A note on the possibility of incomplete theory.

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In the paper it is demonstrated that Bells theorem is an unprovable theorem. This inconsistency is similar to concrete mathematical incompleteness. The inconsistency is purely mathematical. Nevertheless the basic physics requirements of a local model are fulfilled.

Keywords: Negation Incompleteness, Bell's theorem, LHV model.

INTRODUCTION

Let us start our paper with a quote from professor Friedmann's last lecture [3]. (cit ...) Most [mathematicians] intuitively feel that the great power and stability of some "rule book for mathematics" is an important component of their relationship with mathematics. The general feeling is that there is nothing substantial to be gained by revisiting the commonly accepted rule book ... .

In 1964, John Bell wrote a paper [1] about the possibility of hidden variables [2] causing the entanglement correlation $E(a, b)$ between two particles. In the present paper, an inconsistency similar to concrete mathematical incompleteness [4], will be demonstrated from his theorem. The argument for mathematical incompleteness is to proof *and* refute with known concrete mathematical axioms the mathematical statement of Bell's theorem. The author is aware of the scepsis this may raise with certain readers. However, scepsis is simply not enough to push our proof of inconsistency aside and do "business as usual" with Bell's formula.

Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [5]. Bell used hidden variables λ that are elements of a universal set Λ and are distributed with a density $\rho(\lambda) \geq 0$. Suppose, $E(a, b)$ is the correlation between measurements with distant A and B that have unit-length, i.e. $||a|| = ||b|| = 1$, real 3 dim parameter vectors a and b. The basic physics experiment is as follows: Suppose on the A side we have measurement instrument A with parameter vector a. On the B-side we have measurement instrument B with parameter vector b. There is a (Euclidean) distance $d(A, B) > 0$ between instruments A and B which can be large if necessary. In between the two instruments there is a source Σ generating particle pairs. We have, $d(\Sigma, A) = d(\Sigma, B) = \frac{1}{2}d(A, B)$. One particle of the pair is sent to A the other particle of the pair is sent to B. The physics of the two particles of the pair is such that they are entangled, [5],[7].

Then with the use of the λ we can write down the classical probability correlation between the two simultaneously measured particles. This is what we will call Bells formula.

$$E(a, b) = \int_{\lambda\in\Lambda} \rho(\lambda)A(a, \lambda)B(b, \lambda)d\lambda \tag{1}$$

Note that if ℓ is the short-hand notation for the random variable(s), the $E(a, b)$ simply is the expectation value of the product of two $\{-1, 1\}$ functions, $A(a, \lambda)$ and $B(b, \lambda)$. It can be written as $E(a, b) = E_\ell\left(A(a, \ell)B(b, \ell)\right)$. In fact we are looking at a special case of covariance computation [6] with the use of functions A and B, depending on parameters a and b and random variables captured with ℓ, projecting in $\{-1, 1\}$.

In (1) we therefore must have $\int_{\lambda\in\Lambda} \rho(\lambda)d\lambda = 1$. The integration $\int_{\lambda\in\Lambda}$ can be over, as many as we please, variables and over ditto spaces Λ. The density $\rho \geq 0$ also has a very general form.

Proof

From (1) an inequality for four setting combinations, a, b, c and d can be derived as follows

$$E(a,b) - E(a,c) = \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)A(a,\lambda)B(c,\lambda)A(d,\lambda)B(c,\lambda) - \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)A(a,\lambda)B(b,\lambda)A(d,\lambda)B(b,\lambda) + \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)A(a,\lambda)B(b,\lambda) - \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)A(a,\lambda)B(c,\lambda) \quad (2)$$

because, $\{B(c,\lambda)\}^2 = \{B(b,\lambda)\}^2 = 1$. From this it follows

$$E(a,b) - E(a,c) = \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)A(a,\lambda)B(b,\lambda)\{1 - A(d,\lambda)B(b,\lambda)\} + \int_{\lambda\in\Lambda} d\lambda\rho(\lambda)\left(-A(a,\lambda)B(c,\lambda)\right)\{1 - A(d,\lambda)B(c,\lambda)\} \quad (3)$$

Hence, because $1 - A(x,\lambda)B(y,\lambda) \geq 0$ for all x, y with $||x|| = ||y|| = 1$ and $A(a,\lambda)B(b,\lambda) \leq 1$ together with $-A(a,\lambda)B(c,\lambda) \leq 1$, it can be derived that

$$E(a,b) - E(a,c) \leq 2 - E(d,b) - E(d,c) \quad (4)$$

Or,

$$S(a,b,c,d) = E(a,b) + E(d,b) + E(d,c) - E(a,c) \leq 2. \quad (5)$$

Note, no physics assumptions were employed in the derivation of (4). It is pure mathematics. Suppose, further, that if we select for a, b, c and d

$$a = \frac{1}{\sqrt{2}}(1,0,1)\,,\ d = \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$$
$$b = (1,0,0)\,,\ c = (0,0,-1) \quad (6)$$

then $E(x,y)$ cannot be the inner product of the two vectors because, $a \cdot b = \frac{1}{\sqrt{2}}, d \cdot b = \frac{1}{2}, d \cdot c = \frac{1}{2}$ and $a \cdot c = -\frac{1}{\sqrt{2}}$. Hence,

$$S(a,b,c,d) = (a \cdot b) + (d \cdot b) + (d \cdot c) - (a \cdot c) = \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2} - \left(-\frac{1}{\sqrt{2}}\right) = 1 + \sqrt{2} > 2$$

In [7] Peres gives supporting argumentation to the form, $S(a,b,c,d) \leq 2$ derived here. So we can be sure (4) and $S(a,b,c,d) \leq 2$, are a generally valid expression for *all* possible models under the umbrella of (1).

COUNTER PROOF

In this section we will demonstrate that $E(x,y)$ can arbitrarily close approximate $x \cdot y$. As a reminder, both $x \in \mathbb{R}^3$ and $y \in \mathbb{R}^3$ are unit length parameter vectors, hence, $E(x,y) \in [-1,1]$. Although the physical details are unimportant, they can be verified to be within the bounds of applicability of Bell's formula (1).

Preliminaries

The model to be developed here follows the basic physical requirements of a local model. The requirements follow from looking at the physics experiment. In instrument A a set of hidden variables is supposed. Similarly, a set of hidden variables is supposed to reside in instrument B. The instruments are, as in the previous section, represented in the formulae by functions $A(x,\lambda_I,\chi)$ and $B(y,\lambda_{II},\chi)$. The (arrays of) hidden variables λ_I and λ_{II} are independent. A third set of hidden variables, denoted here by χ, are carried by the particles. The χ have a Gaussian density. The χ variables are independent of λ_I and λ_{II}. Moreover, λ_I and λ_{II} are independent. Looking at (1) we see that $\lambda = (\lambda_I, \lambda_{II}, \chi)$. Hence, looking at (1), $A(a,\lambda) = A(a,\lambda_I,\chi)$, $B(b,\lambda) = B(b,\lambda_{II},\chi)$ and $\rho(\lambda) = \rho(\lambda_I,\lambda_{II},\chi)$. This is a local and physically possible situation. Although the proof we deliver here is about a flaw in Bells argumentation, hence is purely mathematics, the necessary basic physical requirements are fulfilled in the model.

It must be stressed that, in anticipation of a more detailed definition below, the mathematical form of the probability density $\rho(\lambda_I, \lambda_{II}, \chi)$ remains *fixed* all the time. This can be easily verified in the section below devoted to the probability density. Clearly, the argumentation of that the inequality *cannot* be violated is then invalid. In the first place we show that clinging on to the inequality is merely attaching believe to one branch of the incompleteness which is demonstrated below. This believe is unfounded. The argumentation "the model is unphysical" is also broken because the basic requirements of a physical model are obeyed. Secondly, we already stated that the probability density remains fixed. Therefore it is possible to rightfully claim a genuine case of rejection of the *validity* of Bells argumentation. It can be verified that we use a model that perfectly fits the physics requirements behind Bells formula (1).

To wrap it up. There is no violation of locality in our model. There is no breach in the constancy of probability density form. The basic physics behind Bells formula are fulfilled. The breakdown of Bells argumentation is purely mathematical.

The opponent has to deliver proof why λ_I in instrument A and λ_{II} in instrument B that are independent and independently distributed variables and the independent and independently distributed χ variables, carried by the particles, is not complying to physical realistic locality.

It will be shown that the argument of Bell is based on negation incompleteness. In other words, we will show $S(a, b, c, d) > 2$ from the same formula that with the same physical requirements gives, along the branch of Bells argumentation, $S(a, b, c, d) \leq 2$. Hence, we will show that Bells formula supports negation incompleteness of the use of statistics in physics experimentation. If readers think otherwise then proof is the route to go. Believe, whoever is expressing it, should be -and actually is- worthless in scientific debate.

Probability density

Let us in the first place define a probability density ρ based upon two separate λ's and on (χ_1, χ_2, χ_3). Suppose, α is a variable to indicate the two separate systems of hidden variables. Let us denote them with I and II, i.e., $\alpha \in \{I, II\}$. Then,

$$\lambda_\alpha = (x_\alpha, \mu_{1,\alpha}, \mu_{2,\alpha}, \mu_{3,\alpha}, \tau_\alpha, n_\alpha) \in \mathbb{R}^6 \tag{7}$$

For λ_I we define a density $\rho_I = \rho_I(\lambda_I)$ and for λ_{II} a density $\rho_{II}(\lambda_{II})$.

The χ variables

For $\vec{\chi} = (\chi_1, \chi_2, \chi_3) \in \mathbb{R}^3$ let us define the Normal Gaussian density

$$\rho_{Norm} = \rho_{Norm}(\chi_1, \chi_2, \chi_3) = \left(\frac{1}{2\pi}\right)^{3/2} \exp\left[-\frac{1}{2}\sum_{k=1}^{3}\chi_k^2\right] \tag{8}$$

The integration of the normal density is, $\int_{-\infty}^{\infty} d\chi_1 \int_{-\infty}^{\infty} d\chi_2 \int_{-\infty}^{\infty} d\chi_3$ and is denoted with brackets, $\langle\cdot\rangle_{Norm}$ such that e.g. $\langle\rho_{Norm}\rangle_{Norm} = 1$. This enables us to formally write the total density as

$$\rho(\lambda_I, \lambda_{II}, \vec{\chi}) = \rho_I(\lambda_I)\rho_{II}(\lambda_{II})\rho_{Norm}(\vec{\chi}) \tag{9}$$

The density defined in (9) should fulfill the requirements alluded to in the previous section devoted to the requirements of the physics behind the model. The χ are mutually independent and are independent of the "instrument variables" λ_I and λ_{II}. Subsequently, let us turn to the use of the χ variables in the model.

Let us, firstly, define the Heaviside function $H(x) = 1 \Leftrightarrow x \geq 0$ and $H(x) = 0 \Leftrightarrow x < 0$. In the second place let us define a sign function from the Heaviside, $\text{sign}(x) = 2H(x) - 1$. Because of the symmetry of the Gaussian in (8), we have in the angular notation of integration for $i, j = 1, 2, 3$ that

$$\langle\text{sign}(\chi_i)\text{sign}(\chi_j)\rho_{Norm}\rangle_{Norm} = \left(\frac{1}{2\pi}\right)^{3/2} \int_{-\infty}^{\infty} d\chi_1 \int_{-\infty}^{\infty} d\chi_2 \int_{-\infty}^{\infty} d\chi_3 \,\text{sign}(\chi_i)\text{sign}(\chi_j) \exp\left[-\frac{1}{2}\sum_{k=1}^{3}\chi_k^2\right] = \delta_{i,j} \tag{10}$$

with, $\delta_{i,j} = 1 \Leftrightarrow i = j$ and $\delta_{i,j} = 0 \Leftrightarrow i \neq j$.

Definition of $\rho_\alpha(\lambda_\alpha),\ \alpha \in \{I, II\}$

Here we turn to the densities, $\rho_\alpha(\lambda_\alpha),\ \alpha \in \{I, II\}$. The $\rho_\alpha(\lambda_\alpha)$ is a product of five factors, ρ_α^r, $r = 0, 1, 2, 3, 4$. We have, for $T \in \mathbb{N}$ and $T >> 16$

$$\begin{aligned}
\rho_\alpha^0 &= \frac{1}{16T\left(1-\frac{4}{T}\right)}, \\
\rho_\alpha^1 = \rho_\alpha^1(x_\alpha) &= H\left(\frac{1}{4}+x_\alpha\right)H\left(-\frac{1}{T}-x_\alpha\right)+H\left(\frac{1}{4}-x_\alpha\right)H\left(-\frac{1}{T}+x_\alpha\right), \\
\rho_\alpha^2 = \rho_\alpha^2(\vec{\mu}_\alpha) &= \prod_{k=1}^{3} H\left(1+\mu_{k,\alpha}\right)H\left(1-\mu_{k,\alpha}\right), \\
\rho_\alpha^3 = \rho_\alpha^3(\tau_\alpha) &= H\left(T+\tau_\alpha\right)H\left(T-\tau_\alpha\right), \\
\rho_\alpha^4 = \rho_\alpha^4(n_\alpha) = 1 \Leftrightarrow n_\alpha \in \{0,1\}\ \&\ \rho_\alpha^4 &= \rho_\alpha^4(n_\alpha) = 0 \Leftrightarrow n_\alpha \notin \{0,1\}.
\end{aligned} \tag{11}$$

Hence, using (11) we then define $\rho_\alpha = \prod_{r=0}^{4} \rho_\alpha^r$.

Subsequently, let us also introduce the angle notation for integration of α densities similar to what we wrote for the Normal density. We have, $T >> 16$

$$\langle \rho_\alpha \dots \rangle_\alpha = \frac{1}{2^4T\left(1-\frac{4}{T}\right)}\left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_\alpha + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_\alpha\right)\prod_{k=1}^{3}\int_{-1}^{1} d\mu_{k,\alpha}\int_{-T}^{T} d\tau_\alpha \sum_{n_\alpha=0}^{1} \dots \tag{12}$$

The previous leads us to $\langle \rho_\alpha \rangle_\alpha = \frac{2}{2^4T\left(1-\frac{4}{T}\right)}\left(2^3 \times 2T\right)\left(\frac{1}{2}-\frac{2}{T}\right) = 1$, and, $T \sim$ sufficiently large number. Looking at the definition of the total density in (9), it can be derived that

$$\int_{\lambda\in\Lambda} d\lambda\rho(\lambda) = \langle\langle\rho_I\rangle_I\,\langle\rho_{II}\rangle_{II}\,\rho_{Norm}\rangle_{Norm} = \langle\rho_I\rangle_I\,\langle\rho_{II}\rangle_{II}\,\langle\rho_{Norm}\rangle_{Norm} = 1 \tag{13}$$

Hence, a valid probability density in (9) is obtained where use is made of (8) and (11). The density given in (9) is a valid fixed form density that is completely local.

Auxiliary functions

The auxiliary function $\Delta_T(y)$: Let us in the first place define

$$\Delta_T(y) = \frac{2/\pi}{1+T^2y^2} \tag{14}$$

Then, because $1 + T^2y^2 \geq 1$ for $y \geq 0$, we find that $-T \leq T\Delta_T(y) \leq T$ is valid and so, $\mathrm{sign}\left(T\Delta_T(y) - \tau_\alpha\right)$ can be meaningfully employed in an integration.

$$\int_{-T}^{T} \mathrm{sign}\left(T\Delta_T(y)-\tau_\alpha\right)d\tau_\alpha = \int_{-T}^{T\Delta_T(y)} d\tau_\alpha - \int_{T\Delta_T(y)}^{T} d\tau_\alpha = \left(T\Delta_T(y)-(-T)\right)-\left(T-T\Delta_T(y)\right) = 2T\Delta_T(y) \tag{15}$$

This is true for arbitrary real y. Hence, also for $y = x_\alpha^2 - \frac{1}{T^2}$ the previous is true.

Elements of the measurement functions: In the second place let us define

$$\begin{aligned}
\sigma_a &= \sum_{k=1}^{3} a_k \mathrm{sign}(\chi_k) \\
\sigma_b &= \sum_{k=1}^{3} b_k \mathrm{sign}(\chi_k)
\end{aligned} \tag{16}$$

It is easily demonstrated that $|\sigma_a| \leq \sqrt{3}$ and $|\sigma_b| \leq \sqrt{3}$.

Indicators: In the third place, let us define three disjoint partitions of the real interval $[-\sqrt{3},\sqrt{3}]$.

$$\begin{array}{l} I_1 = \{x \in \mathbb{R} \,|\, -\sqrt{3} \le x < -1\} \\ I_2 = \{x \in \mathbb{R} \,|\, -1 \le x \le 1\} \\ I_3 = \{x \in \mathbb{R} \,|\, 1 < x \le \sqrt{3}\} \end{array} \tag{17}$$

Clearly, $I_1 \cap I_2 = \emptyset$ together with $I_2 \cap I_3 = \emptyset$ and $I_3 \cap I_1 = \emptyset$. With the use of the three disjoint intervals we may employ the following auxiliary function $\iota_k(x) = 1 \Leftrightarrow x \in I_k$ and $\iota_k(x) = 0 \Leftrightarrow x \notin I_k$. If $\iota_1(x) = 1$, then, $\iota_2(x) = \iota_3(x) = 0$. If $\iota_2(x) = 1$, then, $\iota_1(x) = \iota_3(x) = 0$. If $\iota_3(x) = 1$, then, $\iota_2(x) = \iota_1(x) = 0$.

Auxiliary functions in measurement functions: Let us fourthly also define functions that will be employed together with the $\iota_k(x)$, with, $k = 1, 2, 3$.

$$\begin{array}{l} s_{1,\alpha}(z_\alpha, \lambda_\alpha) = \{n_\alpha \mathrm{sign}\,(z_\alpha + 1 - \mu_{1,\alpha}) - \delta_{0,n_\alpha}\}\,\mathrm{sign}\,(T\Delta_T(f_\alpha(x_\alpha)) - \tau_\alpha) \\ s_{2,\alpha}(z_\alpha, \lambda_\alpha) = \mathrm{sign}\,(z_\alpha - \mu_{2,\alpha}) \\ s_{3,\alpha}(z_\alpha, \lambda_\alpha) = \{n_\alpha \mathrm{sign}\,(z_\alpha - 1 - \mu_{3,\alpha}) + \delta_{0,n_\alpha}\}\,\mathrm{sign}\,(T\Delta_T(f_\alpha(x_\alpha)) - \tau_\alpha) \end{array} \tag{18}$$

The z_α is a short-hand and follows, $z_I = \sigma_a$ and $z_{II} = \sigma_b$, with $\alpha \in \{I, II\}$. The σ_a and σ_b are defined in (16). It is quite easily verifiable that $s_{k,\alpha}(z_\alpha, \lambda_\alpha) \in \{-1, 1\}$, with $k = 1, 2, 3$. Note, $n_\alpha \in \{0, 1\}$. In (18) the short-hand, $f_\alpha(x_\alpha) \equiv x_\alpha^2 - \frac{1}{T^2}$ is employed.

Measurement functions

With the use of the previous definitions we are now able to define the measurement functions A and B.

$$\begin{array}{l} A(a, \lambda_I, \vec{\chi}) = \sum_{k=1}^{3} \iota_k(\sigma_a) s_{k,I}(\sigma_a, \lambda_I), \\ B(b, \lambda_{II}, \vec{\chi}) = \sum_{k=1}^{3} \iota_k(\sigma_b) s_{k,II}(\sigma_b, \lambda_{II}) \end{array} \tag{19}$$

Because the $\iota_k(x)$, $k = 1, 2, 3$ only have one of them unequal to zero, i.e. the I_k of (17) are disjoint, and the s of equation (18) are in $\{-1, 1\}$, we have that both $A(a, \lambda_I, \vec{\chi}) \in \{-1, 1\}$ and $B(b, \lambda_{II}, \vec{\chi}) \in \{-1, 1\}$. Hence the measurement functions in (19) are valid in a Bell correlation $E(a, b)$ such as given in (1). No deeper physics assumption hides behind this because one simply may select functions that project in $\{-1, 1\}$. Bell's formula is general. The A and B are called measurement functions but that is totally unimportant to the mathematics to be developed here.

Clearly, we can conclude that our definitions comply to the basic physical requirements of a local model. Hence, the model is allowed in Bells formula. Note that the measurement representing functions, projecting in $\{-1, 1\}$, also follow the basic physical requirements. The derivation of $S(a, b, c, d) \le 2$ in (5) is therefore possible in this case. We will show that this is just one branch of the argument.

Evaluation

Looking at Bell's correlation in (1) let us write

$$\begin{array}{l} E(a, b) = \left\langle \langle\langle \rho_I \rho_{II} \rho_{Norm} A(a, \lambda_I, \vec{\chi}) B(b, \lambda_{II}, \vec{\chi}) \rangle_I \rangle_{II} \right\rangle_{Norm} = \\ \qquad \left\langle \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I \, \langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II} \, \rho_{Norm} \right\rangle_{Norm} \end{array} \tag{20}$$

Note, λ_I is only found in ρ_I and $A(a, \lambda_I, \vec{\chi})$ while λ_{II} is only found in ρ_{II} and $B(b, \lambda_{II}, \vec{\chi})$. The $\vec{\chi}$, via the σ_a and σ_b dependence is shared between functions A and B. Note for completeness that the function B does not depend on a and A does not depend on b which is in accordance with Einstein's locality condition [2].

In order to have a proper evaluation of the integrals in $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I$, and $\langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II}$ it is sufficient to look at the A side only. The B side evaluations obviously follows similar rules.

We can write explicitly for $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I$

$$\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{1}{2^4 T \left(1 - \frac{4}{T}\right)} \left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \prod_{k=1}^{3} \int_{-1}^{1} d\mu_{k,I} \int_{-T}^{T} d\tau_I \sum_{n_I=0}^{1} A(a, \lambda_I, \vec{\chi}) \tag{21}$$

As it follows from (19), we can have three cases for $\iota_k(\sigma_a)$. Suppose, the selection a and the values of $\vec{\chi}$ are such that σ_a is in I_1. Then $A(a,\lambda_I,\vec{\chi}) = s_{1,I}(\sigma_a,\lambda_I)$. Hence, with the use of (18)

$$\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{1}{2^4 T\left(1-\frac{4}{T}\right)}\left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I\right)\prod_{k=1}^{3}\int_{-1}^{1} d\mu_{k,I}\times$$
$$\int_{-T}^{T} d\tau_I \sum_{n_I=0}^{1}\left\{n_I \mathrm{sign}\left(\sigma_a+1-\mu_{1,I}\right)-\delta_{0,n_I}\right\}\mathrm{sign}\left(T\Delta_T(f_I(x_I))-\tau_I\right) \tag{22}$$

From (15) it already follows that the τ_I integral in (22) equals $2T\Delta_T(f_I(x_I))$. So let us look at the μ integrals and the n_I sum. Before entering into more details let us note that $(\sigma_a+1)\in[-1,1]$ and so

$$\int_{-1}^{1} d\mu\, \mathrm{sign}\left(\sigma_a+1-\mu\right) = \int_{-1}^{\sigma_a+1} d\mu - \int_{\sigma_a+1}^{1} d\mu = 2\left(\sigma_a+1\right) \tag{23}$$

We subsequently see, because $\int_{-1}^{+1} d\mu_{2,I} = \int_{-1}^{+1} d\mu_{3,I} = 2$, together with $\int_{-1}^{+1} d\mu_{1,I} = 2$,

$$\sum_{n_I=0}^{1}\left(\prod_{k=1}^{3}\int_{-1}^{1} d\mu_{k,I}\right)\left\{n_I \mathrm{sign}\left(\sigma_a+1-\mu_{1,I}\right)-\delta_{0,n_I}\right\} = 2^3\sum_{n_I=0}^{1}\left[n_I(\sigma_a+1)-\delta_{0,n_I}\right] =$$
$$2^3[-1+(\sigma_a+1)] = 2^3\sigma_a \tag{24}$$

Hence, if K_T is defined by

$$K_T \equiv \left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I\right)\Delta_T(f_I(x_I)) \tag{25}$$

then, $\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{\sigma_a K_T}{\left(1-\frac{4}{T}\right)}$ when $\sigma_a\in I_1$.

Let us now suppose, $\sigma_a\in I_3$, i.e. $\sigma_a-1\in[-1,1]$. Hence, only $\iota_3(\sigma_a)=1$ and hence, $A(a,\lambda_I,\vec{\chi}) = s_{3,I}(\sigma_a,\lambda_I)$. This implies,

$$\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{1}{2^4 T(1-\frac{4}{T})}\left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I\right)\prod_{k=1}^{3}\int_{-1}^{1} d\mu_{k,I}\times$$
$$\int_{-T}^{T} d\tau_I \sum_{n_I=0}^{1}\left\{n_I \mathrm{sign}\left(\sigma_a-1-\mu_{3,I}\right)+\delta_{0,n_I}\right\}\mathrm{sign}\left(T\Delta_T(f_I(x_I))-\tau_I\right) \tag{26}$$

In the case that $\sigma_a\in I_3$, we also find $\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{\sigma_a K_T}{\left(1-\frac{4}{T}\right)}$. Finally let us look at the case where $\sigma_a\in I_2$. Here we have $\sigma_a\in[-1,1]$. So,

$$\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{1}{2^4 T(1-\frac{4}{T})}\left(\int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I\right)\prod_{k=1}^{3}\int_{-1}^{1} d\mu_{k,I}\int_{-T}^{T} d\tau_I\sum_{n_I=0}^{1}\mathrm{sign}\left(\sigma_a-\mu_{2,I}\right) \tag{27}$$

The result of integration in (27) is that $\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{\sigma_a\left(1-\frac{4}{T}\right)}{\left(1-\frac{4}{T}\right)} = \sigma_a$ and $T\sim$ sufficiently large number.

The integral K_T

In two cases of σ_a we have $\langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I = \frac{\sigma_a K_T}{1-\frac{4}{t}}$ and K_T is defined in (25). For the ease of notation let us write $T=n$. Let us repeat the definition of the K integral

$$K_n = \left(\int_{-\frac{1}{4}}^{-\frac{1}{n}} dx + \int_{\frac{1}{n}}^{\frac{1}{4}} dx\right)\Delta_n\left(x^2-(1/n^2)\right) \tag{28}$$

The integral we want to discuss here is then re-written, using $\Delta_n\left(x^2-(1/n^2)\right)$ defined in (14) as

$$K_n = \frac{2}{\pi}\int_{-\frac{1}{4}}^{-\frac{1}{n}} \frac{dx}{1+n^2(x^2-(1/n^2))^2} + \frac{2}{\pi}\int_{\frac{1}{n}}^{\frac{1}{4}} \frac{dx}{1+n^2(x^2-(1/n^2))^2} \tag{29}$$

Now let us take, $y = x^2-(1/n^2)$. The upper limit of y is, $\frac{1}{4^2}-\frac{1}{n^2}$, $n >> 16$, while the lower limit is 0. Hence, for negative x, we have $x = -\sqrt{y+\frac{1}{n^2}}$. For positive x, we see, $x = \sqrt{y+\frac{1}{n^2}}$. Hence, noting $dx = \pm\frac{dy/2}{\sqrt{y+\frac{1}{n^2}}}$, in terms of y we can write for the two terms in K_n

$$K_n = -\frac{2}{\pi}\int_{\frac{1}{4^2}-\frac{1}{n^2}}^{0} \frac{dy}{1+n^2y^2}\frac{1/2}{\sqrt{y+\frac{1}{n^2}}} + \frac{2}{\pi}\int_{0}^{\frac{1}{4^2}-\frac{1}{n^2}} \frac{dy}{1+n^2y^2}\frac{1/2}{\sqrt{y+\frac{1}{n^2}}} \tag{30}$$

Hence,

$$K_n = \left(\frac{2}{\pi}\right) 2\int_{0}^{\frac{1}{4^2}-\frac{1}{n^2}} \frac{dy}{1+n^2y^2}\frac{1/2}{\sqrt{y+\frac{1}{n^2}}} \tag{31}$$

Let us in the first place try to find the upper limit of K_n from the previous equation. Note, for $n > 4$ that $y+\frac{1}{n^2} \geq \frac{1}{n^2}$, hence, $\frac{1}{\sqrt{y+\frac{1}{n^2}}} \leq n$, given $\frac{1}{4^2}-\frac{1}{n^2} \geq y \geq 0$. This implies

$$K_n \leq \frac{2}{\pi}\int_{0}^{\frac{1}{4^2}-\frac{1}{n^2}} \frac{n dy}{1+n^2y^2} \leq \frac{2}{\pi}\arctan\left[\frac{n}{4^2}-\frac{1}{n}\right] \leq 1,\ (n \sim large). \tag{32}$$

The lower limt in K_n can be found, looking at, $1+n^2y^2 \leq 1+\epsilon^2+n^2y^2$, hence,

$$\frac{1}{1+n^2y^2} \geq \left(\frac{1}{1+\epsilon^2}\right)\left\{\frac{1}{1+n^2\left(\frac{y}{\sqrt{1+\epsilon^2}}\right)^2}\right\}.$$

Let us, in the second place, take $z = y/\sqrt{1+\epsilon^2}$, then, with $dz = dy/\sqrt{1+\epsilon^2}$ we can rewrite the lower limit like

$$K_n \geq \frac{2}{\pi}\frac{1}{\sqrt{1+\epsilon^2}}\int_{0}^{z_{max}} \frac{n\,dz}{1+n^2z^2}\frac{1}{\sqrt{1+n^2z\sqrt{1+\epsilon^2}}} \tag{33}$$

together with, $z_{max} = \left(\frac{1}{4^2}-\frac{1}{n^2}\right)/\sqrt{1+\epsilon^2}$. Note, $-1 \leq \frac{2}{\pi}\arctan(x) \leq 1$ for all $x \in \mathbb{R}\cup\{-\infty,\infty\}$. With arctan the inverse function of the function $-\infty \leq \tan(x) \leq \infty$, with,$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ is intended. Using, $\frac{d}{dz}\arctan(nz) = \frac{n}{1+n^2z^2}$ we are able to write

$$K_n \geq \frac{2}{\pi}\frac{1}{\sqrt{1+\epsilon^2}}\int_{0}^{z_{max}} dz\left(\frac{d}{dz}\arctan(nz)\right)\left[1+n^2z\sqrt{1+\epsilon^2}\right]^{-1/2} \tag{34}$$

Because $\arctan(0)=0$ and we have $0\times n^2 = 0$ when $n \sim$ sufficiently large number, it follows that the constant factor, C_n in a partial integration treatment of the right hand of (34) looks like

$$C_n = \frac{2/\pi}{\sqrt{1+\epsilon^2}}\left\{\arctan\left[\left(\frac{n}{4^2}-\frac{1}{n}\right)/\sqrt{1+\epsilon^2}\right]\left[1+n^2z_{max}\sqrt{1+\epsilon^2}\right]^{-1/2} - \arctan(0)[1+(n^2\times 0)]^{-1/2}\right\} \tag{35}$$

and $1+n^2z_{max}\sqrt{1+\epsilon^2} = 1+\frac{n^2}{4^2}-1$. Hence, $\left[1+n^2z_{max}\sqrt{1+\epsilon^2}\right]^{-1/2} = \frac{1}{\sqrt{\frac{n^2}{4^2}}} = \frac{4}{n}$. This implies, the constant factor

$$C_n = \frac{2/\pi}{\sqrt{1+\epsilon^2}}\arctan\left[\left(\frac{n}{4^2}-\frac{1}{n}\right)/\sqrt{1+\epsilon^2}\right]\frac{4}{n} \approx 0^+$$

for, $n \sim$ sufficiently large number.

So, under a limit, $n \sim$ sufficiently large number, for $z \neq 0$, we see from $-1 \leq \frac{2}{\pi}\arctan(nz) \leq 1$ that the extremes -1 and $+1$ are quickly approximated. In turn, the partial integration of the right hand of (34), finally looks like

$$K_n \geq C_n - \frac{1}{\sqrt{1+\epsilon^2}}\frac{2}{\pi}\int_0^{z_{max}} dz\ \arctan(nz)\left(\frac{d}{dz}\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{-1/2}\right) \tag{36}$$

Hence, when $z > 0$ from $-\frac{2}{\pi}\arctan(nz) \approx -1$ and $C_n \approx 0^+$, for $n \sim$ large,

$$K_n \gtrapprox -\frac{1}{\sqrt{1+\epsilon^2}}\int_0^{z_{max}} dz\left(\frac{d}{dz}\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{-1/2}\right) \tag{37}$$

Note that the step from (36) to (37) is supported by the following.

First let us note that

$$\frac{d}{dz}\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{-1/2} = \left(\frac{-1}{2}\right)\frac{n^2\sqrt{1+\epsilon^2}}{\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{3/2}} \tag{38}$$

Demonstrating the fact that $K_n > 0$ for large n, we can ignore constants in (36). We note that there is a $\Delta z > 0$ beyond which, given n sufficiently large, that the expression in (38) vanishes quickly for $z_{max} \geq z > \Delta z$. In that interval we also may write

$$\arctan(nz) \approx nz \tag{39}$$

For only essential terms we have for the integral in (36)

$$\int_0^{z_{max}} dz\ \arctan(nz)\left(\frac{d}{dz}\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{-1/2}\right) \propto \int_0^{\Delta z} dz\frac{n^3 z}{[1+n^2 z]^{3/2}} \tag{40}$$

Because we may select n large such that for all z in the interval $[0, \Delta z]$, we have $1 + n^2 z \approx n^2 z$. if $n^2 z >> 1$, then, $n^3 z >> 1$. Hence, we may approximate (40) with

$$\int_0^{z_{max}} dz\ \arctan(nz)\left(\frac{d}{dz}\left[1+n^2 z\sqrt{1+\epsilon^2}\right]^{-1/2}\right) \propto \int_0^{\Delta z} dz z^{-1/2} \propto \sqrt{\Delta z} > 0 \tag{41}$$

Because the suppressed constants in this argument, looking at (36) are positive and C_n is positive small, we may conclude that $K_n > 0$ for large n. Interested readers can verify that numerical straightforward proof exists too which shows that the right hand of (36) is positive nonzero.

Equation (37) then gives, using $z_{max} = \left(\frac{1}{4^2} - \frac{1}{n^2}\right)/\sqrt{1+\epsilon^2}$,

$$K_n \gtrapprox -\frac{1}{\sqrt{1+\epsilon^2}}\left[\frac{4}{n} - 1\right] \to \frac{1}{\sqrt{1+\epsilon^2}}, \tag{42}$$

under the condition, $n \sim$ large number. Hence, we may conclude that:

$$1 \geq \lim_{n\to\infty} K_n \gtrapprox \frac{1}{\sqrt{1+\epsilon^2}}.$$

This leads us to, $K_n \approx 1$, where ϵ^2 can be arbitrary small positive real.

RESULT

Returning to $\sigma_a \in I_1$ and $\sigma_a \in I_3$, it is found that approximately we may write $\langle\rho_I A(a, \lambda_I, \vec{\chi})\rangle_I \approx \sigma_a$, because under $T = n$ sufficiently large, $K_T \approx 1$. Moreover under $T \sim$ sufficiently large number we also see that for $\sigma_a \in I_2$ that $\langle\rho_I A(a, \lambda_I, \vec{\chi})\rangle_I \approx \sigma_a$. Hence, because a similar evaluation for B can take place

$$\begin{aligned}\langle\rho_I A(a, \lambda_I, \vec{\chi})\rangle_I &\approx \sigma_a\\ \langle\rho_{II} B(b, \lambda_{II}, \vec{\chi})\rangle_{II} &\approx \sigma_b\end{aligned} \tag{43}$$

Because using (20) and our previous result, we are allowed to write

$$E(a,b) = \langle \langle \rho_I A(a,\lambda_I,\vec{\chi})\rangle_I \langle \rho_{II} B(b,\lambda_{II},\vec{\chi})\rangle_{II} \rho_{Norm}\rangle_{Norm} \approx \langle \sigma_a(\vec{\chi})\sigma_b(\vec{\chi})\rho_{Norm}\rangle_{Norm} \tag{44}$$

This implies, together with (10) that

$$E(a,b) \approx \sum_{i=1}^{3}\sum_{j=1}^{3} a_i b_j \delta_{i,j} = \sum_{j=1}^{3} a_j b_j \tag{45}$$

The latter equation concludes the refutation part of the present paper. In the appendix, to help the reader, an algorithm in R is presented to demonstrate the numerical possibility of $K_n \approx 1$.

CONCLUSION

In our paper, under locality [1], [2], we have construed a model that must, by design, not be able *in any way* to violate the $S(a,b,c,d) \leq 2$. We note that the local hidden variables physical picture is that variables with the index $\alpha = I$ reside in measurement instrument A and $\alpha = II$ reside in measurement instrument B. The χ Gaussian variables can be seen as being carried by the particles to the respective measurement systems. This is a perfectly valid physical possibility.

In the paper it was derived that a model with $S(a,b,c,d) > 2$ can be obtained observing all conditions for a local model. I.e. $E(a,b) \approx \sum_{j=1}^{3} a_j b_j$. was derived using local modeling. In passing we note that, using the random variable notation ℓ, it follows from our model that $E_\ell\left(A(a,\ell)\right) = E_\ell\left(B(b,\ell)\right) = 0$. This easily derives from the symmetry of the Gaussian.

Our result is unrelated to a quantum mechanical violation of the inequality. We can make this claim because, in the first place, a local Bell formula model was used. All the requirements for a local physical model were fulfilled. The probability density has a fixed form. The objection, "the proposed model is unphysical" is clearly invalid. The reader carefully notes that all the basic physical requirements for a local model were fulfilled. In the second place, looking at the derived inequality from Bell's formula, one must mathematically *never* be able, with what kind of a model one cares to select under the umbrella of locality, to obtain $S(a,b,c,d) > 2$.

Subsequently, the reader is reminded that in the paper *no hidden physics assumptions* were used in any step of the derivation. The derivation was completely mathematical. The basic physical requirements are merely there to show that Bells formula is valid physics in both branches of the argument. If the reader thinks it's otherwise he has to *demonstrate* that the mathematics provided in the model cannot be realized in a physical situation. Bells formula is general so in our conception, this form of opposition breaks down. To be more specific, there were *no hidden physics assumptions like non-locality* in the derivation of $E(a,b) \approx \sum_{j=1}^{3} a_j b_j$, hence, $S(a,b,c,d) > 2$, in our model. The reader can easily verify this.

In previous papers, the first author already pointed out that there are inconsistencies in the Bell argumentation [8], [9] and [10]. The presented demonstration shows unequivocally that for fixed density and realizable physics, Bells formula give rise to conflicting conclusions. The believe in a one-branched Bell formula, such as expressed in [**?**] is unfounded because it neglects the possibility of the demonstrated negation incompleteness. Arguments quoted from [**?**] in favor of CHSH or one branch interpretation of Bells formula, are therefore invalid. We note especially the implicit claim about the necessity of a computer violation before due credit can be given to any, more theoretically oriented, criticism. This is, according to us, an unfounded and malignant expression of keeping the faith in only one preferred branch of the argument. The first author has also performed work in the field of computer violation study [11].

To be complete, we note that the $E(x,y) \approx x \cdot y$ for any setting $x \in \mathbb{R}^3$ unit length and $y \in \mathbb{R}^3$ unit length, where the form of the density of the hidden variables remain fixed as given in equation (11) above.

We demonstrated that there exists a necessity to take a closer look at the rulebook of ZFC mathematics [3] and arguments concerning the foundations of quantum mechanics.

REFERENCES

[1] J.S. Bell, "On the Einstein Podolsky Rosen paradox," Physics, 1, 195-200, 1964.
[2] A. Einstein, B., Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," Phys. Rev. 47, 777-780, 1935.
[3] H. Friedman, "Concrete mathematical incompleteness", Lecture at the Andrzej Mostowski Centenary, Warsaw, Poland October 11, 2013
[4] H. Friedman, "Boolean Relation Theory and Incompleteness," Ohio state University, 2010.
[5] D. Bohm, "Quantum Theory," pp 611-634, Prentice-Hall, Englewood Cliffs, 1951.
[6] R.V. Hogg, A.T. Graig,"Introduction to Mathematical Statistics, Third edition" Prentice-Hall, Englewood Cliffs, 1995.
[7] A. Peres, "Quantum Theory: Concepts and Methods," Kluwer Academic, 2002.
[8] H. Geurdes, "A Counter-Example to Bell's Theorem with a Softened Singularity, " Galilean Electrodynamics, 17, 16-20, 2006.
[9] H. Geurdes, "A note on Bells' expression for the QM correlation," AIP Conf. Proc. 1508, 365-369 2012; doi: 10.1063/1.4773149
[10] H. Geurdes, "A probability loophole in the CHSH," Results in Physics, 4, 81-82, 2014.
[11] H. Geurdes, "A computational proof of locality in entanglement.", submitted, 2017.

APPENDIX

With each computable selection of h, a matching n can be performed (in computable boundaries of course) such that $1 \geq K_n \gtrapprox 1$ can be found. We e.g. have $h = 6.25 \times 10^{-6}$ and $n = 4.31 \times 10^{7}$ and $h = 6.25 \times 10^{-7}$ and $n = 1.38 \times 10^{9}$.

```
#
n<-4.31e7
epsilon<-1e-2
sq<-sqrt(1+(epsilon**2))
nIter<-1e4
f<-array(0,nIter)
xA<-array(0,nIter)
gLob<-0
lUpb<-(1/16)-(1/n**2)
h<-(lUpb-gLob)/nIter
k<-0
x<-0
sum<-0
while(x < lUpb){
  k<-k+1
  x<-x+(k*h)
  xA[k]<-x
  f[k]<-(-2/pi)*atan(n*x)
  chi<-(-1/2)*(n**2)*sq
  y<-(1+(x*sq*(n**2)))**(-3/2)
  p<-(x*sq*(n**2))**(-3/2)
 #  print(p-y)
  chi<-chi*y
  f[k]<-f[k]*chi
  sum<-sum+f[k]
}
plot(xA[1:10],f[1:10],type='l')
#
cn<-(2/pi)/sq
cn<-cn*atan(n*lUpb/sq)*4/n
sum<-sum+cn
print(paste0("1 >= Kn >= ",sum))
```