is a
prime number.

**Proof:** When \( x \) is a prime number, since
\[
x^n = z^n - y^n = (z - y)(z^{n-1} + z^{n-2}y + ... + y^{n-2} + y^{n-1}),
\]
so we have
\[
\gcd(z - y, x) = x,
\]
which means
\[z - y \geq x,\]
we have
\[x + y \leq z,
\]
that contradicts against **Theorem 1.1** in which \( x + y > z \), so it is with \( y \), which means there are no positive integer solutions for equation (1-1) when \( x \) or \( y \) is a prime number.

**Theorem 1.5.** There are no positive integer solutions for equation (1-1) when \( z \) is a prime number.

**Proof:** When \( z \) is a prime number, from Theorem 1.12 we only consider the case of \( n > 2 \) is a prime number, since
\[
x^n + y^n = z^n = (x + y)(x^{n-1} + ... + y^{n-1}),
\]
so we have
\[
\gcd(x + y, z) = z,
\]
from **Theorem 1.1** we know \( x + y > z \), so we get
\[x + y \geq 2z,
\]
that contradicts against **Theorem 1.1** in which \( z > x > y \Rightarrow x + y < 2z \), which means there are no positive integer solutions for equation (1-1) when \( z \) is a prime number.

**Theorem 1.6.** There are no positive integer solutions for
\[1^n + y^n = z^n.
\]

**Proof:** Since
\[1 = z^n - y^n = (z - y)(z^{n-1} + z^{n-2}y + ... + y^{n-2} + y^{n-1}),
\]
where
\[
\begin{align*}
z - y &= 1 \\
(z^{n-1} + z^{n-2}y + ... + y^{n-2} + y^{n-1}) &= 1
\end{align*}
\]
that causes \( z, y \) to be non positive integers, so there are no positive integer solutions for
\[1^n + y^n = z^n.
\]
**Theorem 1.7.** There are no positive integer solutions for 

\[ 2^n + y^n = z^n. \]

**Proof:** Since

\[ 2^n = z^n - y^n = (z - y) \left( z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} \right), \]

if

\[
\begin{align*}
&z - y = 1 \\
&z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^n
\end{align*}
\]

then taking the least value for \( y = 2, z = 3 \), we have

\[ 3^{n-1} + 2 \times 3^{n-2} + \ldots + 2^{n-1} > 2^n \]

when \( n > 2 \) that is impossible. If

\[
\begin{align*}
&z - y = 2^j \\
&z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^j \\
&i + j = n \\
&i \geq 1
\end{align*}
\]

then \( z > 2 \) and taking the least value of \( y = 2, z = 3 \), we get

\[ 3^{n-1} + 2 \times 3^{n-2} + \ldots + 2^{n-1} > 2^j \]

with \( n > 2 \) that is also impossible, so there are no positive integer solutions for

\[ 2^n + y^n = z^n. \]

**Theorem 1.8.** There are no positive integer solutions for equation (1-1) when \( n \to \infty \) and \( x, y, z \) in equation (1-1) meet

\[ \sqrt[4]{2}y < z < \sqrt[4]{2}x, x > 2, y > 1, z > 3. \]

**Proof:** Since \( x^n + y^n = z^n \), let \( x > y \), we get

\[ \left( \frac{z}{x} \right)^n - \left( \frac{y}{x} \right)^n = 1, \]

since

\[ z > x > y, \]

so we have

\[ z < \sqrt[4]{2}x, \]

and
\[ \lim_{n \to \infty} \left( \frac{z^n}{x} \right) - \left( \frac{y^n}{x} \right) = \infty > 1 \]

which means there are no positive integer solutions for equation (1-1) when \( n \to \infty \).

According to Theorem 1.6, 1.7 we have \( x > 2, y > 1, z > 3 \).

If \( y^n \geq \frac{z^n}{2} \), since \( x > y \), so we have \( x^n \geq \frac{z^n}{2} \),and

\[ x^n + y^n > z^n, \]

that is impossible. So we have

\[ \sqrt[3]{2}y < z. \]

**Theorem 1.9.** There are no positive integer solutions for equation (1-1) when \( x, y, z \leq 100 \).

**Proof:** From Theorem 1.8, we know \( \sqrt[3]{2}y < z < \sqrt[3]{2}x \), so we have

\[ y < \frac{100}{\sqrt[3]{2}} < x, \]

when \( n = 3 \), we have the smallest values for \( x \), so we get

\[ \left( y < \frac{100}{\sqrt[3]{2}} < x \right) \Rightarrow (y < 79 < x), \]

from Theorem 1.4, 1.5 we know \( x, y, z \) are all not prime numbers. There are below combinations of \( x, y, z \) when \( x, y, z \leq 100 \):

\[
\begin{cases}
(x = 80 \sim 99)^y + (y = 4 \sim (x-1))^y = (z = 81 \sim 100)^y \\
x + y > z \\
x^2 + y^2 > z^2 \\
x^j + y^j > z^j \\
j < n
\end{cases}
\]

Here we take \( 7^a + 9^a = 10^a \) for example to explain how to prove. We plot the graph for this equation as showed in Figure 1-1.
Figure 1-1  Graph of $f(n) = 7^n + 9^n - 10^n$

Obviously for equation $f(n) = 7^n + 9^n - 10^n$ in Figure 1-1, we have $3 < n < 4$ is not an integer so there are no positive integer solutions, using this method we have the conclusion of there are no positive integer solutions for equation (1-1) when $z \leq 100$.

Using the method of which we prove Theorem 1.6, 1.7 we can prove when $x, y \leq 100$, there are no positive integer solutions for equation (1-1).

**Theorem 1.10.** In the equation of (1-1), $x, y, z$ meet

\[ x^{n-i} + y^{n-i} > z^{n-i}, \]

\[ x^{n+i} + y^{n+i} < z^{n+i}, \]

where

\[ n > i \geq 1. \]

This theorem holds true when $x, y, z$ are positive real numbers but $n$ must be a positive integer.

**Proof:** From equation (1-1), since

\[ x^n + y^n = z^n, \]

from Theorem 1.1, since $z > x > y$, we have

\[ x^{n-i} + y^{n-i} > \left( \frac{x}{z} \right)^i x^{n-i} + \left( \frac{y}{z} \right)^i y^{n-i} = z^{n-i}, \]

\[ x^{n+i} + y^{n+i} < \left( z^{n-i} + z^i \right) y^{n-i} = z^{n+i}, \]

so we have

\[ x^{n-i} + y^{n-i} > z^{n-i}. \]
This theorem means given \(x, y, z\) if equation (1-1) has one positive integer solution then this solution is the only one.

**Theorem 1.11.** There are no positive integer solutions for equation (1-1) when
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1.
\]

And in order to have positive integer solutions for equation (1-1),
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 40
\]
must be satisfied.

**Proof:** In equation (1-1), let
\[
\begin{align*}
    a &= x^{n-2}, \\
    b &= y^{n-2}, \\
    c &= z^{n-2}
\end{align*}
\]
we have
\[
\begin{align*}
    ax^2 + by^2 &= cz^2, \\
    a^{n-2}x + b^{n-2}y &= c^{n-2}z
\end{align*}
\]
Since we reduce the order of equation so the method is called “Order reducing method for equations”. Let \(x > y\) and
\[
\begin{align*}
    y &= x - f, \\
    z &= x + e
\end{align*}
\]
we have
\[
\begin{align*}
    ax^2 + b(x - f)^2 &= c(x + e)^2, \\
    a^{n-2}x + b^{n-2}(x - f) &= c^{n-2}(x + e)
\end{align*}
\]
and
\[
\begin{align*}
    \left( a + b - c \right) x^2 - 2(bf + ce)x + \left( bf^2 - ce^2 \right) &= 0, \\
    a^{n-2}x + b^{n-2}(x - f) - c^{n-2}(x + e) &= 0
\end{align*}
\]
the roots are
\[
x = \frac{(bf + ce) \pm \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}, \quad (1-2)
\]
and
\[x = \frac{c^{n-1}e + b^{n-2}f}{a^{n-1} + b^{n-2} - c^{n-2}} = \frac{bfy + cez}{x^{n-1} + y^{n-2} - z^{n-2}}. \tag{1-3}\]

There are two cases for \(bf^2, ce^2\). When \(bf^2 \geq ce^2\) and \(bf^2 < ce^2\).

**Case A:** If \(bf^2 \geq ce^2\), from (1-2) when

\[
x = \frac{(bf + ce) + \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

from **Theorem 1.10** we know \(a + b - c = x^{n-2} + y^{n-2} - z^{n-2} > 0\), so we have

\[
x \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

also from **Theorem 1.10** we have \(x^{n-1} + y^{n-1} - z^{n-1} > 0\), compare to (1-3) we get

\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{2(bf + ce)}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]

When \(\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1\), we have

\[
bfy + cez \leq 2(bf + ce)
\]

that is impossible since from **Theorem 1.8** we know \(y \geq 2\) and \(z > 3\).

When

\[
x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

we have

\[
x \leq \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}},
\]

compare to (1-3) we get

\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]

When \(\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1\), we have
\[ bfy + cez \leq bf + ce \]

that is impossible since from **Theorem 1.8** we have already known \( y \geq 2 \) and \( z > 3 \).

**Case B:** If \( bf^2 < ce^2 \), from (1-2) when
\[
 x = \frac{(bf + ce) + \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-1} + y^{n-2} - z^{n-2}},
\]
we can prove \( (bf + ce)^2 > (a + b - c)(ce^2 - bf^2) \) since if not we have
\[
(bf + ce)^2 \leq (a + b - c)(ce^2 - bf^2)
\]
and
\[
[(2b + a) - c]bf^2 + 2bfce + [2c - (a + b)]ce^2 \leq 0
\]
that is impossible since \( a + b - c > 0 \) and \( c > a, c > b, 2c - (a + b) > 0 \). So we have
\[
x < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-1} + y^{n-2} - z^{n-2}}
\]
compare to (2-4) we get
\[
\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-2} + y^{n-2} - z^{n-2}}.
\]
When \( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1 \), we have
\[
bfy + cez < (bf + ce)(1 + \sqrt{2}) < 2.5(bf + ce)
\]
and
\[
bf(x - f) + ce(x + e) < 2.5(bf + ce)
\]
that leads to
\[
x < \left[ \frac{2.5(bf + ce) + bf^2 - ce^2}{bf + ce} = 2.5 - \frac{ce^2 - bf^2}{bf + ce} \right] < 2.5
\]
where possible values for \( x \) are 1, 2 but according to **Theorem 1.6, 1.7** we know there are no positive integer solutions.

When
\[
x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}
\]
is not possible since \( x \leq 0 \).

So we have the conclusion of there are no positive integer solutions for equation (1-1) when
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1.
\]

Obviously we have
\[
bfy + cez < 2.5 \left( \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \right) (bf + ce),
\]
from Theorem 1.9 we know \( x, y, z \leq 100 \) there are no positive integer solutions for equation (1-1), so we have
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 40,
\]
which must be satisfied to have positive integer solutions for equation (1-1).

2. Proving Method

From Theorem 1.11 we know in order to have positive integer solutions for this equation,
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1
\]
must be satisfied. We give the graph of this equation as showed in Figure 2-1 when
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1,
\]
where \( AB // CD' \).
Figure 2-1  Graph of \( x^n + y^n = z^n \) when \( \frac{x^{n-2} + y^{n-2} - z^{n-2}}{x^{n-1} + y^{n-1} - z^{n-1}} > 1 \)

1. In Figure 2-1 we have
\[
\angle CDE = 360^\circ - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right) - 90^\circ ,
\]

obviously we have
\[
BD = x^{n-1} + y^{n-1} - z^{n-1},
AC = x^{n-2} + y^{n-2} - z^{n-2},
\]
when \( \frac{BD}{AC} > 1 \) we have
\[
\angle ABE - \angle CDE = \angle D'CD + \angle BED > 0 ,
\]
which means
\[
\angle ABE > \angle CDE .
\]
It is also very clear that if \( \angle ABE \leq \angle CDE \) then \( \frac{BD}{AC} < 1 . \)

From Theorem 1.9 we know if \( z \leq 100 \) then there are no positive integer solutions for equation (1-1), when \( n = 3 \) (which is the worst case) we have
\[
\angle CDE = 270^\circ - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right)
= 270^\circ - \arctan\left(100^3 - 100^2\right) - \arctan\left(\frac{1}{100^2 - 100}\right) > 179.99^\circ ,
\]
and
\[
\angle ABE > \angle CDE > 179.99^\circ ,
\]
which means \( \angle ABE, \angle CDE \rightarrow 180^\circ \), so \( ABE, CDE \) are almost lines with \( z > 100, n \geq 3 \),
that leads to \( \frac{BD}{AC} \rightarrow \frac{1}{2} < 1 \), this contradicts against \( BD > AC \). So when \( z \) is large enough
then
\[
\frac{BD}{AC} = \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} < 1 ,
\]
from Theorem 1.11 we know there are no positive integer solutions for equation (1-1).

2. For function
\[
f(z) = \angle CDE = 270^\circ - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right)
= \frac{3}{2} \pi - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right).
\]
we give the function plot for it in Figure 2-2.

\[
\text{Graph of } f(z) = \angle CDE = 270^0 - \arctan\left(\frac{z^n - z^{n-1}}{1}\right) - \arctan\left(\frac{1}{z^{n-1} - z^{n-2}}\right)
\]

where we take \( \pi = 3.1415926535897932 \)

Obviously \( f(z) = \angle CDE \) is a “Monotonically increasing function” when \( z \geq 3 \), and with the increasing of \( z \) the value of \( f(z) = \angle CDE \) is close to \( 180^0 \). It is very clear that \( \angle ABE - \angle CDE \) is decreasing with the increasing of \( z \), since

\[
(\angle ABE - \angle CDE = \angle D'CD + \angle BED) < 180^0 - \angle CDE,
\]

where \( \angle CDE \) is increasing. When \( n = 3 \) since \( \angle CDE > 179.99^0 \), so we have

\[
(\angle D'CD + \angle BED) < 180^0 - \angle CDE < 180^0 - 179.99^0 < 0.01^0,
\]

which means

\[
\angle BED, \angle D'CD < 0.01^0,
\]

and when \( z \) or \( n \) is large enough, we have

\[
\angle ABE - \angle CDE = (\angle BED + \angle D'CD) \to 0,
\]

which means \( BD < AC \) that contradicts against \( BD > AC \). So when \( z \) or \( n \) is large enough then

\[
\frac{BD}{AC} = \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} < 1,
\]

from Theorem 1.11 we know there are no positive integer solutions for equation (1-1).
3. In Figure 2-1 we have

\[ \angle ABE = \frac{3}{2} \pi - \arctan \left( \frac{x^n + y^n - x^{n-1} - y^{n-1}}{1} \right) - \arctan \left( \frac{1}{x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2}} \right), \]

\[ \angle CDE = \frac{3}{2} \pi - \arctan \left( \frac{z^n - z^{n-1}}{1} \right) - \arctan \left( \frac{1}{z^{n-1} - z^{n-2}} \right), \]

so

\[ \angle ABE - \angle CDE = \arctan \left( \frac{z^n - z^{n-1}}{1} \right) + \arctan \left( \frac{1}{z^{n-1} - z^{n-2}} \right) \]

\[ - \arctan \left( \frac{x^n + y^n - x^{n-1} - y^{n-1}}{1} \right) - \arctan \left( \frac{1}{x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2}} \right). \]

From (1-1) we have

\[ z = \left( x^n + y^n \right)^{\frac{1}{n}}, \]

we get

\[ \angle ABE - \angle CDE \]

\[ = \arctan \left( \frac{x^n + y^n - (x^n + y^n)^{\frac{n}{n-1}}}{1} \right) + \arctan \left( \frac{1}{(x^n + y^n)^{\frac{n}{n-1}} - (x^n + y^n)^{\frac{n-2}{n}}} \right) \]

\[ - \arctan \left( \frac{x^n + y^n - x^{n-1} - y^{n-1}}{1} \right) - \arctan \left( \frac{1}{x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2}} \right). \]

We give the plot of \( f(x, y) = \angle ABE - \angle CDE \) using Excel VBA program that showed below:

\[ n = 3 \]

\[ \text{For } x = 1 \text{ To } 10 \text{ \textasciitilde } 5 \text{ Step 1} \]

\[ \text{For } y = 1 \text{ To } x - 1 \text{ Step 1} \]

\[ z = (x \wedge n + y \wedge n) \wedge (1 / n) \]

\[ t1 = z \wedge n - z \wedge (n - 1) \]

\[ t2 = 1 / (z \wedge (n - 1) - z \wedge (n - 2)) \]

\[ t3 = (x \wedge n + y \wedge n) - (x \wedge (n - 1) + y \wedge (n - 1)) \]

\[ t4 = t3 / ((x \wedge (n - 1) + y \wedge (n - 1)) - x \wedge (n - 2) - y \wedge (n - 2)) \]

\[ \text{CDE} = \text{Application}.\text{Atan2}(t1, 1) - \text{Application}.\text{Atan2}(t2, 1) \]

\[ \text{ABE} = \text{Application}.\text{Atan2}(t3, 2) - \text{Application}.\text{Atan2}(t4, 2) \]

\[ R = \text{CDE} - \text{ABE} \]

\[ \text{Cells}(i, 1) = \text{"z=n"} \& z \]

\[ \text{Cells}(i, 2) = \text{"x=y"} \& x \]

\[ \text{Cells}(i, 3) = \text{"y=x"} \& y \]

\[ \text{Cells}(i, 4) = R \]

\[ i = i + 1 \]

\[ \text{If } i > 65536 \text{ Then End} \]

Next y
Figure 2-3 shows the results, obviously $f(x, y) = \angle ABE - \angle CDE, n = 3$ is decreasing.

Figure 2-3  Graph of $f(x, y) = \angle ABE - \angle CDE, n = 3$

4. In Figure 2-1 we have

\[ BD^2 = BE^2 + DE^2 - 2BE \times DE \times \cos(\angle BED) \]
\[ = \left( z^n - z^{n-2} \right)^2 + 1 + \left( x^n + y^n - x^{n-2} - y^{n-2} \right)^2 + 1 - 2\sqrt{\left( z^n - z^{n-2} \right)^2 + 1 \times \left( x^n + y^n - x^{n-2} - y^{n-2} \right)^2 + 1} \times \]
\[ \cos \left[ \arctan \left( \frac{1}{x^n + y^n - x^{n-2} - y^{n-2}} \right) - \arctan \left( \frac{1}{z^n - z^{n-2}} \right) \right] \]

and

\[ AC^2 = AE^2 + CE^2 - 2AE \times CE \times \cos(\angle AEC) \]
\[ = \left( z^n - z^{n-2} \right)^2 + 4 + \left( x^n + y^n - x^{n-2} - y^{n-2} \right)^2 + 4 - 2\sqrt{\left( z^n - z^{n-2} \right)^2 + 4 \times \left( x^n + y^n - x^{n-2} - y^{n-2} \right)^2 + 4} \times \]
\[ \cos \left[ \arctan \left( \frac{2}{x^n + y^n - x^{n-2} - y^{n-2}} \right) - \arctan \left( \frac{2}{z^n - z^{n-2}} \right) \right] \]

from (1-1) we have

\[ y = \left( z^n - x^n \right)^{\frac{1}{n}}. \]

We give the plot of $f(z, x) = \frac{BD}{AC}$ using Excel VBA program that showed below:

\[ n = 3 \]

For $z = 10^7$ To $10^9$ Step 1

For $x = z / (2^{(1/n)})$ To $z - 1$ Step 1

$y = (z^n - x^n) ^{(1/n)}$
\[ t_1 = z^n - z^{n-1} \]
\[ t_2 = x^n + y^n - x^{n-1} - y^{n-1} \]
\[ t_3 = z^n - z^{n-2} \]
\[ t_4 = x^n + y^n - x^{n-2} - y^{n-2} \]

\[ BD = (t_1^2 + t_2^2 + 2 - 2 \cdot \text{Sqr}((t_1^2 + 1) \cdot (t_2^2 + 1)) \cdot \cos(\text{Application.Atan2}(t_2, 1) - \text{Application.Atan2}(t_1, 1))) \]
\[ AC = (t_3^2 + t_4^2 + 8 - 2 \cdot \text{Sqr}((t_3^2 + 4) \cdot (t_4^2 + 4)) \cdot \cos(\text{Application.Atan2}(t_4, 2) - \text{Application.Atan2}(t_3, 2))) \]

\[ R = (BD / AC)^{0.5} \]

Cells(j, 1) = "z=" & z
Cells(j, 2) = "x=" & x
Cells(j, 3) = "y=" & y
Cells(j, 4) = R
j = j + 1
If j > 65536 Then End
Next z
Next x

We give the plot of \( f(z,x) = \frac{BD}{AC}, n = 3 \) when \( z = 10^7, x = \frac{z}{\sqrt{2}} \sim z, \ n = 3 \), it is showed in

**Figure 2-4.**

[Graph showing the plot of \( f(z,x) = \frac{BD}{AC}, n = 3 \)]

With the increasing of \( z, n \) the value of \( f(z,x) = \frac{BD}{AC} \) will be smaller, and we are sure of when \( z, n \to \infty \) or get larger, the conclusion holds. In fact even \( z = 10^6 \), we can still have a result of \( f(z,x) = \frac{BD}{AC} < 40 \).
3. Conclusion

In this paper we first prove there are no positive integer solutions for equation (1-1) when
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1,
\]
and then prove with the increasing of x the conclusion still holds when
\[
\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1 \quad \text{under the assumption of } z < 10^6 \text{ there are no positive integer solutions}
\]
for equation (1-1).