The quantum theory of a closed string.

Johan Noldus

October 9, 2024

Abstract

We look at the theory of closed strings in a novel way as well classically as quantum mechanically and show how the usual Virasoro problem is circumvented.

1 Introduction.

The Virasoro problem in string theory [2] arises most clearly in the covariant quantization where one has hermitian generators L_n with $n \in \mathbb{Z}$ which have to be regarded as constraints; that is physical states have to satisfy $L_n|\Psi\rangle = 0$ for $n \neq 0$ and $L_0|\Psi\rangle = a|\Psi\rangle$ with $a \neq 0$. The Virasoro algebra without central anomalies $c(n)$,

$$[L_n, L_m] = i(n-m)L_{n+m} + c(n-m)1$$

makes this impossible given that

$$0 = [L_n, L_{-n}]\,|\Psi\rangle = 2inL_0|\Psi\rangle = 2ina|\Psi\rangle$$

which contradicts $a \neq 0$. The "fix" of the problem is to keep the constraints $L_n|\psi\rangle = 0$ for $n > 0$ while dropping the others. This leads to physical operators changing particle species, spin and angular momentum causing all known conservation laws of particle physics to fail (but not largely in practice). The downside is that the geometrical description of the theory is totally lost at the quantum level even in a Minkowski background and that everything becomes therefore gauge dependent. This is not expected given that quantum theory works perfectly fine for flat geometries and we shall trace back the problem to the non-geometric character of quantum theory itself. In that context, the worldsheet formulation evaporates and only reparametrisations of the type $t'(t)$ and $s'(s)$ can be made such that the Virasoro problem dissapears giving rise to two mutually commuting symmetry algebra's as the *full* symmetry algebra.

2 Strings from the viewpoint of covariant quantum theory.

In this section, we shall look for the correct clasical equations of motion for a closed string on a generic curved spacetime background. Given a closed string worldsheet $\gamma(t,s)$, we define two vectorfields $\mathbf{V} = \partial_t\gamma(t,s)$ and $\mathbf{Z} = \partial_s\gamma(t,s)$ where $t \in [0,T]$ and $s \in [0,L]$ with periodic boundary conditions in s; obviously

$[\mathbf{V}, \mathbf{Z}] = 0$.

The law one is looking for clearly is of the kind

$$\nabla_{\mathbf{V}}\mathbf{V} = \mathbf{F}(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}}\mathbf{Z}, g(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z})) = \alpha\mathbf{V} + \beta\mathbf{Z} + \delta\mathbf{A}$$

where $\mathbf{A} = \nabla_{\mathbf{Z}}\mathbf{Z}$ is the spatial acceleration and we only include nontrivial gravitaional degrees of freedom which are tangential to the string worldsheet. Because we want to emiminate reparametrizations of the string, we endow $\mathbf{Z}$ with a physical significance. That is, we demand that it corresponds to an arclength, that is

$$g(\mathbf{Z}, \mathbf{Z}) = c$$

where c is a constant. Since this property has to be preserved under time evolution, we compute that

$$0 = \nabla_{\mathbf{V}}g(\mathbf{Z}, \mathbf{Z}) = 2g(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{Z}) = 2\nabla_{\mathbf{Z}}g(\mathbf{V}, \mathbf{Z}) - 2g(\mathbf{V}, \mathbf{A}).$$

To make this equation generic, it is desirable to impose the constraints

$$g(\mathbf{V}, \mathbf{Z}) = d, \; g(\mathbf{V}, \mathbf{A}) = 0.$$

Taking the time evolution of the former gives

$$0 = \nabla_{\mathbf{V}}g(\mathbf{V}, \mathbf{Z}) = g(\nabla_{\mathbf{V}}\mathbf{V}, \mathbf{Z}) + g(\mathbf{V}, \nabla_{\mathbf{V}}\mathbf{Z}) = \alpha g(\mathbf{V}, \mathbf{Z}) + \beta g(\mathbf{Z}, \mathbf{Z}) + \frac{1}{2}\nabla_{\mathbf{Z}}g(\mathbf{V}, \mathbf{V})$$

which suggests that either $\alpha, \beta = 0$ and $g(\mathbf{V}, \mathbf{V}) = e$ or $g(\mathbf{V}, \mathbf{Z}) = 0$ or $g(\mathbf{Z}, \mathbf{Z}) = 0$. Taking the time derivative of our last constraint

$$0 = \nabla_{\mathbf{V}}g(\mathbf{V}, \mathbf{A}) = g(\nabla_{\mathbf{V}}\mathbf{V}, \mathbf{A}) + g(\mathbf{V}, \nabla_{\mathbf{V}}\mathbf{A}) = g(A, A)\delta + g(\mathbf{V}, R(\mathbf{V}, \mathbf{Z})\mathbf{Z}) + g(\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V})$$

which can be rewritten as

$$g(A, A)\delta + g(\mathbf{V}, R(\mathbf{V}, \mathbf{Z})\mathbf{Z}) + \nabla_{\mathbf{Z}}g(\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}) - g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}) = g(A, A)\delta - g(R(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) - g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}).$$

Hence, for consisteny, we demand that $g(A, A) \neq 0$ and

$$\delta = \frac{g(R(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})}{g(A, A)}.$$

There is nothing further to deduce and all constraints are presrved under evolution. This suggests one to put the unknown functions α, β to zero to arrive at the theory

$$g(\mathbf{Z}, \mathbf{Z}) = c, \; g(\mathbf{V}, \mathbf{V}) = e, \; g(\mathbf{V}, \mathbf{Z}) = d, \; g(\mathbf{V}, \mathbf{A}) = 0$$

with as force law

$$\nabla_{\mathbf{V}}\mathbf{V} = \frac{g(R(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})}{g(A, A)}\mathbf{A}.$$

In ordinary string theory on flat Minkowski $\mathbf{F} = \mathbf{A}$ for a Lorentzian flat worldsheet metric and $\mathbf{F} = -\mathbf{A}$ for a Riemannian worldsheet metric supplemented by the conditions that $d = e = c = 0$. The reader immediately notices that in this case δ reduces to

$$\frac{g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})}{g(\mathbf{A}, \mathbf{A}}.$$

and that our constraints then give the ususal Virasoro conditions

$$\partial_t \gamma . \partial_s \gamma = 0 = \partial_t \gamma . \partial_t \gamma = \partial_s \gamma . \partial_s \gamma .$$

The standard equations of motion

$$(\partial_t)^2 \gamma - (\partial_x)^2 \gamma = 0$$

are somewhat more limited than ours, but they *imply* that $\delta = 1$ as an easy calculation shows. So, our theory is soewhat more general than the standard one. A simple calculation reveals that

$$\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}) = 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{V})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{K}) + 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{Z})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{L}) - \sum_{i=1}^{n-4} \eta_{ii} (\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{E}_i))^2$$

where $g(\mathbf{K}, \mathbf{V}) = 1 = g(\mathbf{L}, \mathbf{Z})$ and $g(\mathbf{L}, \mathbf{L}) = g(\mathbf{K}, \mathbf{K}) = g(\mathbf{V}, \mathbf{L}) = g(\mathbf{Z}, \mathbf{K}) = 0$. Moreover, η_{ij} is an $n-4$ dimensional Euclidean vielbein in the remaining orthogonal space directions. The reader notices here that we changed the signature of spacetime to $(+ + - - - -)$ giving two time directions and at least four spatial ones. This must be done for the theory to be nontrivial, indeed, string theory is trivial as the constraint equations imply that $\partial_t \gamma$ is proportional to $\partial_s \gamma$ something which cannot be reconsiled with the Heisenberg commutation relations. This is why they cannot impose the full constraints given that they assume the Heisenberg relations to hold. As will be clear in the next section, we don't quantize in that way but follow an analogy with a procedure hich is fully equivalent for point particles to the standard quantization procedure in flat Minkowxki. From our contraints, it follows directly that this formula reduces to

$$\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}) = -\sum_{i=1}^{n-4} \eta_{ii} (\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{E}_i))^2 .$$

In order not to get into conflict with the usual causality conditions, wwe suggest that the extra time and spacce directions are compactified and way beyond our scale of observation. Hence, a fibre structure is needed for the spacetime manifold with a four dimensional Lorentzian base manifold and a two dimensional Lorentzian fibre. We proceed now to the quantum theory.

3 Quantization of the string.

In ordinary particle theory,ell known that th full quantum theory is provided by the Wightman functions as well as the appropriate Lorentz intertwiners govrning the particle interactions. Th main insight from this author [1] was that the Wigthman functions can be given an entirely geometrical and relational meaning without recourse to any foliation of spacetime given by a class of obervers. The observation is simple that free particles travel on godesics and that the corrct Wightman function is constructed by means of dragging on shell momenta on those godesics which correspond to of shell particle lines. Furthermore, the internal degrees of freedom are associated to representations of the little group of the momentum vector which for massive particles equals $SO(3)$ and for massless particles E_2, the Euclidean group in two dimensions at least if the spacetime dimension equals four. To have a similar thing in our theory, the remaining degrees of freedom constist of rotations in the space perpendicular to $\mathbf{Z}, \mathbf{V}$ which would need $7 = 2 + 5$ dimensions in order to recuperate the $SO(3)$ part. This provides one with a richer particle spectrum and suggests that massive particles can travel at the speed of light in case $\mathbf{Z}$ resides exclusively in the fibre. It seems clear that the string veloity, which is null in the ultrahyperbolic sense, needs to have a timelike component in the base manifold for massive particles to arise there. The reson why string theorists find massive particles in their spectrum is that the Virasoro algebra is not satisfied to begin with. Furthermore, mass quantization can only occur when the fibre momenta are quantized which necessitates closed (timelike) curves in the fibre, hence our compactification. Therefore, mass and in particular the mass gap, are dynamical quantities closely related to the microscopic structure of the fibre.

We now proceed to formulate the correct off shell propagation for strings and the proper dragging law for on shell momenta. To that purpose, let $\zeta(t, s)$ where $t \in \mathbb{R}^+$ and $\zeta(0, s) \sim S^1$ be our off shell string, meaning that for $\mathbf{T} = \partial_t \zeta(t, s)$ and $\mathbf{Z} = \partial_s \zeta(t, s)$, the following constraints hold

$$g(\mathbf{T}, \mathbf{Z}) = g(\mathbf{T}, \mathbf{A}) = g(\mathbf{Z}, \mathbf{Z}) = 0, \; g(\mathbf{T}, \mathbf{T}) = \lambda$$

where λ is not ncessarily zero. We know already that the correct evolution law for the $\mathbf{T}$ field is givn by

$$\nabla_{\mathbf{T}} \mathbf{T} = \mathbf{F}(\mathbf{T}, \mathbf{R}, \mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{T}), \mathbf{g}(\nabla_{\mathbf{Z}} \mathbf{T}, \nabla_{\mathbf{Z}} \mathbf{T}))$$

and that all these constraints are preserved under time evolution. Clearly, our on shell momenta $\mathbf{V}$ have to obey

$$\mathbf{g}(\mathbf{V}, \mathbf{A}) = \mathbf{g}(\mathbf{V}, \mathbf{Z}) = \mathbf{g}(\mathbf{V}, \mathbf{V}) = 0$$

and we look now for the appropriate dragging law

$$\nabla_{\mathbf{T}} \mathbf{V} = \mathbf{G}(\mathbf{T}, \mathbf{V}, \mathbf{Z}, \mathbf{A}, \mathbf{K}, \text{invariants})$$

where K is perpendicular to $\mathbf{T}, \mathbf{V}, \mathbf{Z}, \mathbf{A}$ such that those constraints are preserved under $\mathbf{T}$ evolution. As the reader will notice, it is mandatory to impose two extra constraints on the $\mathbf{V}$ field relating it to $\mathbf{T}$. Considering

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{V}) = 2g(\nabla_{\mathbf{T}} \mathbf{V}, \mathbf{V})$$

indicates one should simply drop the $\mathbf{T}$ dependency in $\mathbf{G}$. Likewise,

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{Z}) = g(\mathbf{V}, \nabla_{\mathbf{T}} \mathbf{Z}) = g(\mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}) = \nabla_{\mathbf{Z}} g(\mathbf{V}, \mathbf{T}) - g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T})$$

suggsts two supplementary constraints, that is

$$g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T}) = 0, \; g(\mathbf{V}, \mathbf{T}) = c$$

where c is a constant. Moreover

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{A}) = \delta g(\mathbf{A}, \mathbf{A}) + g(\mathbf{V}, \nabla_{\mathbf{T}} \mathbf{A}) = \delta g(\mathbf{A}, \mathbf{A}) + g(R(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}) + g(\mathbf{V}, \nabla_{\mathbf{Z}} \nabla_{\mathbf{Z}} \mathbf{T})$$

which can be rewritten as

$$\delta g(\mathbf{A}, \mathbf{A}) + g(R(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}) + \nabla_{\mathbf{Z}} g(\mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}) - g(\nabla_{\mathbf{Z}} \mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}).$$

Upon noticing that

$$\nabla_{\mathbf{Z}} g(\mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}) = \nabla_{\mathbf{Z}} \nabla_{\mathbf{Z}} g(\mathbf{V}, \mathbf{T}) - \nabla_{\mathbf{Z}} g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T}) = 0$$

due to our constraints. Hence, our previous formula reduces to

$$\delta = \frac{g(R(\mathbf{T}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}} \mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T})}{g(\mathbf{A}, \mathbf{A})}.$$

So, we have already determinded two of the five component functions of our dragging field $\mathbf{G}$. Remains to investigate the time evolution of our supplementary constraints

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{T}) = g(\nabla_{\mathbf{T}} \mathbf{V}, \mathbf{T})$$

suggests one to drop the $\mathbf{V}$ dependency in $\mathbf{G}$. Finally,

$$0 = \nabla_{\mathbf{T}} g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T}) = g(\nabla_{\mathbf{T}} \nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T}) + g(\nabla_{\mathbf{Z}} \mathbf{V}, \nabla_{\mathbf{T}} \mathbf{T})$$

which can be further rewritten as

$$g(R(\mathbf{T}, \mathbf{Z})\mathbf{V}, \mathbf{T}) + g(\nabla_{\mathbf{Z}} \nabla_{\mathbf{T}} \mathbf{V}, \mathbf{T}) + g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{A}) \frac{g(R(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}} \mathbf{T}, \nabla_{\mathbf{Z}} \mathbf{T})}{g(\mathbf{A}, \mathbf{A})}.$$

We further investigate

$$g(\nabla_{\mathbf{Z}} \nabla_{\mathbf{T}} \mathbf{V}, \mathbf{T}) = \nabla_{\mathbf{Z}} g(\nabla_{\mathbf{T}} \mathbf{V}, \mathbf{T}) - g(\nabla_{\mathbf{T}} \mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}) == -\kappa g(\mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{T})) - \delta g(\mathbf{A}, \nabla_{\mathbf{Z}} \mathbf{T}) - \gamma g(\mathbf{K}, \nabla_{\mathbf{Z}} \mathbf{T})$$

where we know already δ but κ, γ have not been fixed yet. Further computation yields that

$$g(\mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{T})) = g(\mathbf{Z}, \nabla_{\mathbf{T}} \mathbf{Z})) = \frac{1}{2} \nabla_{\mathbf{T}} g(\mathbf{Z}, \mathbf{Z})) = 0$$

so we have nothing to say about κ and therefore we put it to zero. So, we arrive at the equation

$$0 = g(R(\mathbf{T}, \mathbf{Z})\mathbf{V}, \mathbf{T}) g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{A}) \frac{g(R(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}} \mathbf{T}, \nabla_{\mathbf{Z}} \mathbf{T})}{g(\mathbf{A}, \mathbf{A})} -$$

$$\frac{g(R(\mathbf{T}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}} \mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T})}{g(\mathbf{A}, \mathbf{A})} g(\mathbf{A}, \nabla_{\mathbf{Z}} \mathbf{T}) - \gamma g(\mathbf{K}, \nabla_{\mathbf{Z}} \mathbf{T}).$$

This leaves only for the possibility that $g(\mathbf{K}, \nabla_{\mathbf{Z}} \mathbf{T})$ is nonzero otherwise our theory would become inconsistent. Denoting by

$$(\nabla_{\mathbf{Z}} \mathbf{T})^{\perp}$$

the component of $\nabla_{\mathbf{Z}} \mathbf{T}$ perpendicular to $\mathbf{T}, \mathbf{V}, \mathbf{Z}, \mathbf{A}$ which is usually only determined, in case $g(\mathbf{T}, \mathbf{V}) \neq 0$ up to a multiple of $\mathbf{Z}$, we conclude that we need to add a force term

$$\frac{(\nabla_{\mathbf{Z}} \mathbf{T})^{\perp}}{g((\nabla_{\mathbf{Z}} \mathbf{T})^{\perp}, (\nabla_{\mathbf{Z}} \mathbf{T})^{\perp})} \left(g(R(\mathbf{T}, \mathbf{Z})\mathbf{V}, \mathbf{T}) + g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{A}) \frac{g(R(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}) + g(\nabla_{\mathbf{Z}} \mathbf{T}, \nabla_{\mathbf{Z}} \mathbf{T})}{g(\mathbf{A}, \mathbf{A})} \right)$$

$$-\frac{(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}}{g((\nabla_{\mathbf{Z}}\mathbf{T})^{\perp},(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp})}\left(\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{Z})+g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})}g(\mathbf{A},\nabla_{\mathbf{Z}}\mathbf{T})\right)$$

to our formula for $\mathbf{G}$ which fully dtermines it up to an ambiguity in the definition of $(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}$. This is pretty bad given that this ambiguity propagates in a nontrivial way and our theory would depend upon some convention we have to take at any point of the string. Notice also that ideas where you decouple the bulk from the fiber won't help you; even if initially $\mathbf{Z}$ is in the fiber and $\mathbf{T},\mathbf{V}$ in the bulk, then $\mathbf{A}$ will usually have some nontrivial component in the fiber so that $\mathbf{T},\mathbf{V}$ propagate into the latter and therefore $\mathbf{Z}$ into the bulk. So, $\nabla_{\mathbf{Z}}\mathbf{T}$ does not remain into the bulk and therefore you cannot define an orthognal complement with regards tp $\mathbf{Z}$ pertaining to the bulk alone. So, from this point of view, standard string theory does not make any sense. However, it is easy to save the day by allowing a liberty which we had before and that is to put $g(\mathbf{Z},\mathbf{Z})$ *not* equal to zero; in general, even if $g(\mathbf{V},\mathbf{V})=0=g(\mathbf{V},\mathbf{Z})=g(\mathbf{T},\mathbf{Z})$ the projection above will be uniquely defined as long as $g(\mathbf{V},\mathbf{T})\neq 0$. One can even exclude this exceptional case by imposing $g(\mathbf{V},\mathbf{Z})=g(\mathbf{T},\mathbf{Z})\neq 0$ allowing for $g(\mathbf{Z},\mathbf{Z})$ to be zero and in this case $g(\mathbf{V},\mathbf{T})$ can be anything you like. We shall work from now on with the latter convention, that is $g(\mathbf{V},\mathbf{Z})=g(\mathbf{T},\mathbf{Z})=d\neq 0$ and $g(\mathbf{V},\mathbf{V})=0$.

4 Fourier transform for strings in covariant quantum theory.

In analogy with standard particle physics, we now proceed to construct the Fourier transorm $\phi(S,\mathbf{V}_S,S')$, where S is a null string parametrized as before and $\mathbf{V}_S$ is a null vectorfield defined on the string and satisfying the previous constraints. It is clear that the definition of $\phi(S,\mathbf{V}_S,S')$ should not depend upon the reparametrization freedom hidden in $\mathbf{Z}$ and we define the string length L as the range of this parameter domain. In analogy with particle physics, we start out with the most naive ansatz for a differential equation for $\phi(S,\mathbf{V}_S,t)$; the latter being given by

$$\frac{d}{dt}\phi(S,\mathbf{V}_S,t)=i\frac{\kappa}{L}\left(\int_0^L ds\,\mathbf{g}(\mathbf{V}(t,s),\mathbf{T}(t,s))\right)\phi(S,\mathbf{V}_S;t)$$

with $\phi(S,\mathbf{V}_S,0)=1$ and $\phi(S,\mathbf{V}_S,1)=\phi(S,\mathbf{V}_S,S')$. Here, κ is a dimensionless constant and L is the string length. As is the case in standard particle theory, this factor is a constant in time and simply given by κc where $c=\mathbf{g}(\mathbf{V}_S(t,s),\mathbf{T}(t,s))$. Hence,

$$\phi(S,\mathbf{V}_S,S')=e^{i\kappa c}$$

just as in ordinary particle physics, which we know is already a complete disaster there and leads to distributional propagators. Here, we are going to perform a functional integral over vectorfields instead of vectors and the volume associated with a constant ca is just infinite. There is no other term depending on $\mathbf{V},\mathbf{T}$ one could add since we have the constraint that $g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{T})=0$. Thereforee we will employ the spatial variation of $\mathbf{V}$ contracted with itself $g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V})$. To state it properly, we should add a term

$$\gamma L \int_0^L g(\nabla_{\mathbf{Z}} \mathbf{V}(t,s), \nabla_{\mathbf{Z}} \mathbf{V}(t,s)) ds$$

which is not time independent. This author has proposd similar avnues in standard particle physics. It might be that other terms are required to make a sensible theory but these issues are postponed for further investigation. We now come to the definition of the propagator.

5 The free string propagator.

The particular feature about the string propagator is that it involves an infinite dimensional integration over momentum space $\mathbf{V}_S$ and we limit in the subsequent analysis ourselves to a product manifold $\mathcal{M} \times \mathcal{N}$ where $\mathcal{M}$ is a $3+1$ dimensional Lorentzian base manifold endowed with a $2+1$ dimensional Lorentzian fibre which decouple in the sense that the metric does not mix directions in the fibre and base manifold. The $\mathbf{T}$ field is as such that after parameter time one the string S specified above moves into a string S' with nontrivial projection into $\mathcal{M}$ due to s variations of the $\mathbf{T}$ field; that is, the projection of $\partial_s \mathbf{T}$ on the base is different from zero. Hence we propose,

$$D(S, S') = \int dc \int_{\mathbf{V}_S} d\mu(\mathbf{V}_S) \delta(g(\mathbf{V}_S, \mathbf{V}_S))$$

$$\delta(g(\mathbf{V}_S, \mathbf{Z}) - d) \delta(g(\mathbf{V}_S, \mathbf{T}) - c) \delta(g(\nabla_{\mathbf{Z}} \mathbf{V}_S, \mathbf{Z}) \delta(g(\mathbf{V}_S, \mathbf{A})) \theta_{\mathcal{M}}(\mathbf{V}_S) \phi(S, \mathbf{V}_S, S')$$

where we have chosen a time direction in the base manifold on $\mathcal{M}$ and $\theta_{\mathcal{M}}(\mathbf{V}_S)$ concerns positivity of the projection of $\mathbf{V}_S$ on that time field. The problem here regards the usual definition of the path integral as a limiting measure and much care is required to give this expression a precise meaning. Let us mention that in contrast to string theory on flat spacetime, mass and momentum string states are not easily defined and the angular momentum and spin case are even much more difficult as is the case for ordinary particles on a curved background. We leave further investigations of these ideas to the future.

References

[1] J. Noldus, Generally covariant quantum theory, Vixra 1610.0165

[2] M.B. Green, J.H. Schwarz, E. Witten, Superstring theory Volume I, Cambridge University Press