

Selection of Transportation Companies and Their Mode of Transportation for Interval Valued Data

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Abstract. The paper presents selection of transportation companies and their mode of transportation for interval valued neutrosophic data. The paper focuses on the application of distance measures to select mode of transportation for transportation companies. The paper also presents the application of multi-criteria decision making method using weighted correlation coefficient and extended TOPSIS for transportation companies. The multi-criteria decision making problem (MCDM) is taken in which there are different criteria and different modes. The selection is done among different modes

and then it is done among four transportation companies in which data is taken as Interval Valued Neutrosophic Set (IVNS). The first method is concerned with a multi-criteria fuzzy decision making method based on weighted correlation coefficients under interval valued neutrosophic fuzzy environment. The second method utilizes the extended TOPSIS method to solve the problem with data as IVNS and given attribute weights. The ranking is done and the most appropriate transportation company with the most appropriate mode is selected. The methods are illustrated with numerical examples.

Keywords: multi-criteria decision making problem ; Interval Valued Neutrosophic Set (IVNS); weighted correlation coefficients; TOPSIS; positive ideal solution (PIS).

1 Introduction

Multi criteria decision making (MCDM) problems are focussed at selecting the best alternative among different available alternatives with different criteria. There are different classical methods for different MCDM problems. In real life due to uncertainties and lack of time and knowledge decision makers' preferences are provided as fuzzy data. Fuzzy set theory was introduced by Zadeh [27]. Intuitionistic fuzzy set (IFS) was introduced as a generalization of fuzzy set (FS). IFS was introduced by Atanassov [23] including two membership functions - membership (or called truth-membership) ($T(x)$) and non-membership (or called falsity-membership) ($F(x)$), and satisfying the conditions $T(x), F(x) \in [0,1]$ and $0 \leq T(x) + F(x) \leq 1$.

Atanassov & Gargov [24] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) as a further generalization of IFS. Atanassov [25] also defined some operational laws of IVIFSs. De et al. [39] applied the max-min-max composition to medical diagnosis via IFSs. By following their reasoning, Szmidt & Kacprzyk [6]

applied the distance measures to IFSs in the medical diagnosis.

The concept of neutrosophic set was introduced as a generalization of crisp set, fuzzy set [27], IFS [23] by Smarandache ([7],[9]). The Indeterminacy function (I) was added to the two available parameters: Truth (T) and Falsity (F) membership functions. In neutrosophic set, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and false-membership are completely independent. In intuitionistic fuzzy sets, and the indeterminacy is $1 - T(x) - F(x)$ i.e. hesitancy or unknown degree by default. In neutrosophy, the indeterminacy membership ($I_A(x)$) is introduced as a new subcomponent so as to include the degree to which the decision maker is not sure. This type of treatment of the problem was out of scope of IFSs. The single valued neutrosophic set (SVNS) was introduced for the first time by Wang et al. [15] in 1998. Wang et al. [15] introduced the concept of interval valued neutrosophic set (IVNS) and provided the set-theoretic operators and various properties of SVNS and IVNS. SVNS and IVNS present uncertainty,

imprecise, inconsistent and incomplete information existing in real world.

Bustince & Burillo[13] proposed the concept of correlation and correlation coefficient of IVIFSs along with their properties. They also introduced two decomposition theorems – one in terms of the correlation of interval valued fuzzy sets and entropy of IFS and the other theorem is in terms of correlation of IFSs. Luo et al.[44] proposed a multi-criteria fuzzy decision-making method based on weighted correlation coefficients under interval-valued intuitionistic fuzzy environment with known criterion weight information. Wang et al. [47] proposed an approach to MADM with incomplete attribute weight information where individual assessments are provided as IVIFSs. Elhassouny, and Smarandache[1] used simplified TOPSIS for neutrosophic MCDM problems. Bausys et al. [35] and Bausys et al. [36]) used COPRAS and VIKOR respectively to solve neutrosophic MCDM problems. Ye [20] proposed MADM method with completely unknown weight information. Based on the correlation coefficient studied by Gerstenkorn & Manko [42], Ye [18],[19]) of IVIFSs, Park et al. ([3],[17]) investigated the group decision making problems in which the information about attribute weights is partially known. Ye [20] developed the MCDM method using the correlation coefficient under single-valued neutrosophic environment. Ye [22] also developed an extended TOPSIS method for MADM based on single valued neutrosophic linguistic numbers. Entropy based grey relational analysis method was used for MADM under single valued neutrosophic assessments by Biswas et al. [30]. An MCDM method based on single-valued trapezoidal neutrosophic preference relations with complete weight information was applied by Liang, et al. [37]. Neutrosophic MADM problems with unknown weight information was solved by Biswas et al. [31]. Mondal and Pramanik [26] Pramanik et al. [41] investigated neutrosophic tangent similarity measure and hybrid vector similarity measures respectively and their application to MADM. Sahin [38] also observed cross-entropy measure on interval neutrosophic sets and its applications in MCDM. Xu et al. [5] extended TODIM method for single-valued neutrosophic MADM. Z. Zhang and C. Wu [51] also developed a novel method for single-valued *neutrosophic* MCDM with incomplete weight information.

The technique for order of preference by similarity to ideal solution (TOPSIS) is a well-known method for solving decision making problems proposed by Hwang & Yoon [2]. Lai et al. [46] applied the concept of TOPSIS on multiple objective decision making (MODM) problems. Abo-Sinha & Amer [28] extended TOPSIS method for solving multi-objective large-scale nonlinear programming problems.

Opricovic & Tzeng [40] conducted a comparative analysis of TOPSIS and VIKOR. Many researchers (Chi & Liu [33], Jahanshaloo et al. [10], [11], Kour et al. [4] ; Wang & Lee[47], Opricovic & Tzeng [40] extended TOPSIS approach to fuzzy environment as a natural generalization of TOPSIS models. Chen & Tsao [43] extended the concept of TOPSIS to develop a method for solving MADM problems with interval-valued fuzzy data. Xu [49] developed some geometric aggregation operators, such as the interval-valued intuitionistic fuzzy geometric (IIFG) operator and interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator and applied them to multiple attribute group decision making (MAGDM) with interval-valued intuitionistic fuzzy information. Xu & Chen [50] and Wei & Wang[12] respectively developed some geometric aggregation operators, such as the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator and interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and applied them to MAGDM with interval-valued intuitionistic fuzzy information. However, they used the IIFWG, IIFWOG and IIFHG operators in the situation where the information about attribute weights is completely known. Chi & Liu [33] extended TOPSIS to IVNS environment in which the attribute weights are unknown and the attribute values are presented in terms of IVNS.

Kulak & Kahraman [29] studied a transportation company selection problem using axiomatic design and analytic hierarchy process (AHP) with partially known weight information in fuzzy environment. Kour et al. [4] applied the two methods on multi-criteria fuzzy decision making problems with IVIFS - the first one using correlation coefficient with unknown weights and the second one using TOPSIS method with known weights for the selection of transportation companies. TOPSIS method for MADM under single-valued neutrosophic environment was applied by Biswas et al. [32].

The present paper introduces the relation between the different criteria and different modes of transportation to select mode using distance measures for transportation companies for interval valued neutrosophic data. The present paper also extended the application of multi-criteria fuzzy decision making method with IVNSs to selection of transportation companies with given weights. A transportation company selection problem is taken with four different transportation companies and the data for the different criteria and modes are taken as IVNSs.

The application of distance measures is done to select the best mode of transportation for transportation companies for interval valued neutrosophic data after calculating the minimum distance between the transportation companies and the modes. Then the selection is done for the best

transportation company. The first method involves determining correlation coefficient between an alternative and the ideal alternative. The ranking is then done using this coefficient and the best alternative is selected. The second method focuses the extended TOPSIS method. The weighted collective interval valued neutrosophic decision matrix is constructed. Then the interval valued neutrosophic PIS and NIS are determined using a defined score function. The distance measures are used to calculate the relative closeness of each alternative to the interval valued neutrosophic PIS. The alternatives are ranked and the best one is selected.

No other authors till date have considered the concept of correlation coefficient for IVNSs. Further to find the PIS and NIS for TOPSIS, a new score function has been introduced. And both the methods have been applied to solve a new type of transportation company selection problem in which mode selection is also introduced which has not been done by any other author before.

2 Basic Concept

2.1 Neutrosophic Set

Let X be a space of points (objects), with a generic element in X denoted by x . A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ as by Smarandache [7].

$$A = \{x, T_A(x), I_A(x), F_A(x) \mid x \in X\}$$

The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $[0^-, 1^+]$. That is $T_A(x) : X \in [0^-, 1^+]$, $I_A(x) : X \in [0^-, 1^+]$, and $F_A(x) : X \in [0^-, 1^+]$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2.2 Complement of Neutrosophic set

The complement of a neutrosophic set A is denoted by cA and is defined by Smarandache [7] as $T^c(x) = \{1^+\} - T_A(x)$, $I^c(x) = \{1^+\} - I_A(x)$, and $F^c(x) = \{1^+\} - F_A(x)$ for every x in X .

2.3 Subset of Neutrosophic set

A neutrosophic set A is contained in the other neutrosophic set B , $A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq$

$\inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for every x in X (Smarandache [7]).

2.4 Single Valued Neutrosophic Set (SVNS)

A SVNS [15] A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X , $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

When X is continuous, an SVNS A can be written as

$$A = \int_X \frac{\langle T_A(x), I_A(x), F_A(x) \rangle}{x}, x \in X$$

When X is discrete, an SVNS A can be written as

$$A = \sum_{i=1}^n \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle}{x_i}, x_i \in X$$

2.5 Interval Valued Neutrosophic Set (IVNS)

Let X be a universe of discourse, with a generic element in X denoted by x . An interval neutrosophic set A in X is defined by Wang et al. [14], as $A = \{x, T_A(x), I_A(x), F_A(x) \mid x \in X\}$ where,

$T_A(x)$, $I_A(x)$, $F_A(x)$ are the truth-membership function, indeterminacy-membership function, and the falsity membership function, respectively. For each point x in X , we have $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$ and

$$0 \leq \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \leq 3$$

For convenience, we take an interval-valued neutrosophic set (IVNS), $\tilde{A} = ([a, b], [c, d], [e, f])$ where $[a, b], [c, d], [e, f] \subset [0, 1], 0 \leq b + d + f \leq 3$

2.6 Algebraic Rules of IVNS (Wang et al. [14])

Let

$$\tilde{A} = ([a_1, b_1], [c_1, d_1], [e_1, f_1])$$

and

$$\tilde{B} = ([a_2, b_2], [c_2, d_2], [e_2, f_2])$$

be two IVNS, then

The complement of

$$\tilde{A} = ([a_1, b_1], [c_1, d_1], [e_1, f_1])$$

is given by

$$\tilde{A}^c = ([e_1, f_1], [1 - c_1, 1 - d_1], [a_1, b_1])$$

1. $\tilde{A} \oplus \tilde{B} = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2], [e_1e_2, f_1f_2])$
2. $\tilde{A} \otimes \tilde{B} = ([a_1a_2, b_1b_2], [c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2], [e_1 + e_2 - e_1e_2, f_1 + f_2 - f_1f_2])$
3. $n\tilde{A} = ([1 - (1 - a_1)^n, 1 - (1 - b_1)^n], [c_1^n, d_1^n], [e_1^n, f_1^n]), n > 0$

$$5. \tilde{A}^n = ([a_1^n, b_1^n], [1 - (1 - c_1)^n, 1 - (1 - d_1)^n], [1 - (1 - e_1)^n, 1 - (1 - f_1)^n]), n > 0$$

2.7 Score of IVNS

Let $R = (\tilde{r}_{ij})_{mn}$, where $\tilde{r}_{ij} = [a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}]$ the collective interval - valued neutrosophic decision matrix be. Then $S = (s_{ij})_{mn}$ is defined as the score matrix of

$$R = (\tilde{r}_{ij})_{mn}, \text{ where}$$

$$s_{ij} = s(\tilde{r}_{ij}) = \frac{1}{3}(2 + a_{ij} - c_{ij} - e_{ij} + b_{ij} - d_{ij} - f_{ij}), i = 1, 2, \dots, n$$

(1)

And $s(\tilde{r}_{ij})$ is called the score of \tilde{r}_{ij}

Example 2.7.1 Let

$$\tilde{A} = ([0.3, 0.4], [0.1, 0.2], [0.5, 0.7])$$

$$\tilde{B} = ([0.4, 0.5], [0.2, 0.3], [0.5, 0.6]) \text{ be two IVNSs.}$$

Then by Definition 2.7,

$$s(\tilde{A}_{ij}) = \frac{1}{3}(2 + 0.3 - 0.1 - 0.5 + 0.4 - 0.2 - 0.7) = 0.4$$

$$s(\tilde{B}_{ij}) = \frac{1}{3}(2 + 0.4 - 0.2 - 0.5 + 0.5 - 0.3 - 0.6) = 0.433$$

Hence, $s(\tilde{A}_{ij}) < s(\tilde{B}_{ij})$

Properties 2.7.2 Let $\tilde{r}_{ij} = [a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}]$

be an INVS. Then the score of \tilde{r}_{ij} has some properties as follows:

- (i) $s(\tilde{r}_{ij}) = 0$ if and only if $a_{ij} + b_{ij} = c_{ij} + d_{ij} + e_{ij} + f_{ij} - 2$.
- (ii) $s(\tilde{r}_{ij}) = 1$ if and only if $a_{ij} + b_{ij} = c_{ij} + d_{ij} + e_{ij} + f_{ij} + 1$.
- (iii) $s(\tilde{r}_{ij}) = -1$ if and only if $a_{ij} + b_{ij} = c_{ij} + d_{ij} + e_{ij} + f_{ij} - 1$.

2.8 Distance between two IVNS

Let $X = ([a_{i1}, b_{i1}], [c_{i1}, d_{i1}], [e_{i1}, f_{i1}])$ and $Y = ([a_{i2}, b_{i2}], [c_{i2}, d_{i2}], [e_{i2}, f_{i2}])$ be two IVNSs. The normalized Hamming distance between X and Y is defined by Chi & Liu [33] as

$$d_H(X, Y) = \frac{1}{6n} \sum_{i=1}^n (|a_{i1} - a_{i2}| + |b_{i1} - b_{i2}| + |c_{i1} - c_{i2}| + |d_{i1} - d_{i2}| + |e_{i1} - e_{i2}| + |f_{i1} - f_{i2}|)$$

(2)

3. Problem description and methodology

3.1 Problem Description

The present paper deals with the selection of transportation company and their mode of transportation in interval valued neutrosophic environment. At first the neutrosophic relation Q from a set of different transportation companies T to a set of different criteria C like transportation cost, defective rate, tardiness rate, flexibility, etc. is considered. Then it follows the second relation R from the set of different criteria C to a set of different mode M of transportation like roadways, railways, waterways and airways. The composition of the two neutrosophic relation Q and R is the relation S from the set of transportation companies to the set of different modes which gives the best mode of transportation for each of the transportation companies. Finally, the best transportation company is to be selected among the given different companies. The problem can be solved by different methods available in this context taking into account the different criteria. The present paper focuses on two methods. The first one involves weighted correlation coefficient method. The second one involves extended TOPSIS method. The different weights are given for different criteria.

3.2 Methodology

A. Application of normalized hamming distance for interval valued neutrosophic set

Let there be a neutrosophic relation $X: A_i \rightarrow B_j$ and $Y: B_j \rightarrow C_k$. Using the distance between two IVNSs in Definition 2.8 the normalized Hamming distance for all the elements of the A_i from the C_k is equal to

$$d_H(A_i, C_k) = \frac{1}{6n} \sum_{j=1}^n (|\mu_{jL}(A_i) - \mu_{jL}(C_k)| + |\mu_{jU}(A_i) - \mu_{jU}(C_k)| + |v_{jL}(A_i) - v_{jL}(C_k)| + |v_{jU}(A_i) - v_{jU}(C_k)| + |r_{jL}(A_i) - r_{jL}(C_k)| + |r_{jU}(A_i) - r_{jU}(C_k)|) \quad (3)$$

B. Multi-criteria decision making method based on weighted correlation coefficients in interval valued neutrosophic environment

Let $A = \{A_1, A_2, A_3, \dots, A_m\}$ be a set of alternatives and let $C = \{C_1, C_2, C_3, \dots, C_n\}$ be a set of criteria.

An alternative A_i is represented by the following IVNS:

$$A_i = \{(C_j, [\mu_{A_iL}(C_j), \mu_{A_iU}(C_j)], [v_{A_iL}(C_j), v_{A_iU}(C_j)], [r_{A_iL}(C_j), r_{A_iU}(C_j)] : C_j \in C\}$$

where $0 \leq \mu_{A_iU}(C_j) + v_{A_iU}(C_j) \leq 1$, $\mu_{A_iL}(C_j) \geq 0$

$v_{A_iL}(C_j) \geq 0$, $j = 1, 2, \dots, n$, and $i = 1, 2, \dots, m$.

The IVNS that consists of Intervals $\mu_{A_i}(C_j) = [a_{ij}, b_{ij}]$, $v_{A_i}(C_j) = [c_{ij}, d_{ij}]$

$r_{A_i}(C_j) = [e_{ij}, f_{ij}]$ for $C_j \in C$ is denoted

by $\alpha_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}])$ for convenience.

We can express an interval-valued neutrosophic decision matrix $D = (\alpha_{ij})_{mn}$.

Ye ([18],[19]) established a model for weighted correlation coefficient between each alternative and the ideal alternative for single valued neutrosophic sets (SVNSs) using known weights of the criterion. Though the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives. Ye ([18],[19]) defined the ideal alternative for SVNSs as $\alpha^* = (a_{ij}^*, b_{ij}^*, c_{ij}^*) = (1, 0, 0)$.

If the information about weight w_j of the criterion C_j ($j = 1, 2, \dots, n$) is completely known, for determining the criterion weight from the decision matrix D we can establish an exact model for the weighted correlation coefficient between an alternative A_i and the ideal alternative A^* represented by the IVNS as in Equation (4). We define the ideal alternative A^* as the IVNS

$$\alpha^* = ([a_{ij}^*, b_{ij}^*], [c_{ij}^*, d_{ij}^*], [e_{ij}^*, f_{ij}^*]) = ([1, 1], [0, 0], [0, 0])$$

$$W_i(A_i, A^*) = \frac{\sum_{j=1}^n w_j [a_{ij} a_{ij}^* + b_{ij} b_{ij}^* + c_{ij} c_{ij}^* + d_{ij} d_{ij}^* + e_{ij} e_{ij}^* + f_{ij} f_{ij}^*]}{\sqrt{\sum_{j=1}^n w_j [a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2]} \sqrt{\sum_{j=1}^n w_j [a_j^{*2} + b_j^{*2} + c_j^{*2} + d_j^{*2} + e_j^{*2} + f_j^{*2}]}} \quad (4)$$

Then the bigger the value of the weighted correlation coefficient W_i is, the better the alternative A_i is. Therefore all the alternatives can be ranked according to the value of the weighted correlation coefficients so that the best alternative can be selected.

C. TOPSIS method to solve the multi-attribute decision making problem with the given information about attribute weights in interval valued neutrosophic environment

In the situations where the information about weights is completely known, that is, the weights $w_i = (w_1, w_2, \dots, w_m)^T$ of the c_j ($j = 1, 2, \dots, n$) can be completely determined in advance, then we can construct the weighted collective interval-valued neutrosophic decision matrix

$$R^* = (\tilde{r}_{ij}^*)_{mn} \text{ where}$$

$$\tilde{r}_{ij}^* = w_i \tilde{r}_{ij} = \{[1 - (1 - a_{ij})^{w_i}, 1 - (1 - b_{ij})^{w_i}], [c_{ij}^{w_i}, d_{ij}^{w_i}], [e_{ij}^{w_i}, f_{ij}^{w_i}]\} \quad (5)$$

is the weighted IVNS, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$, and w_i is weight of the attribute u_i such that $w_i \geq 0$ and $\sum_{i=1}^m w_i = 1$.

Now, we denote by

$$\tilde{r}_{ij}^* = ([a_{ij}^*, b_{ij}^*], [c_{ij}^*, d_{ij}^*], [e_{ij}^*, f_{ij}^*]) \text{ where } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (6)$$

Let J_1 be a collection of benefit attributes (i.e., the larger u_i , the greater preference) and J_2 be a collection of cost attributes (i.e., the smaller u_i , the greater preference). The interval-valued neutrosophic PIS, denoted by A^+ , and the interval-valued neutrosophic NIS, denoted by A^- , are de-

defined as follows:

$$A^* = \{ \{c_j, (\max_i \tilde{r}_{ij}^* : j \in J_1), (\min_i \tilde{r}_{ij}^* : j \in J_2)\} : (7)$$

$$j = 1, 2, \dots, n\}^T = (\tilde{r}_1^+, \tilde{r}_2^+, \dots, \tilde{r}_n^+)^T$$

$$A^- = \{ \{c_j, (\min_i \tilde{r}_{ij}^* : i \in J_1), (\max_i \tilde{r}_{ij}^* : i \in J_2)\} : (8)$$

$$j = 1, 2, \dots, n\}^T = (\tilde{r}_1^-, \tilde{r}_2^-, \dots, \tilde{r}_n^-)^T$$

where $\tilde{r}_i^+ = ([a_i^+, b_i^+], [c_i^+, d_i^+][e_i^+, f_i^+]$ and

$\tilde{r}_i^- = ([a_i^-, b_i^-], [c_i^-, d_i^-][e_i^+, f_i^+]$, $i=1, 2, \dots, m$

Burillo & Bustince [13] method has been extended to find the separation measures for interval valued intuitionistic fuzzy numbers in Park et al. [17] and in Kour et al, [4]. The extension of this in IVNS has been used here to find separation measures based on the Hamming distance.

$$S_{i^+}^{d_1} = \frac{1}{6} \sum_{j=1}^n \left[|a_{ij}^* - a_i^+| + |b_{ij}^* - b_i^+| + |c_{ij}^* - c_i^+| + |d_{ij}^* - d_i^+| + |e_{ij}^* - e_i^+| + |f_{ij}^* - f_i^+| \right] \quad (9)$$

$$S_{i^-}^{d_1} = \frac{1}{6} \sum_{j=1}^n \left[|a_{ij}^* - a_i^-| + |b_{ij}^* - b_i^-| + |c_{ij}^* - c_i^-| + |d_{ij}^* - d_i^-| + |e_{ij}^* - e_i^-| + |f_{ij}^* - f_i^-| \right] \quad (10)$$

The relative closeness of an alternative A_i with respect to interval-valued neutrosophic PIS A^* is defined as the following:

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-} \text{ where } i=1, 2, \dots, m \quad (11)$$

The bigger the closeness coefficient C_i^+ , the better the alternative A_i will be, as the alternative A_i is closer to the interval-valued neutrosophic PIS A^* . Therefore, the alternatives A_i ($i = 1, 2, \dots, m$) can be ranked according to the closeness coefficients so that the best alternative can be selected.

3.3 Solution Procedure:

A. Algorithm for the method based on normalized hamming distance

Let $T = \{T_1, T_2, T_3, \dots, T_m\}$ be a set of transportation companies, $C = \{C_1, C_2, C_3, \dots, C_n\}$ be a set of criteria and $M = \{M_1, M_2, M_3, \dots, M_p\}$ be a set of modes of transportation where each of the C_j of T_i and M_k is represented by IVNS.

$$C(T_i) = ([\mu_{jL}(T_i), \mu_{jU}(T_i)], [v_{jL}(T_i), v_{jU}(T_i)], [r_{Lj}(T_i), r_{Uj}(T_i)])$$

$$M_k = ([\mu_{jL}(M_k), \mu_{jU}(M_k)], [v_{jL}(M_k), v_{jU}(M_k)], [r_{Lj}(M_k), r_{Uj}(M_k)])$$

Using the distance between two IVNSs in Definition 2.8 the Normalized Hamming distance for all the criteria of the i -th transportation company from the k -th modes is equal to

$$d_H(C(T_i), M_k) = \frac{1}{30} \sum_{j=1}^5 (|\mu_{jL}(T_i) - \mu_{jL}(M_k)| + |\mu_{jU}(T_i) - \mu_{jU}(M_k)| + |v_{jL}(T_i) - v_{jL}(M_k)| + |v_{jU}(T_i) - v_{jU}(M_k)| + |r_{Lj}(T_i) - r_{Lj}(M_k)| + |r_{Uj}(T_i) - r_{Uj}(M_k)|) \quad (12)$$

The minimum distance determines the appropriate mode of each transportation company.

B. Algorithm for the method based on weighted correlation coefficients using given weights

Step 1: Calculate the weighted correlation coefficient

$W_i(A^*, A_i)$ ($i = 1, 2, \dots, m$) by using Eq. (4).

Step 2: Rank the alternatives according to the obtained correlation coefficients, and then obtain the best choice.

C. Algorithm for TOPSIS method with the given information about attribute weights

Step1. Calculate the weighted collective interval-valued neutrosophic decision matrix $R^* = (\tilde{r}_{ij}^*)_{mn}$

Step 2: Calculate the score matrix $S = (s_{ij})_{m \times n}$ of the collective interval-valued neutrosophic decision matrix R using Equation(1) from Definition 2.7.

Step3. Determine the interval-valued neutrosophic PIS A^* , and interval-valued neutrosophic NIS A^- using Equations(7), (8) and score matrix S obtained above in Step 2.

Step 4. Calculate the separation measures S_i^+ and S_i^- of each alternative A_i ($i = 1, 2, \dots, m$) from interval-valued neutrosophic PIS A^* and interval-valued neutrosophic NIS A^- , respectively using Equations (9) and (10).

Step 5: Calculate the relative closeness C_i^+ of each alternative A_i ($i = 1, 2, \dots, m$) to the interval-valued neutrosophic PIS A^* using Equation(11).

Table 1. Data of transportation companies and their criteria in form of interval valued neutrosophic fuzzy numbers

Step 6. Rank the alternatives A_i ($i = 1, 2, \dots, m$), according

to the relative closeness to the interval-valued neutrosophic PIS A^* and then select the most desirable one (s).

4. Numerical Illustration:

4.1 Example

An international company needs a freight transportation company to carry its goods. The company determined four possible transportation companies. The criteria considered in the selection process are transportation costs, defective rate, tardiness rate, flexibility and documentation ability. Transportation cost is the cost to carry one ton along one kilometre. Tardiness rate is computed as “the number of days delayed/the number of days expected for delivery. In Kulak & Kahraman [29], Transportation costs, defective rate and tardiness rate are taken to be crisp variables and the other criteria “flexibility” and “documentation ability” are taken as linguistic variables just to find only the best transportation company. In Kour et al. [4], the problem is taken in Interval valued Intuitionistic fuzzy environment in which each element of the decision matrix is taken as interval valued intuitionistic fuzzy numbers and the best appropriate transportation company is selected.

In the present paper, the problem is modified as the best transportation company and also their mode of transportation is selected under interval valued neutrosophic

Let the set of transportation companies be $T = \{TC1, TC2, TC3, TC4\}$. Let the set of different criteria of the transportation companies be denoted by $C = \{\text{Transportation cost (TC), Defective rate (DR), Tardiness rate (TR), Flexibility (F), Documentation ability (DA)}\}$. The data of degree of satisfaction, indeterminacy and rejection of each criterion by each transportation company is represented by an IVNS in Table 1. The IVNS is denoted by a set of Intervals $T_i = (C_j, [\mu_{T,L}, \mu_{T,U}], [v_{T,L}, v_{T,U}], [r_{T,L}, r_{T,U}])$:

$$C_j \in C = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}])$$

Table 2. Data of criteria of transportation companies and their mode of transportation in form of interval valued neutrosophic fuzzy numbers

The IVNS is usually elicited from the evaluated score to which the alternative TC_i satisfies the criterion C_j by means of a score law and data processing or from appropriate membership functions in practice. Therefore,

Alter-native	Criteria				
	Transporta-tion Cost	Defective Rate	Tardiness Rate	Flexibility	Documenta-tion Ability
Trans- portation Com- pa- nies					
Trans- portation Com- pa- nies .1	[[0.7,0.8],[0.01,0.02],[0.2,0.4]]	[[0.8,0.85],[0.02,0.03],[0.3,0.5]]	[[0.3,0.4],[0.2,0.4],[0.1,0.2]]	[[0.6,0.8],[0.01,0.02],[0.2,0.3]]	[[0.4,0.5],[0.1,0.3],[0.1,0.2]]
Trans- portation Com- pa- nies .2	[[0.8,0.85],[0.01,0.03],[0.2,0.3]]	[[0.01,0.03],[0.8,0.9],[0.3,0.5]]	[[0.8,0.92],[0.01,0.04],[0.2,0.3]]	[[0.01,0.02],[0.4,0.6],[0.2,0.3]]	[[0.85,0.9],[0.01,0.02],[0.2,0.4]]
Trans- portation Com- pa- nies .3	[[0.85,0.89],[0.02,0.05],[0.3,0.5]]	[[0.4,0.6],[0.1,0.3],[0.2,0.4]]	[[0.9,0.95],[0.01,0.02],[0.3,0.4]]	[[0.9,0.92],[0.01,0.03],[0.3,0.5]]	[[0.7,0.8],[0.02,0.04],[0.2,0.4]]
Trans- portation Com- pa- nies .4	[[0.8,0.9],[0.01,0.02],[0.2,0.5]]	[[0.2,0.4],[0.6,0.7],[0.3,0.4]]	[[0.2,0.3],[0.3,0.6],[0.3,0.4]]	[[0.5,0.6],[0.1,0.2],[0.2,0.3]]	[[0.7,0.8],[0.3,0.4],[0.02,0.1]]

environment.

Alter-native Criteria	Mode of transportation			
	Road- ways	Railways	Water- ways	Airways
Trans- portation Cost	[[0.7,0.85],[0.02,0.03],[0.1,0.15]]	[[0.8,0.9],[0.02,0.03],[0.01,0.04]]	[[0.5,0.6],[0.1,0.2],[0.3,0.35]]	[[0.3,0.4],[0.2,0.3],[0.4,0.5]]
Defec- tive Rate	[[0.3,0.4],[0.1,0.2],[0.5,0.6]]	[[0.6,0.7],[0.03,0.04],[0.2,0.25]]	[[0.65,0.75],[0.02,0.05],[0.1,0.2]]	[[0.8,0.9],[0.01,0.02],[0.01,0.1]]
Tardiness Rate	[[0.3,0.5],[0.02,0.04],[0.4,0.45]]	[[0.5,0.65],[0.01,0.02],[0.2,0.25]]	[[0.4,0.5],[0.01,0.05],[0.2,0.3]]	[[0.75,0.85],[0.02,0.03],[0.1,0.15]]
Flexibil- ity	[[0.8,0.9],[0.2,0.3],[0.01,0.08]]	[[0.6,0.7],[0.1,0.2],[0.2,0.25]]	[[0.5,0.6],[0.01,0.02],[0.15,0.2]]	[[0.4,0.5],[0.02,0.04],[0.2,0.3]]
Docu- menta- tion Ability	[[0.6,0.7],[0.01,0.02],[0.2,0.25]]	[[0.65,0.8],[0.03,0.05],[0.15,0.2]]	[[0.7,0.8],[0.2,0.4],[0.1,0.15]]	[[0.75,0.85],[0.03,0.04],[0.05,0.1]]

we can express an interval-valued neutrosophic decision matrix $D = (\alpha_{ij})_{m \times n}$.

Similarly let the set of different transportation modes is denoted by $M = \{\text{Roadways, Railways, Waterways, Airways}\}$. The data of degree of satisfaction, indeterminacy and rejection of each criterion for each mode is represented by an IVNS in Table 2.

$$C_j = (M_k, [\mu_{C_jL}, \mu_{C_jU}], [\nu_{C_jL}, \nu_{C_jU}], [r_{C_jL}, r_{C_jU}]) :$$

$$M_k \in M = ([a_{jk}, b_{jk}], [c_{jk}, d_{jk}], [e_{jk}, f_{jk}])$$

And it can be denoted by an interval-valued neutrosophic decision matrix $D' = (\beta_{jk})_{n \times p}$.

The weights are taken as $w_1=0.38, w_2=0.17, w_3=0.21, w_4=0.24, w_5=0.00$

4.2 Solution

The given problem is a multi criteria decision making problem in interval valued neutrosophic environment and is solved in two sections. The first section follows up with selecting the best mode of transportation for each transportation company using distance measures. The second section includes the selection of the most appropriate transportation company by the two above mentioned methods. The results are obtained as follows:

A. Solution with method based on Application of Normalized Hamming Distance for Interval valued neutrosophic set

The Equation (3) is used to find the distance for all the criteria of the i-th transportation company from the k-th modes using the normalised Hamming distance as in Table 3. In the definition 2.8, the normalized hamming distance between X and Y (defined by Chi & Liu [33]) is given in Equation (2) which means the distance between any two IVNS. This definition is utilized to calculate the minimum distance between two IVNS in two different but related tables with IVNS as in Equation (3). Then the Equation (3) is utilized to find the Normalized Hamming distance for all the criterion of the i-th transportation company from the k-th modes as in Equation (12) taking data from the related tables Table 1 and Table 2. The minimum distance determines the appropriate mode of each transportation company. For Example - The minimum distance for all the criteria of the **transportation company TC2** is **0.2337** from the **Railways** mode. That means the appropriate mode for transportation companyTC2 is Railways. Similarly, the appropriate mode for each transportation company is given in Table 4.

Table 3. Data of distances for each transportation company from the considered set of their possible modes of transportation

Alternative Transportation Companies	Mode of transportation			
	Roadways	Railways	Waterways	Airways
Trans.Comp.1	0.1737	0.1333	0.1283	0.1847
Trans.Comp.2	0.2393	0.2337	0.361	0.292
Trans.Comp.3	0.172	0.1303	0.1727	0.2087
Trans.Comp.4	0.194	0.1923	0.1887	0.2743

Table 4. Appropriate Mode for each transportation company

Transportation companies	Minimum Distance	Appropriate Mode
Trans.Comp.1	0.1283	Waterways
Trans.Comp.2	0.2337	Railways
Trans.Comp.3	0.1303	Railways
Trans.Comp.4	0.1887	Waterways

B. Solution with method based on weighted correlation coefficients

The attribute weights are taken as $w_1=0.38, w_2=0.17, w_3=0.21, w_4=0.24, w_5=0.00$

Step 1: The weighted correlation coefficient between an alternative A_i and the ideal alternative A^* represented by the IVNS

Is given by Equation (4).

Then taking weight attributes as $w_1=0.38, w_2=0.17, w_3=0.21, w_4=0.24, w_5=0.00$, the weighted correlation coefficient can be calculated for the data mentioned in Table 1 by applying Equation (4).

By applying Equation (4), we can compute $W_i(A^*, A_i)$ ($i = 1, 2, 3, 4$) as

$$W_1(A^*, A_1) = 0.6737 ; W_2(A^*, A_2) = 0.4811 ;$$

$$W_3(A^*, A_3) = 0.8942; W_4(A^*, A_4) = 0.7076$$

Step 2: From the weighted correlation coefficients between the alternatives and the ideal alternative, the ranking order

$$\text{is } A_3 \prec A_4 \prec A_1 \prec A_2$$

which is given in Table 5.

Table 5 Ranking based on Weighted Correlation Coefficient

Alternatives	Value of $W_i(A^*, A_i)$	Rank
Trans.Comp.1	0.6737	3
Trans.Comp.2	0.4811	4
Trans.Comp.3	0.8942	1
Trans.Comp.4	0.7076	2

Therefore, we can see that the alternative *TC3* is the best choice, which is the same result as Kulak & Kahraman [29] and by method of weighted correlation coefficient in Kour et al.[4].

C. Solution with TOPSIS method with the given information about attribute weights

The attribute weights are taken as $w_1=0.38, w_2=0.17, w_3=0.21, w_4=0.24, w_5=0.00$

Step 1: The weighted collective interval-valued neutrosophic decision matrix $R^* = (\tilde{r}_{ij}^*)_{mn}$ is calculated (Table 6) applying Equation (5).

Step 2: The score matrix $S = (s_{ij})_{mn}$ of the collective interval-valued neutrosophic decision matrix R is calculated using Equation (1) from Definition 2.7 as in Table 7.

Step 3: Using Equations (7), (8) and score matrix obtained above, the interval-valued neutrosophic PIS A^* and interval-valued neutrosophic NIS A^- is determined as in Table 8.

Step 4: The separation measures S_i^+ and S_i^- of each alternative A_i ($i = 1, 2, 3, 4$) are calculated from interval-valued neutrosophic PIS A^* and interval-valued neutrosophic NIS A^- , respectively, based on the Hamming distance using Equations (9) - (10) (Table 9).

Step 5: The relative closeness C_i^+ of each alternative A_i ($i = 1, 2, 3, 4$) to the interval-valued neutrosophic PIS A^* is calculated with the different separation measures, based on the Hamming distance, using Eq. (11) (Table 10).

Step 6. Rank the preference order of alternatives A_i ($i = 1, 2, 3, 4$) (Table 6), according to the relative closeness to the

interval-valued neutrosophic PIS A^* and the ranking order is $A_4 \prec A_3 \prec A_1 \prec A_2$.

Therefore, we can see that the alternative *TC4* is the best choice and then the most desirable alternative is Transportation company *TC4* as by TOPSIS in Kour et al. [4].

Table 6 Weighted collective interval valued neutrosophic fuzzy decision matrix

Alternative	Criteria				
	Transportation Cost	Defective Rate	Tardiness Rate	Flexibility	Documentation Ability
Trans. Comp.1	[[0.37,0.46], [0.17,0.22], [0.54,0.71]]	[[0.24,0.28], [0.51,0.55], [0.81,0.89]]	[[0.07,0.10], [0.7,0.83], [0.62,0.71]]	[[0.2,0.32], [0.33,0.39], [0.68,0.75]]	[[0,0], [1,1], [1,1]]
Trans. Comp.2	[[0.46,0.51], [0.17,0.26], [0.54,0.63]]	[[0.0017,0.005], [0.963,0.982], [0.815,0.888]]	[[0.29,0.41], [0.38,0.51], [0.71,0.78]]	[[0.002,0.005], [0.8,0.88], [0.68,0.75]]	[[0,0], [1,1], [1,1]]
Trans. Comp.3	[[0.51,0.57], [0.23,0.32], [0.63,0.77]]	[[0.08,0.14], [0.68,0.81], [0.76,0.86]]	[[0.38,0.47], [0.38,0.44], [0.78,0.83]]	[[0.42,0.45], [0.33,0.43], [0.75,0.85]]	[[0,0], [1,1], [1,1]]
Trans. Comp.4	[[0.46,0.58], [0.17,0.23], [0.54,0.77]]	[[0.04,0.08], [0.92,0.94], [0.81,0.86]]	[[0.05,0.07], [0.78,0.9], [0.78,0.83]]	[[0.15,0.2], [0.58,0.68], [0.68,0.75]]	[[0,0], [1,1], [1,1]]

Table 7 Score matrix of the Weighted collective interval valued neutrosophic fuzzy decision matrix

Alternative	Criteria	
	Minimize	Maximize
Transportation		

tion companies	Transportation Cost	Defective Rate	Tardiness Rate	Flexibility	Documentation Ability
Trans.Comp.1	0.3967	-0.08	-0.2333	0.1233	-0.6667
Trans.Comp.2	0.45667	-0.5473	0.1067	-0.3677	-0.6667
Trans.Comp.3	0.3767	-0.2967	0.14	0.17	-0.6667
Trans.Comp.4	0.4433	-0.47	-0.39	-0.1133	-0.6667

Table 8 Interval valued PIS and NIS

	Minimize			Maximize	
	Transportation Cost	Defective Rate	Tardiness Rate	Flexibility	Documentation Ability
PI S	[(0.51,0.57], [0.23,0.32], [0.63,77])	[(0.0017,0.005], [0.963,0.982], [0.815,0.888])	[(0.05,0.07], [0.78,0.9], [0.78,0.83])	[(0.42,0.45], [0.33,0.43], [0.75,0.85])	[(0,0], [1,1], [1,1])
NI S	[(0.46,0.51], [0.17,0.26], [0.54,0.63])	[(0.24,0.28], [0.51,0.55], [0.81,0.89])	[(0.38,0.47], [0.38,0.44], [0.78,0.83])	[(0.002,0.005], [0.8,0.88], [0.68,0.75])	[(0,0], [1,1], [1,1])

Table9 Separation measures based on Hamming distance

Alternatives	S_i^+	S_i^-
Trans.Comp.1	0.4997	0.5688
Trans.Comp.2	0.6505	0.29073
Trans.Comp.3	0.39033	0.5372
Trans.Comp.4	0.287	0.6372

Table 10 Relative closeness C_i^+ based on Hamming Distance

Alternatives	Value of C_i^+	Rank
Trans.Comp.1	0.53234	3
Trans.Comp.2	0.30888	4
Trans.Comp.3	0.57917	2
Trans.Comp.4	0.68946	1

5. Results and comparison

In this paper, the distance measures on interval valued neutrosophic set using the normalized hamming distance help to find the best modes of transportation for each transportation company as in Table 4. The paper helps to find the appropriate transportation company. It follows with two methods. The first method which is based on weighted correlation coefficient gives the best transportation company as TC3. The result is same as in the Kour et al. [4] for the method to find the best transportation company based on weighted correlation coefficient under interval valued intuitionistic fuzzy environment. The second method which is the extended TOPSIS gives the best transportation company as TC4. The result is same as in the Kour et al. [4] for the extended TOPSIS method to find the best transportation company under interval valued intuitionistic fuzzy environment. In addition, this paper also helps to find the best mode of transportation for the selected transportation companies. In the first result, the selected transportation company TC3 opt for Railways whereas in the second result, the selected transportation company TC4 chooses Waterways as their mode of transportation. The present paper also deals with degree of indeterminacy along with the degree of acceptance and rejection of the different attributes as in Kour et al. [4]. The results can be compared with the help of the below mentioned tables (Table 11, Table 12, Table 13 and Table 14).

Table11 Solution as in [4] under interval valued intuitionistic fuzzy environment

Alternatives	Rank with Weighted Correlation Coefficient Method(unknown weights)	Rank with Extended TOPSIS(known weights)

	weights)	
Trans.Comp.1	3	3
Trans.Comp.2	4	4
Trans.Comp.3	1	2
Trans.Comp.4	2	1

Table12 Appropriate Transportation Company in [4] under interval valued intuitionistic fuzzy environment

Weighted Coefficient	Correlation Method(unknown weights)	ExtendedTOPSIS(known weights)
Trans Comp 3		Trans Comp 4

Table13 Solution as in the present paper under interval valued neutrosophic environment

Alternatives	Rank with Weighted Correlation Coefficient Method(known weights)	Rank with Extended TOPSIS(known weights)
Trans.Comp.1	3	3
Trans.Comp.2	4	4
Trans.Comp.3	1	2
Trans.Comp.4	2	1

Table14 Appropriate Transportation Company and their mode in the present paper under interval valued neutrosophic environment

Methods	Weighted Correlation Coefficient Method (unknown weights)	Extended TOPSIS (known weights)

	weights)	
Best Transportation Company	Trans Comp 3	Trans Comp 4
Best Transportation Mode	Railways	Waterways

6. Conclusion

- A new type of transportation company selection problem is constructed in which the mode of transportation is also selected along with the best transportation company which gives a greater scope of its application in real life circumstances to achieve better requirements of the transportation companies.
- The method for the application of normalized hamming distance on interval valued neutrosophic set helps the users to relate the given two different relational tables consisting of transportation companies, their criteria and their mode of transportation and thus to find the appropriate mode of each transportation companies for the first time.
- The weighted correlation coefficient method helps the users to solve the multi-criteria decision making problems with given weight information which has been done for the first time in Interval valued neutrosophic environment
- The extended TOPSIS method provides us an effective and practical way to solve the same type of problems, where the data is characterized by IVNSs and the information about weights is completely known. A score function has been defined for interval valued neutrosophic sets for the first time and is used to find the interval valued neutrosophic PIS and NIS.
- The interval valued neutrosophic set data can be seen as real life uncertainties and so represents more practical solutions of the problem where the degree of acceptance, indeterminacy and rejection of the different attributes are taken into account.

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