ON THE GRAVITATIONAL CONSTANT

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ABSTRACT

In 1687 Isaac Newton published “Philosophiæ Naturalis Principia Mathematica”[i], which established the three laws of motion and the law of Universal Gravitation which laid the groundwork for classical mechanics. The relationship between force, mass and distance inspired by Kepler’s earlier work gave rise to the value what is now more commonly referred to as the gravitational constant “*G*”. The value of this constant has since become a fundamental part of almost every cosmological theory to the current day. This paper analyzes the profound effect Newton had upon modern cosmology.

Keywords: cosmology: observations — methods: observational — supernovae: general

1. Introduction

A common misconception is that the equations of Newton do not contain a dependency on time. This view is borne from a naïve superficial understanding of the Gravitational constant itself. It is worthy of note, that Newton formulated his law of “Universal Gravitation” from the work of Kepler, which certainly takes into account a dependency on time. Throughout this paper, the dimension of time will not only be included, but attention will be drawn to the very nature of this parameter and why Newton deemed it necessary to include it in the first place. To recognize that time forms a part of Newton’s equations requires nothing more than the correct observation of the dimensions of his equations which clearly states the dimension of time in seconds;

$$f = \frac{GMm}{r^2} m^3 kg^{-1} s^{-2} \tag{1}$$

It is the intention of this paper to definitively prove that the Gravitational Constant “G” is not actually a natural constant but rather a representation of a more fundamental geometric construct and its subsequent effect on current cosmological theories. The terms and use of Newton’s equation are very well understood, with one small exception, being that of the Gravitational Constant “*G*”. From the outset as its name suggests, it is commonly believed to be a mysterious scalar value, a natural constant of nature with a value that can only be determined by experiment of $6.674 * 10^{-11}\ m^3 kg^{-1} s^{-2}$. This assumption would appear <u>not</u> to be the case as the dimensions of “G” itself, clearly contain components of time, distance and weight. Without doubt, this indicates that “G” cannot be a constant but rather a compound value, containing additional, not normally referenced information.

2. Basic Theory

In order to discover the actual contents of the gravitational constant, basic dimensional analysis can be performed on Newton’s equation;

$$[G] \cdot \left[\frac{Mm}{r^2}\right] \tag{2}$$

Clearly, the right hand component of the equation represents nothing more than a ratio between the masses of two objects and their separation, which is operated upon by a proportionality term namely “G”, initially appearing to be much simpler than in actuality. When analyzing the dimensions, it rapidly becomes apparent that there is indeed, much more to the Gravitational Constant than initially thought and that the following must be correct when derived from equation (1);

$$G = [L^3] \cdot \left[\frac{1}{M}\right] \cdot \left[\frac{1}{T^2}\right] = \frac{L^3}{MT^2} \tag{3}$$

$$G = \frac{L^3}{MT^2} \tag{4}$$

This being entirely consistent with physics text books and the fact that Newton first formulated his laws of gravitation from Kepler’s law of Periods, when reorganized produces a generally accepted ratio of proportionality;

$$G_{\odot} = \frac{4\pi^2 {r_{\oplus}}^3}{M_{\odot} {T_{\oplus}}^2} \tag{5}$$

The denominator ${T_{\oplus}}^2$ is the time taken to sweep an orbital path in radians per second. The value of ${r_{\oplus}}^3$ in the numerator being the semi-major axis of the earth from the sun whose mass is $M_\odot$. Consequently substituting the following currently accepted values;

$T_\oplus =$ $3.155815 * 10^7$ period in seconds.
$M_\odot =$ $1.98855 * 10^{30}$ mass of the sun in kgs.
$r_\oplus =$ $1.496 * 10^{11}$ earths orbit in meters.

It comes as no surprise that the equation results in the familiar and universally accepted value for the gravitational constant "G" of $6.674 * 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$.

As a consequence of this rather basic calculation it can be seen that the value of "G" is not actually a constant but rather a generic geometrical function of proportionality which is more accurately represented by;

$$G_\odot = \frac{4\pi^2 r^3}{MT^2} \qquad (6)$$

In the interests of consistency, this can be taken one step further by substituting this function back into Newton's equation and the orbital velocities of the planets in the solar system calculated, as is common practice with Kepler's law of periods. This, as anticipated in every case returns the currently accepted value for "G".

It is possible to improve upon the current accuracy of "G" from first principles. There exists one condition inasmuch as the Planck constant should be considered valid. Clearly, if its value was to change it would necessarily negate all of the additional constants in physics that rely upon it, as such this assumption is considered reasonable.

In order to calculate the exact value of "*G*" it was decided that only the minimum of established verifiable parameters should be used with care taken to not create circular references to variables. Consequently, a simple example is proposed using the Schwarzschild equation and Planck length. The point where the Schwarzschild radius is equal to the Planck length can be found when the mass is the Planck mass and the length from both equations is equal to the Planck length, bearing in mind that the Schwarzschild equation is a direct derivation from Newton's basic laws.

$$l_c = \frac{h}{m_p c} = \frac{2Gm_p}{c^2} \qquad (7)$$

Evaluating the above and including the known values returns the exact value of π which strongly suggests that the statement is indeed correct noting that the Plank constant h is not represented by a reduced value;

$$\frac{hc}{2Gm_p{}^2} = \pi \qquad (8)$$

Returning to the original equation (7) and simplifying by moving the 2π to the left hand side of the equation results in the reduced Planck constant, (Dirac constant) which can subsequently be used to establish an accurate value of "G".

$$\frac{h}{2\pi} * \frac{c}{m_p{}^2} = G \qquad (9)$$

$$\frac{\hbar c}{m_p{}^2} = 6.674081 * 10^{-11} \qquad (10)$$

It can be seen that the calculated value for "G" in equation (10) is accurate to six decimal places, which is accurate to one order of magnitude greater than the current measured value.

3. Summary and Conclusions

It has been shown that "G" is a function of proportionality and is nothing more than Kepler's third law of periods. The value of "G" is obtained using the mass of the sun and the semi-major axis of the Earth's rotation. On the basis that "G" contains a parameter which represents an orbital period, it is therefore implicit that care must be taken that its value is used within the context of orbital systems. The calculated value of "G" using an alternative method returned a result of $6.674081 * 10^{-11}$. This value represents an accuracy of one order of magnitude greater than current CODATA[ii] measurements of "G"[iii] the value being $6.67408(31) * 10^{-11}$.

References

[i] *Isaac Newton "Principia", Andrew Motte's English translation published 1729*

[ii] *https://physics.nist.gov/cgi-bin/cuu/Value?bg|search_for=gravitational+constant*

[iii] *Abou Layla, A.E.S. (2017) Precise Ideal Value of the Universal Gravitational Constant G. Journal of High Energy Physics , Gravitation and Cosmology , 3, 248-253. https://doi.org/10.4236/jhepgc.2017.32020*